Test Review #1 Rationals and Radicals Name Key

1

1. Identify any holes, vertical asymptotes, horizontal asymptotes, and slant asymptotes of each graph. If they do not exist write **none** in the space provided. Accurately graph each rational function.

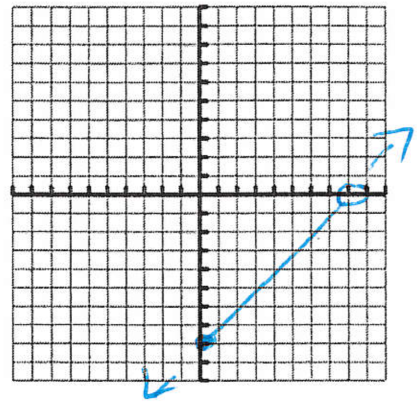
a. $f(x)=\dfrac{x^2-16x+64}{x-8}$ $x-8$

Domain $x \neq 8 \Rightarrow (-\infty, 8) \cup (8, \infty)$

Hole(s) $(8, 0)$ Vertical Asymptotes —

X-intercepts $(8, 0)$ Horizontal Asymptotes —

Y-intercepts $(0, -8)$ Slant Asymptotes —



b. $f(x)=\dfrac{2x^2-5x-12}{x^2+x-6}$ $\dfrac{(2x+3)(x-4)}{(x+3)(x-2)}$

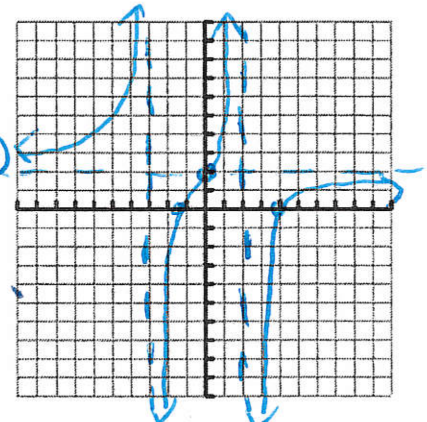
Domain $x \neq -3,\ x \neq 2$ $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

Hole(s) —   Vertical Asymptotes $x=-3$ $x=2$

X-intercepts $(4, 0)$ $(-3/2, 0)$ Horizontal Asymptotes $y=2$

set $y=0$

Y-intercepts $(0, 2)$ Slant Asymptotes —



Check $x=1$

$y=\dfrac{5(-3)}{4(-1)}=\dfrac{-15}{-4}=$ pos

c. $f(x)=\dfrac{3x^2-12x+15}{3x-6}$

$y=\dfrac{3(x^2-4x+5)}{3(x-2)}=\dfrac{3(x-5)(x+1)}{3(x-2)}$

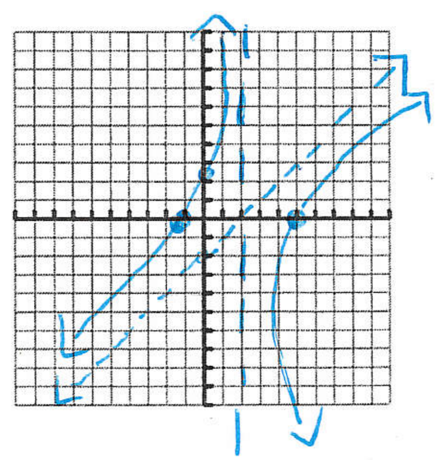
$$\begin{array}{r} x-2 \\ x-2\,\overline{)\,x^2-4x+5} \\ \underline{x^2-2x} \\ -2x+5 \end{array}$$

