Key

Unit 6 Summative Review Graphing & Modeling with Trig Fns v3

CP Pre-Calculus

No Calculator

Name: _____

Identify the indicated information. Then, graph 2 periods of each function.

1) $y = -2\cos\left(3x - \frac{\pi}{2}\right) + 5$

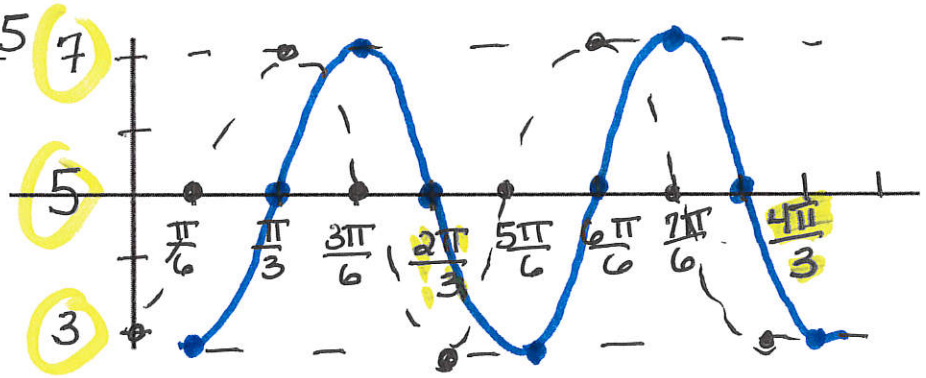
IN factored form: $y = -2\cos\left[3\left(x - \frac{\pi}{6}\right)\right] + 5$

Amplitude 2 Period $\frac{2\pi}{B} = \frac{2\pi}{3}$

Intervals: $\frac{2\pi}{3} \div 4 = \frac{\pi}{6}$

P.S. $\frac{\pi}{6}$ V.S. 5

range: [3, 7]



2) $y = -3\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$

In factored form: $y = -3\sin\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + 1$

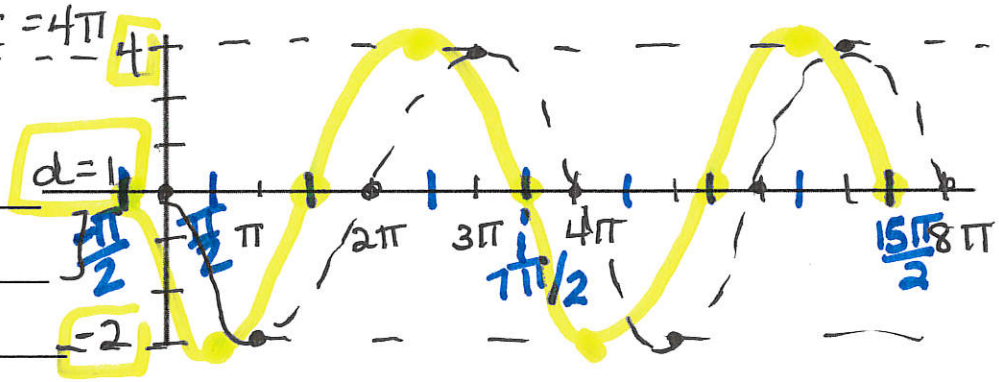
Amplitude 3 Period $2\pi \div \frac{1}{2} = 4\pi$

Intervals: π

P.S. π/2 V.S. 1

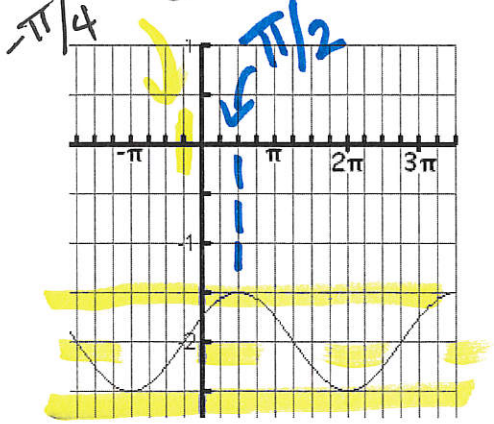
domain: $(-\infty, \infty) \Rightarrow$ [

range: [-2, 4]



Write a sine and cosine equation for each graph.

