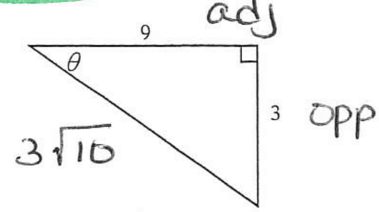
Name: ______________________ per. ____

Key

PART 2

1. Find the indicated information. Leave answers in EXACT (simplest radical) form.



a) sin θ= $\frac{3}{3\sqrt{10}} = \frac{\sqrt{10}}{10}$

b) sec θ= $\frac{3\sqrt{10}}{9} = \frac{\sqrt{10}}{3}$

$\frac{r}{x}$ $\frac{h}{a}$

c) tan θ= $\frac{3}{9} = \frac{1}{3}$

d) csc θ = $3\sqrt{10}/3 = \sqrt{10}$

$\frac{1}{\sin} = \frac{r}{y} = \frac{h}{o}$

$3^2 + 9^2 = c^2$

$9 + 81 = c^2 \quad c = \sqrt{90} = 3\sqrt{10}$

e) Find θ to the nearest tenth of a degree using your calculator set on degree mode (Think – what key might you use?)

$\tan^{-1}(1 \div 3) = 18.43° \approx 18.4°$

$18.4°$

2. Look up the definition of co-function. Then, find a co-function with the same value as the one stated.

a. sin 76° $\cos 14°$

.97 .97

b. cos 12° $\sin 78°$

$\cos\theta = \sin(90 - \theta)$

3. Let β be in standard position. In which quadrant(s) can β lie under the given conditions? Don't guess! Use your ASTC to help you!

a. Cos β > 0 and Tan β < 0 IV

b. cot β and csc β have the same sign I, IV

Tan Sin

4. Find the exact value. **DO NOT USE A CALCULATOR.**

a. $\cot 60° = \frac{\sqrt{3}}{3}$

$\cot\frac{\pi}{3}$

b. $\cos 30° = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{6}$

c. $\sec 45° = \sqrt{2}$

$\frac{1}{\sin\frac{\pi}{4}} = 1 \div \frac{\sqrt{2}}{2} = \frac{2}{\sqrt{2}} = \sqrt{2}$

d. $\csc 60° = \frac{2\sqrt{3}}{3}$

$\csc\frac{\pi}{3}$

$\frac{1}{\sin\pi/3} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

e. $\tan 30° = \frac{\sqrt{3}}{3}$

$\tan\frac{\pi}{6}$

f. $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

g. $\cot 330° = -\sqrt{3}$

$\frac{11\pi}{6} \rightarrow Q4$

neg

$-\sqrt{3}$

h. $\csc\frac{5\pi}{4} = -\sqrt{2}$

$\frac{1}{\sin}$

neg.

$\frac{-2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2} = -\sqrt{2}$

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i. $\cos 210° =$ $-\frac{\sqrt{3}}{2}$

$\div 30 = 2$

$\frac{7\pi}{6}$ −,−

Cos neg →

j. $\sec 300° =$ 2

$\div 60°$

$= 5$

so $\frac{5\pi}{3}$ $\frac{1}{\cos} = 2$

k. $\tan\frac{5\pi}{2} =$ undef

$= \tan\frac{\pi}{2}$

(0, 1)

$= \frac{1}{0}$

l. $\sin\frac{3\pi}{4}$ $\frac{\sqrt{2}}{2}$

m. $\csc 270°$ −1

$\div 30$

$= \frac{9\pi}{6} = \frac{3\pi}{2}$

(0, −1)