Domain $x \neq 2$

Hole(s) none   Vertical Asymptotes $x=2$

X-intercepts $(5,0)$ $(-1,0)$ Horizontal Asymptotes NO

Y-intercepts $(0, \frac{5}{2})$ Slant Asymptotes $y=x-2$



(2)

d. $f(x)=\frac{x^2-x-6}{x-2}$ $\frac{(x-3)(x+2)}{(x-2)}$

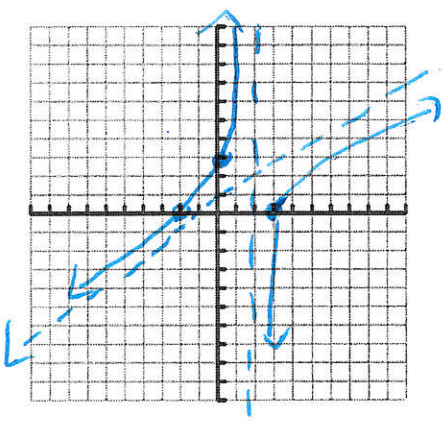
Domain $x \neq 2$

Hole(s) — Vertical Asymptotes $x=2$

X-intercepts (3,0)(-2,0) Horizontal Asymptotes NO

Y-intercepts (0, 3) Slant Asymptotes $y=x+1$

$$\begin{array}{r} x+1 \\ x-2\overline{)x^2-x-6} \\ x^2-2x \\ \hline x-6 \\ x-2 \\ \hline \end{array}$$



2. Simplify each of the following. Write the restrictions on the domain.

a. $\frac{x^2-5x+6}{x+4} \bullet \frac{3x+12}{x-2}$

$\frac{(x-3)(x-2)}{(x+4)} \cdot \frac{3(x+4)}{(x-2)}$

$= 3x-9$

$x \neq -4, 2$

a2. $\frac{36-x^2}{2x^3-2x^2-60x} \bullet \frac{3x^3+15x^2}{6x^3}$

$\frac{-1(x+6)(x-6)}{2x(x^2-x-30)} \cdot \frac{3x^2(x+5)}{2 \cdot 6x^3}$

$2x(x-6)(x+5)$

$= -\frac{(x+6)}{4x^2}$

$= \frac{-x-6}{4x^2}$

$x \neq 0, 6, -5$

3

Simplify. Recall, division can be rewritten as multiplication by the reciprocal

b. $\dfrac{\dfrac{x^2+10x-11}{x^2+6x+5}}{\dfrac{x^2+9x-22}{x^2+3x-10}} = \dfrac{(x+11)(x-1)}{(x+5)(x+1)} \div \dfrac{(x+11)(x-2)}{(x+5)(x-2)}$

$= \dfrac{(x+11)(x-1)}{(x+5)(x+1)} \cdot \dfrac{(x+5)}{(x+11)}$

$= \dfrac{x-1}{x+1}$

c. $\dfrac{\dfrac{2}{x}+1}{\dfrac{1}{x+2}-\dfrac{1}{2}} = \dfrac{\dfrac{2+x}{x}}{\dfrac{2-x-2}{2(x+2)}}$

$= \dfrac{x+2}{x} \cdot \dfrac{2(x+2)}{-x}$

$= \dfrac{2(x+2)^2}{-x^2}$

RECALL, TO ADD OR SUBTRACT FRACTIONS YOU MUST HAVE A common den

Simplify:

c. $\left(\dfrac{x-5}{x-5}\right) \dfrac{x+5}{x^2+10x+25} - \dfrac{2x}{x^2-25} \left(\dfrac{x+5}{x+5}\right)$