3) Sine: $y = \frac{1}{2}\cos\left[\frac{2}{3}\left(x + \frac{\pi}{4}\right)\right] + -2$ Cosine: $y = \frac{1}{2}\cos\left[\frac{2}{3}\left(x - \frac{\pi}{2}\right)\right] + -2$



$d = 2$

$|a| = \frac{1}{2}$

pos cos →

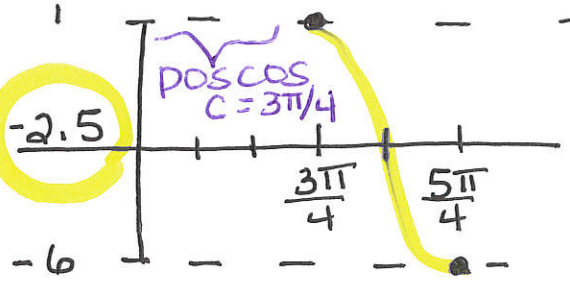
$c = \pi/2$

$per = \frac{3\pi}{1} = \frac{2\pi}{B}$

$3B = 2\pi$

$B = 2/3$

4) A. Sketch the graph and write a sine and a cosine equation with a maximum at $\left(\frac{3\pi}{4}, 1\right)$ and a minimum at $\left(\frac{5\pi}{4}, -6\right)$.



$d = \frac{\max + \min}{2}$

$d = \frac{1 + -6}{2}$

$= \frac{-5}{2} = -2.5 = d$

$\frac{1}{2}$ per $= \frac{2\pi}{4}$

Per $= \pi = \frac{2\pi}{B}$

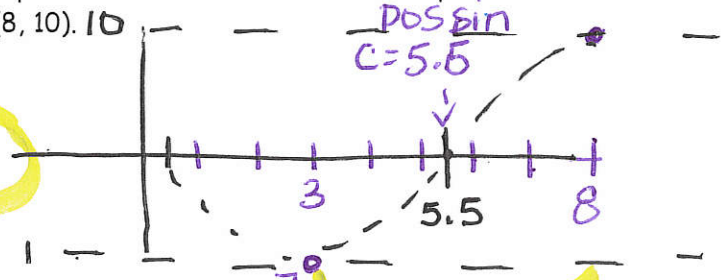
$B = 2$

$|a| = \max - \text{mid} = 1 - -2.5 = |3.5| = |a|$

$y = 3.5\cos\left[2\left(x - \frac{3\pi}{4}\right)\right] - 2.5$

$y = 3.5\sin\left[2\left(x - \frac{\pi}{2}\right)\right] - 2.5$

4) B. Sketch the graph and write a sine and a cosine equation for the sinusoidal function with a min at (3, 1) and a max at (8, 10).



$d = \frac{10+1}{2} = 5.5$

$|a| = 4.5$

neg cos

$c = 3$

$\frac{1}{2}$ per $= 5$

1 per $= 10 = \frac{2\pi}{B}$

$10B = 2\pi$

$B = \frac{2\pi}{10} = \frac{\pi}{5}$

$y = -4.5\cos\left[\frac{\pi}{5}(x - 3)\right] + 5.5$

$y = 4.5\sin\left[\frac{\pi}{5}(x - 5.5)\right] + 5.5$

$y = -4.5\sin\left[\frac{\pi}{5}(x - .5)\right] + 5.5$

4C). Given that the function h(t) = -50cos[(pi/2)(x)] + 54 represents the height, in feet, of a person riding a Ferris wheel at t seconds, determine the following:

