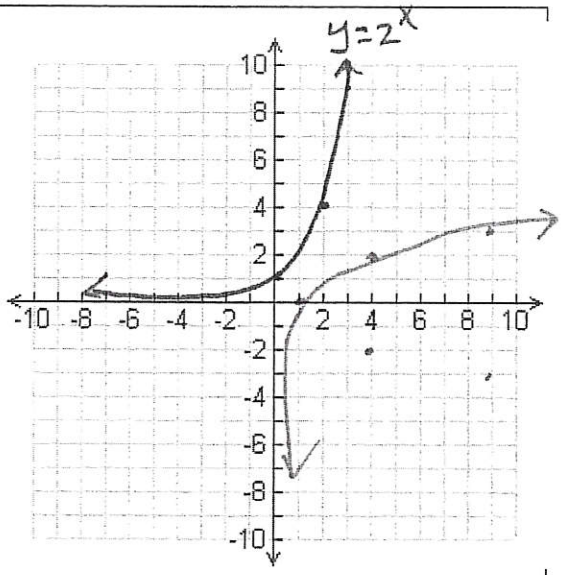
Pre-Calc CP Exponentials & Logarithms UNIT 3 SUMMATIVE REVIEW

Name: Key Date:

Exponential Functions $f(x) = b^x$ vs. Logarithmic Functions $f(x) = \log_b x$

1a.) Sketch and label the graph of $f(x) = 2^x$



1b.) Sketch the inverse of the above function

2.) What is the equation of the graph sketched in 2?

$y = \log_2 x$

3). What is the asymptote of the function sketched in #2?

$x = 0$

4A). What is the domain? $(0, \infty)$

4B). What is the range? $(-\infty, \infty)$

5A.) Explain why (0,1) is a point on the graph of every function of the form $f(x) = b^x$.

$b^0 = 1$

5B). Kelly said that (1,0) is always a point on the graph of $y = \log_b x$. Do you agree with Kelly? Explain why or why not.

true for parent function
not true for transformed log functions

$y = \log_b x$ iff $b^y = x$ $\therefore$ $x = 1$, $y = 0$

$b^0 = 1$

6. Use transformations to sketch the graphs of the following. Be sure to write a rule to generate the transformed order pair or complete a table to help you.

A. $y = -2^{x+1} + 3$ (←1, ↑3)

reflect x-axis

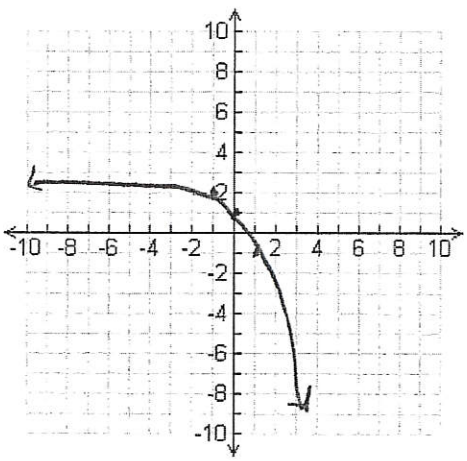
$x-1, -y+3$

(0,1)
(1,-1)
(-1,2)

B. $y = (2)^{x-1} - 3$ (↓3)

D.N.E

4.



NO CALCULATOR

Write in exponential form. Solve for x.

7) $\log_2 (1/8) = x$ $2^x = 1/8 \quad x = -3$

8) $\log_7 7\sqrt{7} = x$ $\log_7 7^{3/2} = x$ $x = 3/2$

9) $\log 1000 = x$ $10^x = 1000$ $x = 3$

Write in logarithmic form.

10) $3^x = 27$ $\log_3 27 = x$

11) $x^5 = 98$ $\log_x 98 = 5$

12) $4^{2.5} = 32$ $\log_4 32 = 2.5$

Write as a single logarithm.

