

SEQUENCES AND SERIES Lesson #1 Introduction to Arithmetic Sequences

The following numbers are the first few terms of a sequence.

$$-3, 1, 5, 9, \dots$$

a. What are the next two terms of the sequence? 13, 17

b. How did you calculate them? + 4

Find the next three terms of each sequence and describe the pattern.

| | Next 3 terms | Pattern |
|------------------------|----------------------|-----------|
| a. 1, -4, -9, -14, ... | <u>-19, -24, -29</u> | <u>-5</u> |

| | | |
|------------------------|-----------------------------|-----------------------------|
| b. 10, 21, 31, 40, ... | <u> </u> | <u> </u> |
|------------------------|-----------------------------|-----------------------------|

| | | |
|--|---|----------------------------------|
| c. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \dots$ | <u>$\frac{3}{2}, \frac{7}{4}, 2$</u> | <u>$+\frac{1}{4}$</u> |
|--|---|----------------------------------|

| | | |
|------------------------------|-------------------|-----------------------------|
| d. 2, 6, 12, 20, 30, 42, ... | <u>56, 72, 90</u> | <u>+4, +6, +8, +10, +12</u> |
|------------------------------|-------------------|-----------------------------|

A sequence in which a constant is added to the previous term to get the next term is called an **arithmetic sequence**. The constant is called the **common difference**, d . Decide whether each sequence given in #2 is arithmetic and, if so, identify the common difference.

| Arithmetic? | Common Difference |
|---------------|-------------------------------------|
| a. <u>yes</u> | <u>$d = -5$</u> |
| b. <u>no</u> | <u> </u> |
| c. <u>yes</u> | <u>$d = \frac{1}{4}$</u> |
| d. <u>no</u> | <u> </u> |

The symbol for the first term of sequence is a_1 , the second term is a_2 , and so forth. Thus, for the sequence $-3, 1, 5, 9, \dots$, $a_1 = -3$ and $a_2 = 1$.

a. What is a_5 for the sequence given above? 13

b. What is a_8 for the sequence given above? 25

$$d = +4$$

$$a_n = 4n - 7$$

$$a_8 = 4(8) - 7 = 25$$

The formula for the n^{th} term of an ARITHMETIC SEQUENCE:

$$a_n = a_1 + d(n-1)$$

Where, $a_n =$

$a_1 =$

$(n-1) =$

$d =$

1. Use the pattern you found above to determine the indicated term and the general term, a_n .

1 2 3 4
 a. -2, 4, 10, 16, ... $a_{31} = \underline{178}$ $a_n = \underline{6n-8}$

+6
 $d = 6$
 $a_1 = -2$
 $a_n = -2 + 6(n-1) = 6n-8$
 $a_{31} = 178$

b. -8, -3, 2, 7, ... $a_{25} = \underline{112}$ $a_n = \underline{5n-13}$

$d = +5$
 $a_1 = -8$
 $a_{25} = 5(25) - 13$
 $125 - 13 = 112$
 $a_n = a_1 + d(n-1)$
 $a_n = -8 + 5d - 5$
 $a_n = 5d - 13$

2. Find each indicated term for the given information.

a. $a_1 = 5$ $d = 4$ $a_6 = \underline{25}$
 $a_n = 5 + 4(n-1)$
 $= 4n + 1$

$a_6 = 4(6) + 1 = 25$

c. $a_1 = 7$ $d = 10$ $a_{11} = \underline{113}$
 $a_n = 7 + 10(n-1)$
 $a_n = 10n + 3$
 $a_{11} = 110 + 3 = \underline{113}$

b. $a_1 = -2$ $d = 5$ $a_{21} = \underline{98}$
 $a_n = -2 + 5(n-1)$
 $a_n = 5n - 7$
 $a_{21} = 98$

d. $a_1 = -\frac{1}{2}$ $d = \frac{1}{4}$ $a_{10} = \underline{\frac{7}{4}}$
 $a_n = -\frac{1}{2} + \frac{1}{4}(n-1)$
 $a_n = \frac{1}{4}n - \frac{3}{4}$
 $a_{10} = \frac{10}{4} - \frac{3}{4} = \underline{\frac{7}{4}}$

3. Find a_1 and d for each sequence.

a. $a_8 = 24$ $a_{15} = 45$ $a_1 = \underline{3}$ $d = \underline{3}$

$(8, 24)$ $(15, 45)$
 $d = \frac{45-24}{15-8} = \frac{21}{7} = 3$
 $a_n = a_1 + 3(n-1)$
 $24 = a_1 + 3(8-1)$

$n=8$
 $a_8 = 24$
 c. $a_3 = 11$ $a_8 = 26$ $a_1 = \underline{5}$ $d = \underline{3}$ $a_1 = 3$

$d = \frac{26-11}{8-3} = \frac{15}{5} = 3$
 $a_n = a_1 + 3(n-1)$
 $11 = a_1 + 3(3-1)$
 $5 = a_1$

b. $a_5 = -1$ $a_{12} = 34$ $a_1 = \underline{-21}$ $d = \underline{5}$

$d = \frac{34-(-1)}{12-5} = \frac{35}{7} = 5$
 $a_n = a_1 + 5(n-1)$
 $-1 = a_1 + 5(5-1)$
 $-1 = a_1 + 20$
 $-21 = a_1$

d. $a_{21} = 13$ $a_{27} = 16$ $a_1 = \underline{3}$ $d = \underline{\frac{1}{2}}$
 $d = \frac{16-13}{27-21} = \frac{3}{6} = \frac{1}{2}$
 $a_n = a_1 + \frac{1}{2}(n-1)$
 $13 = a_1 + \frac{1}{2}(21-1)$
 $3 = a_1$

