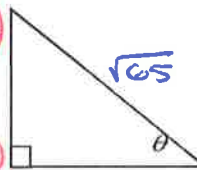
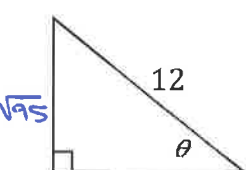


Precalculus
4.3-4.4 Review

Name Key



Find the exact values of the six trigonometric functions of the angle theta shown in the figure.

<p>1.</p>  <p> $\sin \theta = \frac{8}{\sqrt{65}} = \frac{8\sqrt{65}}{65}$ $\csc \theta = \frac{\sqrt{65}}{8}$ $\cos \theta = \frac{1}{\sqrt{65}} = \frac{\sqrt{65}}{65}$ $\sec \theta = \sqrt{65}$ $\tan \theta = 8$ $\cot \theta = \frac{1}{8}$ </p> <p> $(1)^2 + (8)^2 = r^2$ $1 + 64 = r^2$ $65 = r^2$ $r = \sqrt{65}$ </p>	<p>2.</p>  <p> $\sin \theta = \frac{\sqrt{95}}{12}$ $\csc \theta = \frac{12\sqrt{95}}{95}$ $\cos \theta = \frac{7}{12}$ $\sec \theta = \frac{12}{7}$ $\tan \theta = \frac{\sqrt{95}}{7}$ $\cot \theta = \frac{7\sqrt{95}}{95}$ </p> <p> $y^2 + (7)^2 = (12)^2$ $y^2 + 49 = 144$ $y^2 = 95$ $y = \sqrt{95}$ </p>
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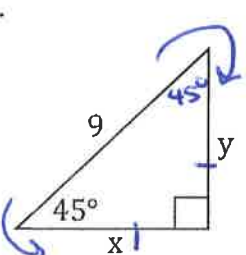
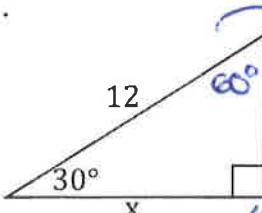
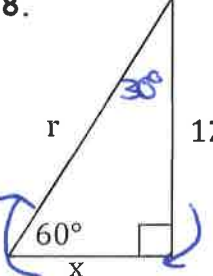
3. List the three Pythagorean Identities.

$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$

Sketch a right triangle corresponding to the trigonometric function of the acute angle theta. Find the remaining five trigonometric functions of theta.

<p>4. $\sin \theta = \frac{5}{6}$</p>  <p> $\sin \theta = \frac{5}{6}$ $\csc \theta = \frac{6}{5}$ $\cos \theta = \frac{\sqrt{11}}{6}$ $\sec \theta = \frac{6\sqrt{11}}{11}$ $\tan \theta = \frac{\sqrt{11}}{5}$ $\cot \theta = \frac{5\sqrt{11}}{11}$ </p> <p> $x^2 + (5)^2 = (6)^2$ $x^2 + 25 = 36$ $x^2 = 11$ $x = \sqrt{11}$ </p>	<p>5. $\tan \theta = 3$</p>  <p> $\sin \theta = \frac{3\sqrt{10}}{10}$ $\csc \theta = \frac{\sqrt{10}}{3}$ $\cos \theta = \frac{\sqrt{10}}{10}$ $\sec \theta = \sqrt{10}$ $\tan \theta = 3$ $\cot \theta = \frac{1}{3}$ </p> <p> $(1)^2 + (3)^2 = r^2$ $1 + 9 = r^2$ $10 = r^2$ $r = \sqrt{10}$ </p>
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Find the exact values of the missing variables - x, y, or r.

<p>6.</p>  <p> $x = \frac{9}{\sqrt{2}} = \frac{9\sqrt{2}}{2}$ $y = \frac{9\sqrt{2}}{2}$ </p>	<p>7.</p>  <p> $y = \frac{12}{2} = 6$ $x = 6\sqrt{3}$ </p>	<p>8.</p>  <p> $x = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ $r = (4\sqrt{3})(2) = 8\sqrt{3}$ </p>
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Draw a picture to represent the situation and then solve.

Degree mode!

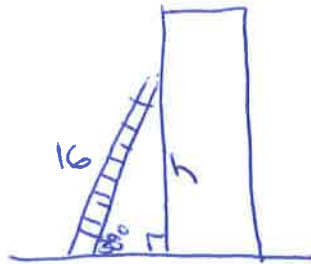
9. A 16-foot ladder is leaning against a building. If the ladder forms a 38° angle with the ground, determine how high the ladder meets the building.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 38^\circ = \frac{h}{16}$$

$$h = 16 \sin(38)$$

$$h \approx 9.851 \text{ ft.}$$



10. Michael is looking at the top of a mountain. It is 2,050 feet high. If Mike is 6 feet tall (eye level) and he is looking up at a 10° angle, how far is he from the mountain?