$\csc = \frac{1}{-1}$

n. $\tan\frac{11\pi}{4}$ −1

$= \frac{8\pi}{4} + \frac{3\pi}{4}$

↓

Q2

Tan neg

o. $\cos\frac{10\pi}{4}$ 0

$= \frac{5\pi}{2}$

$= \frac{4\pi}{2} + \frac{\pi}{2}$

(0, 1)

p. $\sin\frac{13\pi}{4}$ $-\frac{\sqrt{2}}{2}$

$= \frac{8\pi}{4} + \frac{5\pi}{4}$

Q3 →

sin neg

q. $\cos\frac{3\pi}{4}\tan\frac{7\pi}{3} =$ $-\frac{\sqrt{6}}{2}$

$-\frac{\sqrt{2}}{2} \cdot \sqrt{3}$

r. $\csc 120° + \cot 315° =$ ________

$\frac{2\pi}{3}$ Sin pos

45° Q4 neg

$\frac{2}{\sqrt{3}} + -1$

$\frac{2\sqrt{3}}{3} + \frac{-3}{3} = \frac{2\sqrt{3}-3}{3}$

s. $4\cos 60° + 3\tan\frac{\pi}{3} =$ $2+3\sqrt{3}$

$4\cos\frac{\pi}{3} + 3\sqrt{3}$

$4\left(\frac{1}{2}\right)$

$2 + 3\sqrt{3}$

t. $6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right) =$ ________

$6\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\sqrt{3}\right)$

$-3\sqrt{2} - 2\sqrt{3}$

u. $\cos 540° - \tan(-405°) =$ 0

$= 360 + 180$

$+360$

-45 → Tan neg in Q4

(−1, 0)

$-1 - (-1) = 0$

******<u>Use your knowledge of reciprocal trig functions to simplify. Rewrite each reciprocal function in terms of its base function. Think reference angles as well!</u>

<u>Example:</u>

w. tan 68° • cot 68°=____________

$$= \tan 68^0 \bullet \frac{1}{\tan 68^0} = 1$$

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y. tan 20° • cos (−20°) • csc 20° = 1

$\frac{\sin\theta}{\cos\theta} \cdot \cos\theta \cdot \frac{1}{\sin\theta}$

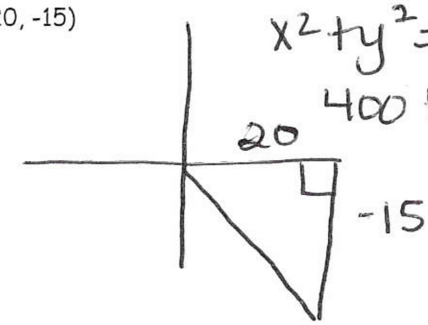
z. sin 45° • sec (−45°) = 1

$\frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}}$ cos pos

5. The terminal side of an angle α in standard position passes through point G. Sketch α and find the exact values of the six trigonometric functions of α.

a) G(20, -15)

$x = 20$
$y = -15$
$r = 25$



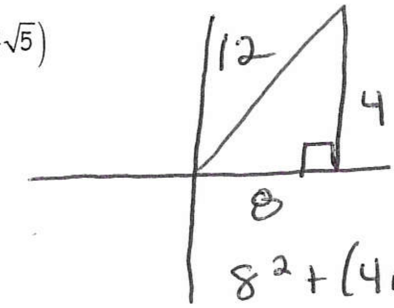
$x^2 + y^2 = r^2$
$400 + 225 = r^2$
$625 = r^2$
$r = \sqrt{625}$
$r = \pm 25$
$r = 25$

sin α $\frac{-15}{25} = \frac{-3}{5}$ csc α $-\frac{5}{3}$

cos α $\frac{20}{25} = \frac{4}{5}$ sec α $\frac{5}{4}$

tan α $\frac{-15}{20} = \frac{-3}{4}$ cot α $-\frac{4}{3}$

b) G $(8, 4\sqrt{5})$



$8^2 + (4\sqrt{5})^2 = r^2$
$64 + 80 = r^2$
$r = \pm 12$
$r = 12$

sin α $\frac{4\sqrt{5}}{12} = \frac{\sqrt{5}}{3}$ csc α $\frac{3\sqrt{5}}{5}$

cos α $\frac{8}{12} = \frac{2}{3}$ sec α $\frac{3}{2}$

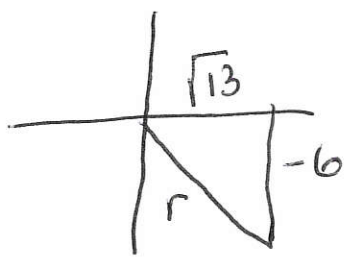
tan α $\frac{\sqrt{5}}{2}$ cot α $\frac{2\sqrt{5}}{5}$