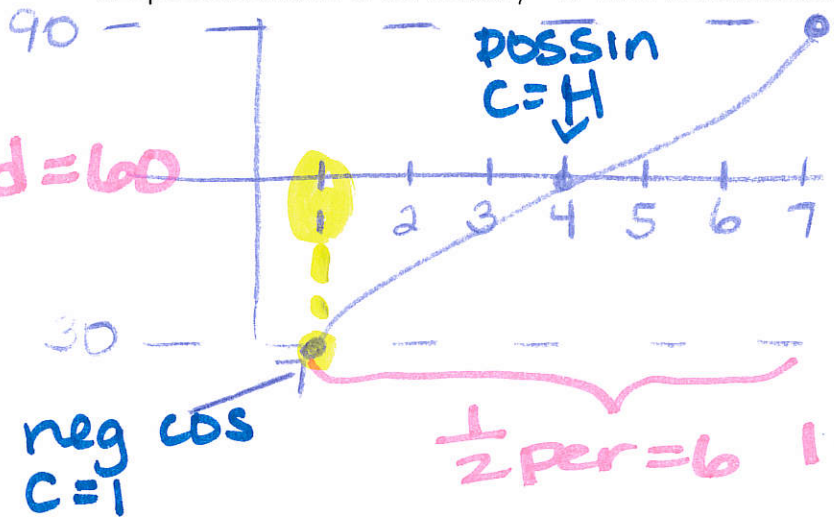
A. Diameter of the wheel = 100 ft radius = amplitude

B. Height above the ground at which the person gets on the Ferris wheel 4 feet

C. Time it takes to go around once: period $= \frac{2\pi}{B} = 2\pi \div \frac{\pi}{2} = 4$ seconds

D. The highest height the wheel will reach: $54 + 50 = 104$ feet.

5) Seasonal temperatures in Barcelona are cyclical. The lowest average temperature in January in Barcelona, Spain was 30 degrees. The highest average temperature for the year was 90 degrees in July. Calculate the amplitude, phase shift, vertical shift, and period for the past year, given that temperatures are periodic in nature. Let January = 1. Then write a sinusoidal equation to represent the situation.



pos sin
C=H

|a| = 30

$y = -30\cos\left[\frac{\pi}{6}(x-1)\right]+60$

$y = 30\sin\left[\frac{\pi}{6}(x-4)\right]+60$

neg cos
C=1

$\frac{1}{2}$ per = 6 | per = 12 $= \frac{2\pi}{B}$

$12B = 2\pi$

$B = \pi/6$

Use your model to predict the average temperature in March.

March → t = 3

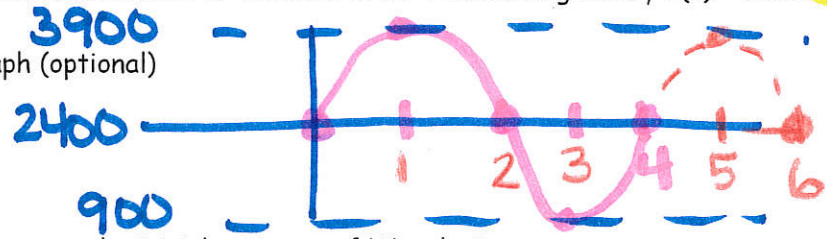
$y = -30\cos\left[\frac{\pi}{6}(3-1)\right]+60 = -30\left(\frac{1}{2}\right)+60$

$-15+60 = 45°$

$\pi/3$

6) The population estimate of Wuzzles after t weeks is given by P(t) = 1500sin((pi/2)t) + 2400:

a) Graph (optional)



per = $2\pi \div \frac{\pi}{2}$

= 4

b) What was the initial estimate of Wuzzles?

t = 0 2400

c) Between what numbers does the Wuzzle population vary?

900 and 3900

d) How many Wuzzles would you expect there to be 6 weeks into the population study?

Look at graph or sub in 6 for t

2400 or $P(6) = 1500\left(\sin\frac{\pi}{2}\cdot 6\right)+2400$

e) How many Wuzzles would you expect there to be 4 weeks into the study?