$(x+5)^2 \quad (x+5)(x-5)$

$= \dfrac{x^2-25-2x^2-10x}{\text{den}}$

$= \dfrac{-1x^2-10x-25}{\text{den}}$

$= \dfrac{-1(x+5)(x+5)}{(x+5)(x+5)(x-5)} = \boxed{\dfrac{-1}{x-5}}$

d. $\dfrac{2x-3}{3x^2-13x-10} - \dfrac{2x+1}{5-x} + \dfrac{1}{3x+2}$

$(3x+2)(x-5) \quad 5-x \to -1(x-5)$

$\dfrac{2x-3-(2x+1)(3x+2)+(x-5)}{\text{den}}$

$= \dfrac{2x-3-[6x^2+7x+2]+x-5}{\text{den}}$

$= \dfrac{2x-3-6x^2-7x-2+x-5}{\text{den}}$

$= \dfrac{-6x^2-4x-10}{\text{den}}$

$= \dfrac{-2(3x^2+2x+5)}{(x-5)(3x+2)}$

D2. $\dfrac{3-x}{x^2-6x+9} - \dfrac{x+5}{3-x}$

$-1(x-3)$

$= \dfrac{-x+3}{(x-3)(x-3)} + \dfrac{x+5}{x-3}\left(\dfrac{x-3}{x-3}\right)$

$= \dfrac{-x+3+x^2+2x-15}{\text{den}}$

$= \dfrac{x^2+x-12}{(x-3)(x-3)} = \dfrac{(x+4)(x-3)}{(x-3)(x-3)}$

4

RECALL, WHEN SOLVING EQUATIONS< OUR GOAL IS TO ELIMINATE THE FRACTIONS!
WE DO THIS by multiplying both sides of the equation (numerators only), by the ___________!
This caused the denominators to cancel!

3. Solve each equation.

a. $\frac{1}{9}+\frac{1}{2x}=\frac{1}{x^2}$ $x \neq 0$

LCD = $18x^2$

$2x^2 + 9x = 18$

$2x^2 + 9x - 18 = 0$

$(2x - 3)(x + 6) = 0$

$x = 3/2 \quad x = -6$

b. $\frac{1}{2y+1}+\frac{1}{y+1}=\frac{8}{15}$ $y \neq -1$, $y \neq -1/2$

LCD = $15(2y+1)(y+1)$

$15y + 15 + 30y + 15 = 8(y+1)(2y+1)$

$45y + 30 = 8(2y^2 + 3y + 1)$

$45y + 30 = 16y^2 + 24y + 8$

$0 = 16y^2 - 21y - 22$

$0 = (y-2)(16y+11)$

$y = 2 \quad y = \frac{-11}{16}$

c. $\frac{6}{y^2+2y}-\frac{y+1}{y+2}=\frac{2}{y}$

LCD = $y(y+2)$

$6 - (y+1)\cdot y = 2(y+2)$

$6 - y^2 - y = 2y + 4$

$0 = y^2 + 3y - 2$

$0 = (y-2)(\,?\,)$ DNF

QF $x = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

$x \neq 0$, $x \neq -2$

d. $\frac{2}{x-3}-\frac{4}{x+3}=\frac{8}{x^2-9}$

$2(x+3) - 4(x-3) = 8$

$2x + 6 - 4x + 12 = 8$

$-2x + 18 = 8$

$-2x = -10$ $x \neq 3, -3$

$x = 5$

4. Solve each inequality. Recall, you must set the inequality equal to ___0___ and turn the other side into ONE quotient in order to solve rational inequalities. !