6. Find the exact values of the other 5 trigonometric functions for an angle α in standard position lying in the given quadrant

a) Cot α = $\frac{-\sqrt{13}}{6}$; IV

$x = \sqrt{13}$
$y = -6$

$\text{Cot} = \frac{x}{y}$



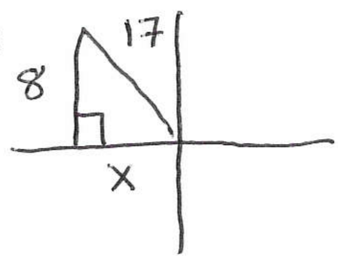
$r^2 = 13 + 36$
$r^2 = 49$
$r = \pm 7$ $r = 7$

Sin α $-6/7$ Csc α $-\frac{7}{6}$

Cos α $\sqrt{13}/7$ Sec α $\frac{7\sqrt{13}}{13}$

Tan α $\frac{-6\sqrt{13}}{13}$

b) Sin α = $\frac{8}{17}$; II



$x^2 + 64 = 289$
$x^2 = 225$
$x = \pm 15$
$x = -15$

Csc α $\frac{r}{y} = \frac{17}{8}$

Cos α $\frac{-15}{17}$ Sec α $\frac{r}{x} = -17/15$

Tan α $-\frac{8}{15}$ Cot α $\frac{-15}{8}$

Pre-Calculus CP Review – Unit 5 Part 2 UNIT CIRCLE/TRIG FNS/IDENTITIES

Name: ______________________________ per. ______

7. Given the following, find tan, cot, sec, csc. Think: x = 8 y = 5 r = 13

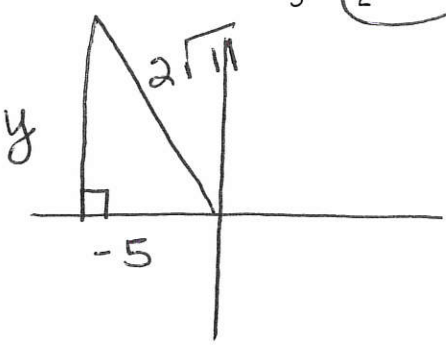
$\sin\theta = \frac{5}{13}, \cos\theta = \frac{8}{13}$

Tan = $\frac{y}{x} = \frac{5}{8}$ cot = $\frac{8}{5}$ sec = $\frac{13}{8}$ csc = $\frac{13}{5}$

8. Use your knowledge to find the missing value. Be sure to sketch a right triangle in the correct quadrant first. Then, identify what you know (x, y, r) and label your triangle. Use the PT to find the length of the missing side. Use the quadrant to determine the sign. Recall, the radius is always positive!

a. $\sec\theta = -\frac{2\sqrt{11}}{5}, \left(\frac{\pi}{2} < \theta < \pi\right)$ tan Θ = $\frac{y}{x} = -\frac{\sqrt{19}}{5}$ → Q2

b. $\cos t = -\frac{3}{\sqrt{10}},\ \pi < \theta < \frac{3\pi}{2}$ csc Θ = $\frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$ → Q3



$\sec = \frac{1}{\cos} = \frac{r}{x}$

cos neg.

$r = 2\sqrt{11}$

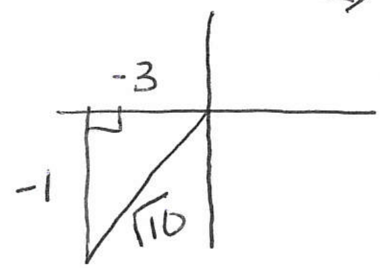
$x = -5$

$(-5)^2 + y^2 = (2\sqrt{11})^2$

$25 + y^2 = 44$ $y^2 = 19$

$y = \pm\sqrt{19}$

Q3 → $y = \sqrt{19}$



$\csc = \frac{r}{y}$

$\cos = \frac{x}{r}$

$x = -3$

$r = \sqrt{10}$

$9 + y^2 = 10$

$y^2 = 1$

$y = -1$

9. Use trig identities to simplify each expression

a. $\frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta - \sin\theta\cos\theta}$