13) $2\log_a 3 + 3\log_a 2$ $\log_a 3^2 + \log_a 2^3$ $\log_a 72$

14) $\frac{1}{2}\log_a 16 + \frac{1}{3}\log_a 27$ $= \log_a 4 + \log_a 3$ $= \log_a 12$

15) $\log x^3 - 2\log x$ $= \log \frac{x^3}{x^2} = \log x$

C. Simplify each expression. NO CALCULATOR!

16. $\log_2 32 = 5$

17. $\log_2 (1/8)$ $= \log_2 2^{-3}$ $x = -3$

18. $\log_5(-25)$ D.N.E.

19. $\log \frac{1}{100}$ $= \log 10^{-2}$ $x = -2$

20. $\log_x \sqrt[3]{x^2}$ $= \log_x x^{2/3}$ $x = 2/3$

21. $\log_{\frac{1}{3}} 81$ $= \log_{3^{-1}} 3^4 = x$ $3^{-x} = 3^4$ $x = -4$

D. Condense. Simplify if possible.

22. $\log_b p + \log_b t - \frac{1}{2}\log_b q$ $\log_b \frac{pt}{q^{1/2}}$

23. $\log_a k - \left(\log_a m + \frac{1}{2}\log_a s\right)$ $\log_a K - \log_a ms^{1/2}$ $= \log_a \frac{K}{ms^{1/2}}$

24. $4\ln x - \ln y - 3\ln w$ $\ln \frac{x^4}{y} - \ln w^3$ $\ln \frac{x^4}{w^3 y}$

25. $2\log 3 - 4\log x - \log 100$ $\log \frac{9}{x^4} - \log 100 = \log \frac{9}{100x^4}$

E. Expand :

26. $\log \frac{x^2 y^3}{\sqrt{z}}$ $\log x^2 + \log y^3 - \log z^{1/2}$ $2\log x + 3\log y - \frac{1}{2}\log z$

27. $\log \frac{\sqrt[4]{a^2}}{\sqrt[4]{b}}$ $\log \sqrt[4]{a^2} - \log \sqrt[4]{b}$ $= \log a^{1/2} - \log b^{1/4}$ $= \frac{1}{2}\log a - \frac{1}{4}\log b$

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WITH CALCULATOR

F. Solve. Be sure to check solutions as needed to make sure they do not make the arguments negative.

28. $\log_{14} x + \log_{14}(x-5) = 1$

$\log_{14}(x^2-5x) = 1$

$14 = x^2-5x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x=7$ ✓

29. $\log_2(2x+2) + \log_2(x-3) - \log_2(x-5) = 5$ $x=\{7, 11\}$

$\log_2 \frac{(2x+2)(x-3)}{x-5} = 5$

$(x-5)2^5 = (2x+2)(x-3)$

$32x-160 = 2x^2-4x-6$

$2x^2-36x+154=0$

$x^2-18x+77=0$

$(x-11)(x-7)=0$

30. $\log_2(x+6)+\log_2(2x)=5$

$\log_2(x+6)(2x)=5$

$2^5 = 2x^2+12x$

$2x^2+12x-32=0$

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$x=-8$

$x=2$ ✓

31. $\log_2(x+5) + \log_2(x-2) = 3$

$\log_2(x+5)(x-2)=3$

$2^3 = x^2+3x-10$

$x^2+3x-18=0$

$(x+6)(x-3)=0$

$x=-6, 3$ ✓

32. $\log_2(x-6)-\log_2(x)=4$

$\log_2\left(\frac{x-6}{x}\right)=4$

$2^4 = \frac{x-6}{x}$

$16x = x-6$

$15x=-6$

$x=-6/16$

n.R.S.

33. $\log_6(2x-5)+1=\log_6(7x+10)$ What do you see? What could you do?