a. $\frac{6}{x}+3>\frac{2}{x}$

$\frac{6}{x} + 3 - \frac{2}{x} > 0$

$\frac{4}{x} + \frac{3x}{x} > 0$

$\frac{3x+4}{x} > 0$

den $\neq 0$ $x \neq 0$ - open

num = 0 $3x + 4 = 0$

$x = -4/3$ - open

+ + + (−4/3) − − − − (0) + + + → $(-\infty, -4/3) \cup (0, \infty)$

b. $\frac{2x+1}{3x+1}<\frac{x-1}{3x+1}$

$\frac{2x+1-(x-1)}{3x+1} < 0 \rightarrow \frac{x+2}{3x+1} < 0$

$x \neq -1/3$ open

$x = -2$ open

+ (−2) − (−1/3) +

$(-2, -1/3)$

5

c. $\frac{1}{4x}+\frac{5}{8x}\geq\frac{1}{2}$

$\frac{2}{8x}+\frac{5}{8x}-\frac{4x}{8x}\geq 0$

$\frac{7-4x}{8x}\geq 0$

$x=0$ open

$x=\frac{7}{4}$ closed

$\ominus \quad \oplus \quad \ominus$

$0 \quad 7/4$

$(0, \frac{7}{4}]$

d. $\frac{x+3}{x-5}\leq 5$

$\frac{x+3}{x-5}-\frac{5(x-5)}{(x-5)}\leq 0$

$\frac{-4x+28}{x-5}\leq 0$

$x=5$ open

$x=7$ closed

$\ominus \quad \oplus \quad \ominus$

$5 \quad 7$

$\cancel{(5,7]}$

$(-\infty,5)\cup[7,\infty)$

5\. A. You have created a new type of jelly bean that you would like to market to Harry Potter World. They have requested that the packages contain 6.25 cubic centimeters of jelly beans, and that they be packaged in cylindrical containers. You, of course, want to minimize your packaging costs! What dimensions should you use for the container?

Constraint: $V=\pi r^2 h=6.25$ $\quad h=\frac{6.25}{\pi r^2}$

Function to be optimized: $SA=2\pi r^2+2\pi r h$

Real world domain: $(0,2)$

Work:

$SA=2\pi r^2+2\pi r\cdot\frac{6.25}{\pi r^2}$

$S.A.=2\pi r^2+\frac{12.5}{r}$

$r=x \quad =.998 \quad \therefore \boxed{1\text{ cm}}=r$ $\quad$ so $h=\frac{6.25}{\pi(1)^2}=\boxed{2\text{ cm}}$

$y=S.A.$

$r=1\text{cm}$

$h=2\text{cm}$

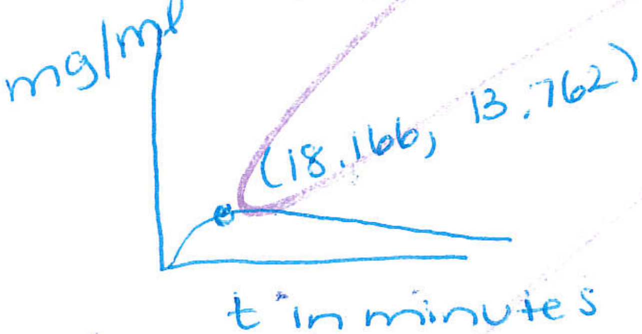
5B.

see other p. 6

(6)

The function $C(t) = \dfrac{5t}{0.01t^2 + 3.3}$ describes the concentration of a drug in the blood stream over time. In this case, the medication was taken orally. C is measured in micrograms per milliliter and t is measured in minutes.

a. Sketch a graph of the function over the first two hours after the dose is given. Label axes.
b. Determine when the maximum amount of the drug is in the body and the amount at that time.



18 minutes after ingesting, there are 13.76 micrograms/ml in bloodstream.

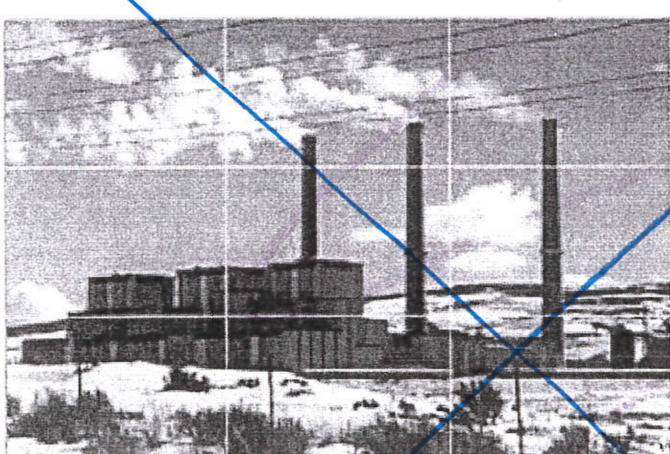
c. What is the concentration of the drug in the bloodstream after 30 minutes?

$x = 30$ Calc Value $x = 30$

$y = 12.2$ mc/ml.

5C.