2400

$= 1500\sin 3\pi + 2400$

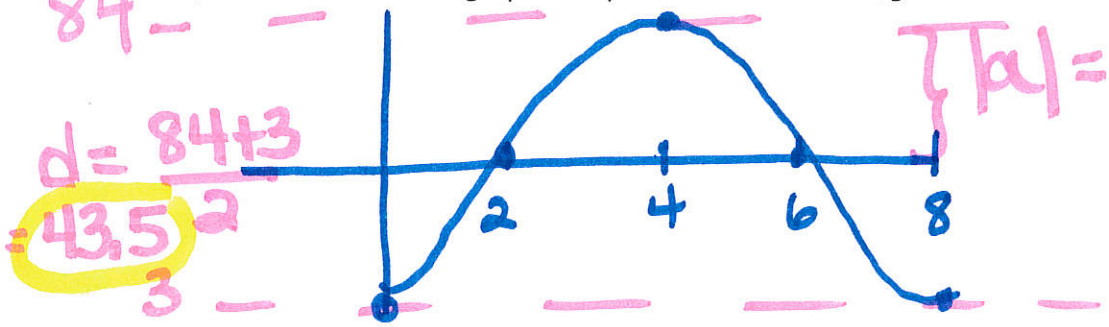
$= 1500(0) + 2400 = 2400$

No Calculator

7) Riders board the Ferris wheel at its lowest height of 3 feet off the ground. The amusement park advertises that riders will reach a height of 84 feet, and that they will complete 30 full rotations on the wheel during the 4 minute ride.

$\frac{30 rev}{240 sec} = \frac{1 rev}{8 sec}$

a) Sketch and label a graph to represent this situation, given t = time in seconds.



$d = \frac{84+3}{2} = 43.5$

$|a| = 40.5$

$Per = \frac{8}{1} = \frac{2\pi}{B}$

$8B = 2\pi$

$B = \frac{\pi}{4}$

b) Write a cosine model to represent this situation

$y = -40.5 \cos\left[\frac{\pi}{4}(x)\right] + 43.5$

c) Write a corresponding sine model to represent this situation.

$y = +40.5 \sin\left[\frac{\pi}{4}(x-2)\right] + 43.5$

d) How many seconds after the ride starts will a rider first reach a height of 40 feet?

$y_2 = 40$ feet Calc int.

1.89 seconds

e) Your friends want to take a picture of you on the Ferris wheel. They have calculated that they could get the best shot when you are 12 feet off the ground. When should they take the picture?

$y_2 = 12$ calc int.

.87 seconds into the ride or 7.13 seconds into the ride.

f) What is the diameter of the wheel?

$|a| = r$

$2r = d$

81 feet

No Calculators

9) $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\sin \frac{\pi}{3}$

$\sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

10) $\sin\left(\arctan(-1)\right)$

$\sin \frac{-\pi}{4}$

$-\frac{\sqrt{2}}{2}$

11) $\cos\left(\frac{5\pi}{3} - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \cos\left(\frac{5\pi}{3} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{10\pi}{6} - \frac{\pi}{6}\right)$

$= \cos\left(\frac{9\pi}{6}\right) = \cos\frac{3\pi}{2} = 0$

12) $\cos\left(\cos^{-1}1 + \sin^{-1}\left(-\frac{1}{2}\right)\right)$

$= \cos\left(0 + \frac{-\pi}{3}\right)$

$= \cos\frac{-\pi}{3}$

$= \frac{1}{2}$

13) $\tan\left(\frac{11\pi}{4} + \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$= \tan(-1)$