$\log_6(7x+10)-\log_6(2x-5)=1$

$\log_6 \frac{(7x+10)}{(2x-5)}=1$

$6^1 = \frac{7x+10}{2x-5}$; $12x-30=7x+10$

$5x=40$

$x=8$ ✓

34. $140 = 7e^{3k}+28$

$112 = 7e^{3K}$

$16 = e^{3K}$

$\ln 16 = 3K$

$K = .9242$

35. $3(\ln 5x) = 24$

$\ln 5x = 8$

$e^8 = 5x$

$\frac{e^8}{5} = x$

$x = 596.19$

36. $4^{x+1} = 8^{2x+3}$

$(2^2)^{x+1} = (2^3)^{2x+3}$

$2x+2 = 4x+9$

$-7 = 4x$

$x = -7/4$

37. $3^{2x-1} = \frac{1}{9}$

$3^{2x-1} = 3^{-2}$

$2x-1 = -2$

$2x = -1$

$x = -1/2$

$25^x = 5^{x^2-15}$

(38) $125^x = 5^{x^2-15}$

$5^{3x} = 5^{x^2-15}$

$3x = x^2 - 15$

$x^2 - 3x - 15 = 0$

$(x-5)(x+3) = 0$

$x = \{5, -3\}$

39. $8^{2x-5} = 5^{x+1}$

$(2x-5)\log 8 = (x+1)\log 5$

$2x\log 8 - x\log 5 = 5\log 8 + \log 5$

$x = \frac{5\log 8 + \log 5}{2\log 8 - \log 5}$

$x = 4.71$

40. $3^{4x-7} = 4^{2x+3}$ take log or ln of both sides!

$(4x-7)\ln 3 = (2x+3)\ln 4$

$4x\ln 3 - 7\ln 3 = 2x\ln 4 + 3\ln 4$

$4x\ln 3 - 2x\ln 4 = 7\ln 3 + 3\ln 4$

$x(4\ln 3 - 2\ln 4) = 7\ln 3 + 3\ln 4$

$x = \frac{3\ln 4 + 7\ln 3}{4\ln 3 - 2\ln 4}$

$x = 7.3060$

let $m = e^{2x}$

41. $e^{2x} + 3e^x - 4 = 0$

$m + 3m - 4 = 0$

$(m+4)(m-1) = 0$

$m = -4, \quad m = 1$

$e^{2x} = -4 \quad e^{2x} = 1$

$2x = \ln(-4)$

$2x = \ln 1$

$x = 0$

42. $e^{4x} + 6e^{2x} + 5 = 0$

let $m = e^{2x}$

$m^2 + 6m + 5 = 0$

$(m+5)(m+1) = 0$

$m = -5, \quad m = -1$

$e^{2x} = -5 \quad e^{2x} = -1$

$2x = \ln(-5) \quad 2x = \ln(-1)$

no real solution

43. $6(\ln 3x) = 36$

$\ln(3x) = 6$

$e^6 = 3x$

$x = \frac{e^6}{3}$

$x = 134.48$

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Compound Interest	$A = P\left(1+\frac{r}{n}\right)^{nt}$	P is the initial investment, r is the annual interest rate, t is the time in years, and n is the number of times per year the interest will be compounded
Continuously Compounded Interest or Exponential Growth	$A = Pe^{rt}$	P is the initial investment, r is the annual interest rate, and t is the time in years
Exponential Growth or Decay	$y = ne^{kt}$ $A = A_0e^{kt}$	
Newton's Law of Cooling	$T = C + (T_0 - C)e^{kt}$	T is the temperature of a heated object at time t, C is the constant temperature of the surrounding medium, and k is a negative constant that is associated with the cooling object.

P r

44. Siya plans to invest \$500 at 8.25% interest, compounded continuously. How long will it take for her money to triple?