This application is a **Cost-Benefit Model**. A utility company burns coal to generate electricity. The cost C (in dollars) of removing p amount (percent) of the smokestack pollutants is given by:



$$C = \frac{80{,}000p}{(100 - p)}$$

Is it possible for the company to remove 100 percent of the pollutants? Discuss why or why not, and support your response by using algebraic analysis on the given model.

What happens if the company does try to remove 100 percent of the pollutants? Will the company be successful at doing so, or will the attempt end in failure, that is, will it be to much expense for the company?

NO $100 - 100 = 0$. At this value of p, the fn is undefined. There is an asymptote. If $p = 99.999\%$ of polutants $C = \dfrac{80{,}000(99.99\bar{9})}{(100 - 99.99\bar{9})}$

Fn → ↑ +∞ → too costly.

6.

6

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7. The fixed cost of production is \$20,000 per month. The cost per unit is \$300 .

A. Write an equation to represent the average cost of producing n units/

$\bar{C} = \dfrac{20{,}000 + 300n}{n}$

B. What is the average cost of producing 100 units?

$\bar{C}(100) = \dfrac{20{,}000 + 30000}{100} = \dfrac{50{,}000}{100} = 500$ per unit

C. What is the average cost of producing 100,000 units?

$\bar{C} = \dfrac{20{,}000 + 30000000}{100{,}000}$ = ~~300,200~~ = \$ 300.50 per unit.

D. What is the horizontal asymptote of the average cost function?

H.A BOSCCO y = 300

E. What does the horizontal asymptote represent?

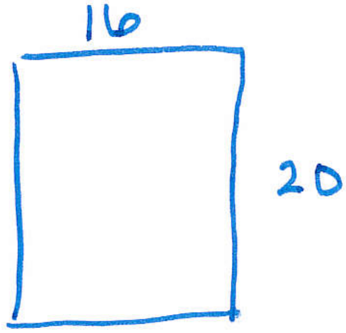
Limit
As $x \to \infty^+$; $y \to 300$

Avg cost won't drop below \$300.

7

8. You wish to create a rectangular box with an open top by cutting squares out of a piece of cardboard that measures 16 inches by 20 inches.

What should the side length of the square be if you want to MAXIMIZE volume?



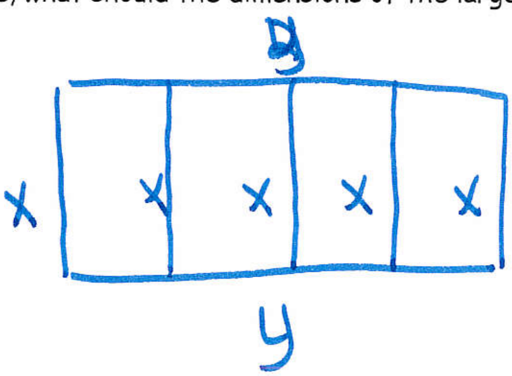
$V = LWH$

$V = (20-2x)(16-2x)x$

Calc max

9. You have a plot of land that you want to cut using fences of the same width, in order to create 4 rectangular gardens.

If you want the most available space available for planting, and you have 1200 feet of fencing available to use, what should the dimensions of the large rectangular space be?



$5x + 2y = 1200$

$y = -\frac{5}{2}x + 600$

$A = L \cdot W = xy$

$= x\left(-\frac{5}{2}x + 600\right)$

Calc max.

8

10. You have 1200 square inches of fabric available to create a cylindrical container with NO TOP.

If you want the container to hold the greatest amount possible, what should the dimensions be?

S.A. $= \pi r^2 + 2\pi r h = 1200$

$2\pi r h = 1200 - \pi r^2$

$h = \frac{(1200 - \pi r^2)}{(2\pi r)}$

$V = \pi r^2 h$

$V = \pi r^2 \left(\frac{1200 - \pi r^2}{2\pi r} \right)$

$V = r(1200 - \pi r^2) \div 2$

Calc max: 11.28, 4513.52

r, Volume

r = 11.28 in

h = 11.29 in

V = 4513.52 in^3