$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$= \frac{(\sin\theta + \cos\theta)\cancel{(\sin\theta - \cos\theta)}}{\sin\theta\cancel{(\sin\theta - \cos\theta)}}$

$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

$= 1 + \cot\theta$

b. $\tan^2\theta\left(\csc^2\theta - 1\right)$

↓

$= \tan^2\theta(\cot^2\theta)$

$= \tan^2\theta\left(\frac{1}{\tan^2\theta}\right)$

$= 1$

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c. $(\sin t+\cos t)(\sin t-\cos t)+1$

$\sin^2 t-\cos^2 t+1$

$= 1+\sin^2 t-\cos^2 t$

$= 1+\sin^2 t-(1-\sin^2 t)$

$= 1+\sin^2 t-1+\sin^2 t$

$= 2\sin^2 t$

d. $(\sin t-\cos t)^2$

$= \sin^2 t - 2\sin t\cos t + \cos^2 t$

$= 1-2\sin t\cos t$

e. $\cos^2 x+\cos^2 x\cot^2 x$

$= \cos^2 x(1+\cot^2 x)$

$= \cos^2 x(\csc^2 x)$

$= \cos^2 x\left(\frac{1}{\sin^2 x}\right)$

$= \cot^2 x$

f. $(\cot\theta+\csc\theta)(\cot\theta-\csc\theta)$

$\cot^2\theta-\csc^2\theta$

$= -1$

g. $\tan^4\theta+2\tan^2\theta+1$

$= (\tan^2\theta+1)^2$

$= (\sec^2\theta)^2$

$= \sec^4\theta$

h. $\frac{\sin^2 x-\tan^2 x}{\tan^2 x\sin^2 x}$

$= \frac{\sin^2 x}{\tan^2 x\sin^2 x} - \frac{\tan^2 x}{\tan^2 x\sin^2 x}$

$= \frac{1}{\tan^2 x} - \frac{1}{\sin^2 x}$

$= \cot^2 x - \csc^2 x$

$= -1$

Name: ______________________________ per. ______

i. sec x cot x – cot x cos x

$\cot x\,(\sec x - \cos x)$

$= \frac{\cos x}{\sin x}\left(\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}\right)$

$= \frac{\cos x}{\sin x}\left(\frac{\sin^2 x}{\cos x}\right)$

$= \sin x$

j. $\frac{1-\sin}{1-\sin}\left(\frac{\cos x}{1+\sin x}\right) + \left(\frac{\cos x}{1-\sin x}\right)\frac{1+\sin}{1+\sin}$

$\frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{(1+\sin x)(1-\sin x)}$

$= \frac{2\cos x}{1-\sin^2 x} = \frac{2\cos x}{\cos^2 x}$

$= \frac{2}{\cos x} = 2 \cdot \frac{1}{\cos x}$

$= 2\sec x$

k. $\dfrac{\cot x \sec^2 x - \cot x}{\sin x \tan x + \cos x}$

$\frac{\cot x\,(\sec^2 x - 1)}{\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}}$

$= \frac{\cot x\,(\tan^2 x)}{\frac{1}{\cos x}}$ $= \left(\frac{1}{\tan x} \cdot \frac{\tan^2 x}{1}\right) \cdot \cos x$

$= \tan x \cos x$

$= \frac{\sin x}{\cos x} \cdot \cos x$

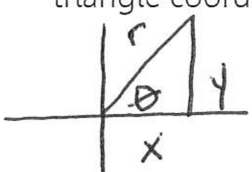
$= \sin x$

L. $\dfrac{\sec x}{\csc x}$

$\frac{1}{\cos x} \div \frac{1}{\sin x}$

$= \frac{\sin x}{\cos x} = \tan x$

m. Generate the 3 Pythagorean Identities from your knowledge of the unit circle right triangle coordinates, the PT, and definition of quotient and reciprocal identities.



A.

Unit circle $\rightarrow r = 1$

$x^2 + y^2 = r^2$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$

$x = \cos\theta$

$y = \sin\theta$

B. $\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\boxed{1 + \tan^2\theta = \sec^2\theta}$

OR $\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\boxed{\cot^2\theta + 1 = \csc^2\theta}$