SERIES AND SEQUENCES Lesson #1 Arithmetic Sequence Practice

1. Find the next three terms of each sequence and describe the pattern.

| | Next 3 terms | Pattern |
|--------------------------|---------------------|---|
| a. 4, 8, 12, 16, ... | <u>20, 24, 28</u> | <u>add 4</u> |
| b. 400, 200, 100, ... | <u>50, 25, 12.5</u> | <u>multiply by $\frac{1}{2}$</u> |
| c. -6, -5, -3, 0, 4, ... | <u>9, 15, 22</u> | <u>add 1, then 2, then 3, ...</u> |
| d. 3, 4, 7, 12, 19, ... | <u>28, 39, 52</u> | <u>add 1 then 3 then 5</u> |
| e. 1, 1, 2, 3, 5, 8, ... | <u>13, 21, 34</u> | <u>add 2 terms that come before</u> |

2. Decide whether each sequence is arithmetic and, if so, identify the common difference.

| | | |
|---------------------------|------------|-------------|
| a. 1, 3, 9, 27, ... | <u>no</u> | |
| b. 11, 16, 21, 26, ... | <u>yes</u> | <u>d=5</u> |
| c. 4.5, 9, 18, 36, ... | <u>no</u> | |
| d. 3, -1, -5, -9, ... | <u>yes</u> | <u>d=-4</u> |
| e. 0, 10, 21, 33, 46, ... | <u>no</u> | |

3. Consider the sequence 15, 18, 21, ... Find each indicated value.

a. $d = \underline{3}$ b. $a_2 = \underline{18}$ c. $a_5 = \underline{27}$ d. $a_7 = \underline{33}$

$a_n = 3n + 12$ $a_5 = 15 + 12$ $a_7 = 3(7) + 12$

4. Find a_1 and d for each sequence.

a. $a_6 = -19$ $a_8 = -29$
 $d = \frac{-29 - (-19)}{8 - 6} = \frac{-10}{2} = -5$
 $a_n = a_1 + -5(n-1)$
 $-19 = a_1 + -5(6-1)$
 $-19 = a_1 - 25$
 $6 = a_1$
 $d = -5$ | $a_1 = 6$

b. $a_5 = 24$ $a_{12} = 45$
 $d = \frac{45 - 24}{12 - 5} = \frac{21}{7} = 3$
 $a_n = a_1 + 3(n-1)$
 $24 = a_1 + 3(5-1) \rightarrow 24 = a_1 + 12$
 $12 = a_1$
 $d = 3$ | $a_1 = 12$

c. $a_7 = -23$ $a_{14} = -37$
 $d = \frac{-37 - (-23)}{14 - 7} = \frac{-14}{7} = -2$
 $a_n = a_1 - 2(n-1)$
 $-23 = a_1 - 2(7-1)$
 $-23 = a_1 - 12$
 $-11 = a_1$
 $d = -2$ | $a_1 = -11$

d. $a_{50} = 5.4$ $a_{75} = 7.9$
 $d = \frac{7.9 - 5.4}{75 - 50} = \frac{2.5}{25} = \frac{1}{10}$
 $a_n = a_1 + \frac{1}{10}(n-1)$
 $5.4 = a_1 + \frac{1}{10}(49)$
 $5.4 = 4.9 + a_1$
 $.5 = a_1$
 $a_1 = 1.5$
 $\frac{1}{10} = d$

Arithmetic Sequences Quick Check

Name: Key

Show all work to receive full credit. Good luck!

1) Consider the sequence 3, 8, 13, Find each value. (Be sure to show your work)

a) $d = 5$ b) $a_1 = 3$ c) $a_{29} = 143$ d) $a_n = 5n - 2$

$$a_n = a_1 + d(n-1)$$

$$a_n = 3 + 5(n-1)$$

$$a_{29} = 5(29) - 2$$

$$= 143$$

2) Find a_1 and d for each sequence. Then write the explicit formula for the sequence.

a) $a_{12} = 36$ $a_{20} = 100$

$(12, 36)$ $(20, 100)$

$$d = \frac{100 - 36}{20 - 12} = \frac{64}{8} = 8 = d$$

$$a_n = a_1 + 8(n-1)$$

$$36 = a_1 + 8(12)$$

$$a_1 = -52$$

3) Find the n^{th} term of each sequence.

a) $a_1 = -3$ $d = 5$ $n = 26$

$$a_n = a_1 + d(n-1)$$

$$a_n = -3 + 5n - 5$$

$$a_n = 5n - 8$$

$$a_{26} = 5(26) - 8$$

$$= 130 - 8 = 122$$

b) $a_1 = 7$ $d = -4$ $n = 86$

$$a_n = 7 + -4(n-1)$$

$$a_n = -4n + 11$$

$$a_{86} = -4(86) + 11$$

$$= -320 + 11$$

$$= -309$$

$$= -309$$

5) Complete the statement below.

-98 is the _____th term of the sequence 1, -2, -5, -8, ...

$$a_1 = 1$$

$$d = -3$$

$$a_n = a_1 + d(n-1)$$

$$a_n = 1 + -3(n-1)$$

$$a_n = -3n + 4$$

$$-98 = -3n + 4$$

$$-102 = -3n$$

$$n = 34$$

$$a_n = -30 - 2.5(n-1)$$

$$a_n = -30 - 2.5n + 2.5$$

$$a_n = -27.5 - 2.5n$$

$$34^{\text{th}}$$