

Key Extra Practice C.

Extra Practice

Verifying Trig Identities

Section 5.1

Name: _____

Date: _____

$$1. (\cot^2 x + 1)(\sin^2 x - 1) = -\cot^2 x$$

$$(\csc^2 x - 1 + 1)(1 - \cos^2 x - 1) =$$

$$\csc^2 x \cdot -\cos^2 x =$$

$$\frac{1}{\sin^2 x} \cdot \frac{-\cos^2 x}{1} =$$

$$-\cot^2 x =$$

$$2. (\sec x - \tan x)^2 = \frac{1 - \sin x}{1 + \sin x}$$

$$\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 =$$

$$\frac{(1 - \sin x)^2}{\cos^2 x} =$$

$$\frac{(1 - \sin x)(1 - \sin x)}{1 - \sin^2 x} = \frac{(1 - \sin x)(1 - \sin x)}{(1 + \sin x)(1 - \sin x)}$$

$$\frac{1 - \sin x}{1 + \sin x} =$$

$$3. \frac{\sin x}{1 - \cos x} - \frac{\sin x \cos x}{1 + \cos x} = \csc x (1 + \cos^2 x)$$

$$\frac{\sin x(1 + \cos x) - \sin x \cos x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)} =$$

$$\frac{\sin x + \sin x \cos x - \sin x \cos x + \sin x \cos^2 x}{1 - \cos^2 x} =$$

$$\frac{\sin x(1 + \cos^2 x)}{\sin^2 x} = \frac{1}{\sin x} + \frac{\cos^2 x}{\sin x} =$$

$$\csc x (1 + \cos^2 x) =$$

$$4. \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1}$$

$$= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x(1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} (\sin^2 x)$$

$$= \tan^2 x \sin^2 x$$

$$5. \frac{\csc x}{1 + \csc x} - \frac{\csc x}{1 - \csc x} = 2 \sec^2 x$$

$$\frac{\csc x(1 - \csc x) - \csc x(1 + \csc x)}{(1 + \csc x)(1 - \csc x)} =$$

$$\frac{\csc x - \csc^2 x - \csc x - \csc^2 x}{(1 - \csc^2 x)} =$$

$$\frac{-2 \csc^2 x}{\cot^2 x} = \frac{-2}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} =$$

$$\frac{-2}{\cos^2 x} = -2 \sec^2 x =$$

$$6. \frac{\sin x \cos x}{\cos^2 x - \sin^2 x} = \frac{\tan x}{1 - \tan^2 x}$$

$$= \frac{\frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\sin x}{\cos x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{\cos^2 x - \sin^2 x}$$

$$= \frac{\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$7. \frac{\sec x \sin x}{\tan x + \cot x} = \sin^2 x$$

$$\frac{\frac{1}{\cos x} \cdot \frac{\sin x}{1}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} =$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\sin x}{\cos x} \cdot \frac{\cos x \sin x}{1} = \sin^2 x =$$

$$9. \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$$

$$\frac{(1 + \sin x)(1 + \sin x) - (1 - \sin x)(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} =$$

$$\frac{1 + 2\sin x + \sin^2 x - (1 - 2\sin x + \sin^2 x)}{1 - \sin^2 x} =$$

$$\frac{4\sin x}{\cos^2 x} =$$

$$\frac{4\sin x}{\cos x} \cdot \frac{1}{\cos x} = 4 \tan x \sec x =$$

$$11. \frac{\cos x + 1}{\cos x - 1} = \frac{1 + \sec x}{1 - \sec x}$$

$$= \frac{1 + \frac{1}{\cos x}}{1 - \frac{1}{\cos x}}$$

$$= \frac{\frac{\cos x + 1}{\cos x}}{\frac{\cos x - 1}{\cos x}}$$

$$= \frac{\cos x + 1}{\cos x - 1}$$

$$8. \frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$\frac{\tan x + \tan y}{\frac{1}{\tan x} + \frac{1}{\tan y}} = \frac{\tan x + \tan y}{\frac{\tan x + \tan y}{\tan x \tan y}} =$$

$$\frac{\tan x + \tan y}{1} \cdot \frac{\tan x \tan y}{\tan x + \tan y} =$$

$$\tan x \tan y =$$

$$10. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\cos x + 1}{\sin x (\sin^2 x)} =$$

$$\frac{\cos x + 1}{\sin x (1 - \cos^2 x)} =$$

$$\frac{(\cos x + 1)}{\sin x (1 + \cos x)(1 - \cos x)} = \frac{1}{\sin x} \cdot \frac{1}{1 - \cos x}$$

$$\frac{\csc x}{1 - \cos x}$$

$$12. \frac{(\sec x - \tan x)^2 + 1}{\csc x (\sec x - \tan x)} = 2 \tan x$$

$$\frac{\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 + 1}{\frac{1}{\sin x} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)} = \frac{\left(\frac{1 - \sin x}{\cos x}\right)^2 + 1}{\frac{1}{\sin x} \left(\frac{1 - \sin x}{\cos x}\right)}$$

$$\frac{1 - 2\sin x + \sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}$$

$$\frac{1 - 2\sin x + 1}{\cos^2 x} = \frac{2(1 - \sin x)}{\cos^2 x} \cdot \frac{\sin x \cos x}{(1 - \sin x)}$$

$$\frac{1 - \sin x}{\sin x \cos x} = \frac{2 \sin x}{\cos x} = 2 \tan x$$