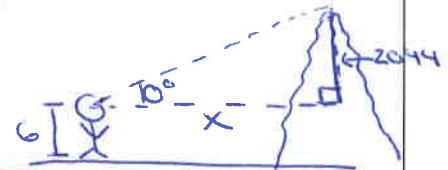
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 10^\circ = \frac{2044}{x}$$

$$x \tan 10^\circ = 2044$$

$$x = \frac{2044}{\tan(10)}$$

$$x \approx 11592.100 \text{ ft.}$$



Determine the value(s) of theta in degrees ($0^\circ \leq \theta < 360^\circ$) and radians ($0 \leq \theta < 2\pi$).

11. $\sin(\theta) = \frac{\sqrt{3}}{2}$
 $\theta' = 60^\circ = \frac{\pi}{3}$

$$\theta = 60^\circ = \frac{\pi}{3}$$

$$\theta = 120^\circ = \frac{2\pi}{3}$$

12. $\tan(\theta) = 1$
 $\theta' = 45^\circ = \frac{\pi}{4}$

$$\theta = 45^\circ = \frac{\pi}{4}$$

$$\theta = 225^\circ = \frac{5\pi}{4}$$

13. $\csc(\theta) = 2$
 $\sin \theta = \frac{1}{2}$
 $\theta' = 30^\circ = \frac{\pi}{6}$

$$\theta = 30^\circ = \frac{\pi}{6}$$

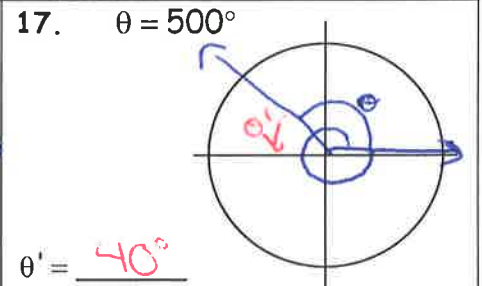
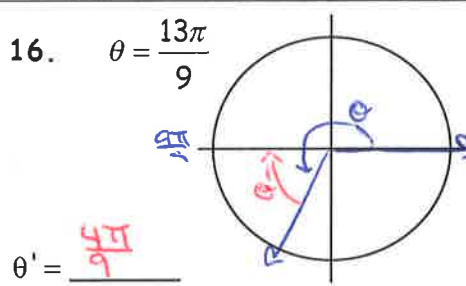
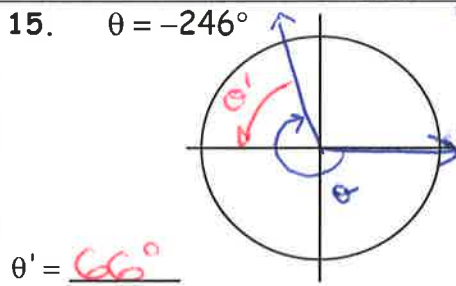
$$\theta = 150^\circ = \frac{5\pi}{6}$$

14. $\cot(\theta)$ is undefined
 $\tan \theta = 0$

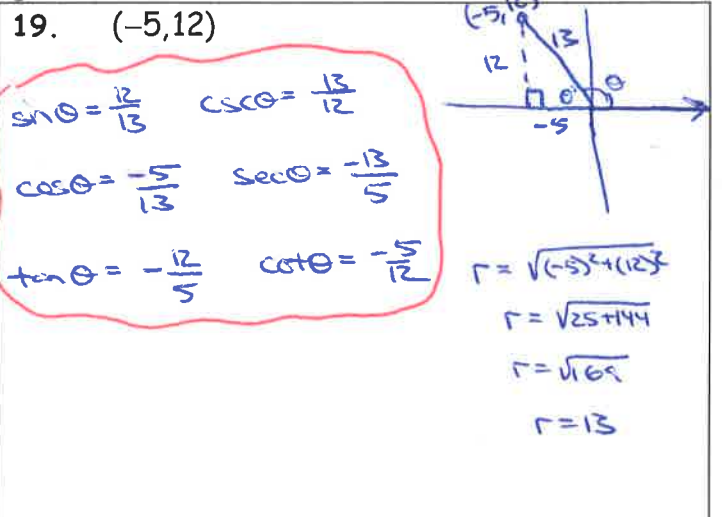
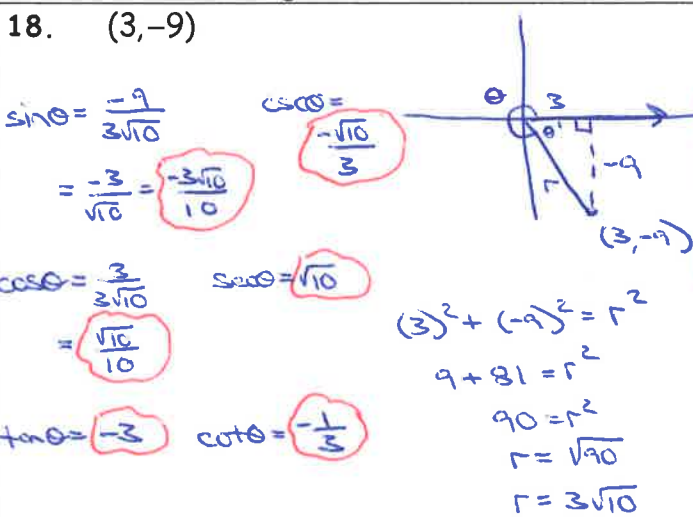
$$\theta = 0^\circ = 0$$