$= \tan\left(\frac{11\pi}{4} + \frac{-\pi}{4}\right)$

$= \tan(10\pi/4)$

$= \tan(5\pi/2) = \frac{y}{x}$

undefined

14) $\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{2}\right)$

$= \sin\left(\frac{\pi}{6} + \frac{\pi}{2}\right)$

$= \sin\frac{4\pi}{6}$

$= \sin\frac{2\pi}{3}$

$= \sqrt{3}/2$

15) $\cos\left(2\tan^{-1}\sqrt{3}\right)$

$\cos(2\pi/3)$

$-\frac{1}{2}$

16) $\cos\left(\arctan\left(\frac{-\sqrt{3}}{3}\right) - \arcsin\left(\frac{-\sqrt{3}}{2}\right)\right)$

$= \cos\left(\frac{-\pi}{6} - \frac{-\pi}{3}\right)$

$= \cos\left(\frac{-\pi}{6} + \frac{2\pi}{6}\right)$

$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

17) $\sin\left(\tan^{-1}\sqrt{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

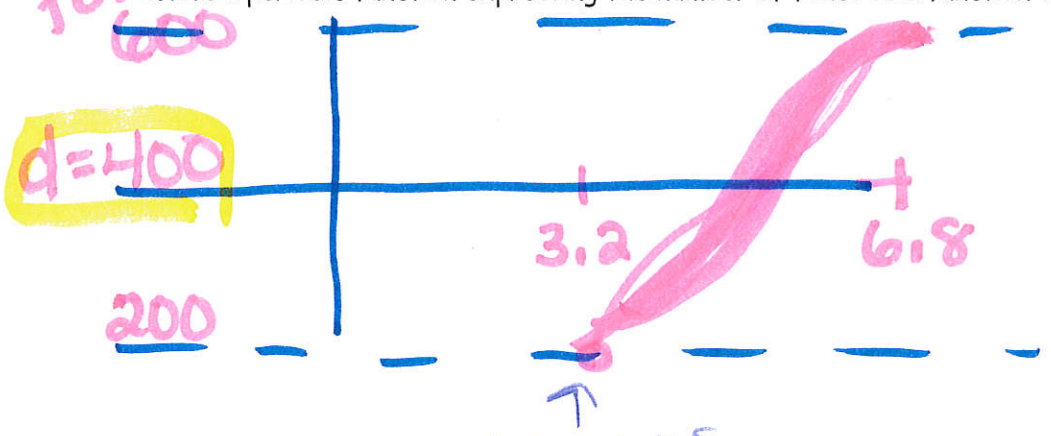
$= \sin\left(\frac{\pi}{3} - \frac{-\pi}{3}\right)$

$= \sin\frac{2\pi}{3}$

$= \frac{\sqrt{3}}{2}$

8. Naturalists find that the populatios of some kinds of predatory animals vary periodically. Such is the case with the fox and the chipmunks._ The collection of data shows that the mínimum number of 200 foxes existed 3.2 years after data began being kept, while the subsequent maximim of 600 foxes occurred 6.8 years after the beginning of data collection.

Write a periodic function expressing the number of foxes as a function of time.



$|a| = 200$

$\frac{1}{2}$ per $= 6.8 - 3.2$

$\frac{1}{2}$ per $= 3.6$

per $= 7.2$.

$\frac{7.2}{1} = \frac{2\pi}{B}$

$7.2B = 2\pi$

$B = \frac{2\pi}{7.2}$

$B = \frac{20\pi}{72}$

$B = \frac{10\pi}{36}$

$B = \frac{5\pi}{18}$

$$y = -200\cos\left[\frac{5\pi}{18}(x-3.2)\right] + 400$$

b. Predict the fox population 8 years after records began being kept.

$t = 8$

$x = 8$

$$y = -200\cos\left[\frac{5\pi}{18}(8-3.2)\right] + 400$$

$$y = -200\cos\left[\left(\frac{5\pi}{18}\right)(4.8)\right] + 400$$

$y = -200\cos\left(\frac{24\pi}{18}\right) + 400$

Use calc.

$y = 500$

$y = -200\cos\frac{4\pi}{3} + 400$

$y = -200\left(-\frac{1}{2}\right) + 400 = 500$