A=3P

$1500 = 500e^{.0825t}$

$3 = e^{.0825t}$

$\ln 3 = .0825t$

$t \approx 13.32$ yrs or 13 yrs 4 mos

A_0 K is neg r

45. A \$40,000 car depreciates at a constant rate of 12% per year. In how many years will the car be worth \$12,000?

A

$12{,}000 = 40{,}000e^{-.12t}$

$\ln(.3) = -.12t$

$t \approx 10$ yrs.

r

46. Suppose TJ invested \$500 at 6% annual interest compounded twice a year. When will his investment be worth \$1000?

$1000 = 500\left(1+\frac{.06}{2}\right)^{2t}$

2t

$2 = 1.03$

$\ln 2 = (\ln 1.03)\,2t$

$23.45 = 2t$

$t \approx 11.7$ yrs

A_0 A

47. An organism of a certain type can grow from 30 to 195 organisms in 5 hours. Find k for the growth formula.

① $195 = 30e^{K5}$

$6.5 = e^{K5}$

$\ln 6.5 = 5K$

$.374 = K$

$A_0 : A \rightarrow 1:3$

48. Grant Investments Inc. promises to triple your money in 12 years. Assuming continuously compounding of interest, what rate of interest is needed?

$3 = 1e^{12K}$

$\ln 3 = \ln e^{12K}$

$\ln 3 = 12K$

$K = .0916$

$\approx 9.16\%$ rate

t

49. A substance decomposes radioactively. Its half-life is 32 years. Find the constant k in the decay formula.

$A = \frac{A_0}{2}$

$\frac{1}{2} = e^{32k}$

$\ln(.5) = 32K$

$K = -.0217$

A_0

49b. It is reported that 100 grams of the substance was spilled. Safety standards say that the environment is safe when there are 10 grams or less of the substance. How long will it take for the amount to reach safe levels?

A

$A = A_0 e^{Kt}$

$10 = 100\, e^{-.0217t}$

$\frac{1}{10} = e^{-.0217t}$

$\ln(.10) = -.0217t$

$t = 106$ yrs

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50. A object is heated to 100 °C (T_0) and is then allowed to cool in a room whose air temperature is 30 °C (C).

a) If the temperature of the object is 80 °C (T) after 5 (t) minutes, when will its temperature be 50 °C?

What must you find first? K

(1)

$T = C + (T_0 - C)e^{Kt}$

$80 = 30 + (100-30)e^{K5}$

$50 = 70e^{K5}$

$\ln\left(\frac{5}{7}\right) = 5K$

$K = -.0673$

(2) T

$50 = 30 + 70e^{-.0673t}$

$\frac{20}{70} = e^{-.0673t}$

$\ln(2/7) = -.0673t$

$t = 18.615$ minutes

b) Determine the elapsed time before the temperature of the object is 35 °C. (T)

$35 = 30 + 70e^{-.0673t}$

$\ln\left(\frac{5}{70}\right) = -.0673t$

$t \approx 39.213$ minutes

51. The homicide unit arrives at the crime scene to find a body.
When they first arrive, it is 10 a.m. and the temperature of the body is 90 degrees.
The thermostat shows that the temperature in the room has been steady for the past 12 hours, at 70 degrees.
The medical examiner takes another temperature reading in the same location, 1 hour later, at 11 a.m.
The temperature of the body at this time is 86 degrees.

A witness claims to have seen the victim alive at 8 a.m. that morning. Could the witness be telling the truth?

Clearly show all math work done to answer this question. Your work has to be admissible in court!

10 a.m. → 90° let $t = 0$ → start time is 10 a.m.

$T_0 = 90$
$C = 70$
$t = 1$
$T = 86$

$T = C + (T_0 - C)e^{kt}$

#1. Find K using data given

$86 = 70 + (90-70)e^{1k}$
$86 = 70 + 20e^{k}$
$16 = 20e^{k}$
$\frac{16}{20} = e^{k}$
$\ln\left(\frac{8}{10}\right) = \ln e^{k}$
$\ln(.8) = k$ ≈ $-.2231 = k$

Now → think 8 am → $t = -2$

So $T = 70 + 20e^{-.2231(-2)}$
$T = 101.25$

It is possible ... if the person had a 101.25° fever!