$$\theta = 180^\circ = \pi$$

Determine the reference angle θ' . Using the blank coordinate planes is optional.



The point given is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions on the angle.



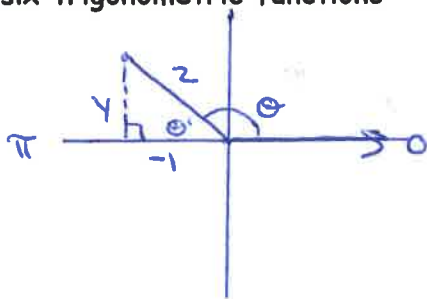
20. Given $\sec\theta = -2$ and $0 \leq \theta \leq \pi$, find the values of the six trigonometric functions of θ .

$$\cos\theta = -\frac{1}{2}$$

$$y^2 + (-1)^2 = (2)^2$$

$$y^2 = 3$$

$$y = \sqrt{3}$$



$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\csc\theta = \frac{2\sqrt{3}}{3}$$

$$\cos\theta = -\frac{1}{2}$$

$$\sec\theta = -2$$

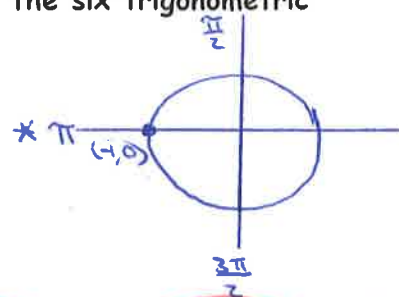
$$\tan\theta = -\sqrt{3}$$

$$\cot\theta = -\frac{\sqrt{3}}{3}$$

21. Given $\cot\theta$ is undefined and $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$,

find the values of the six trigonometric functions of θ .

$$\tan\theta = 0$$



$$\sin\theta = 0$$

$\csc\theta$ is undefined

$$\cos\theta = -1$$

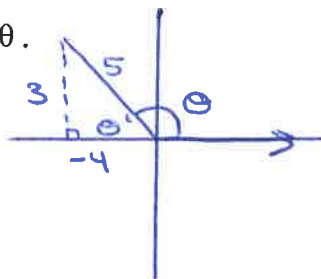
$$\sec\theta = -1$$

$$\tan\theta = 0$$

$\cot\theta$ is undefined

22. Given $\sin\theta = \frac{3}{5}$ and $\cot\theta < 0$, determine $\cos\theta$ and $\tan\theta$.

*Pythagorean triple
3-4-5



$$\cos\theta = -\frac{4}{5}$$

$$\tan\theta = -\frac{3}{4}$$

23. Given $\cos\theta = -\frac{3\sqrt{10}}{10}$ and $\tan\theta > 0$, determine $\csc\theta$ and $\tan\theta$.

$$(-3\sqrt{10})^2 + y^2 = (10)^2$$

$$(9 \cdot 10) + y^2 = 100$$

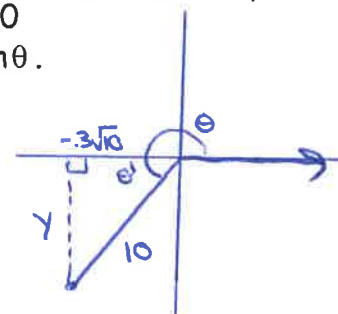
$$90 + y^2 = 100$$

$$y^2 = 10$$

$$y = \pm\sqrt{10} \rightarrow y = -\sqrt{10}$$

$$\csc\theta = \frac{10}{-\sqrt{10}} = -\frac{10\sqrt{10}}{10} = -\sqrt{10}$$

$$\tan\theta = \frac{-\sqrt{10}}{-3\sqrt{10}} = \frac{1}{3}$$

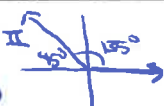


Evaluate the following trigonometric functions.

24. $\cos(135^\circ)$

$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Ref L: 45°



$$\cos(135^\circ) = -\frac{\sqrt{2}}{2}$$

25. $\sec\frac{\pi}{2}$



$\sec\frac{\pi}{2}$ is undefined

26. $\sin(-390^\circ)$

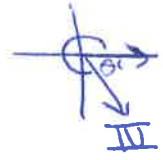
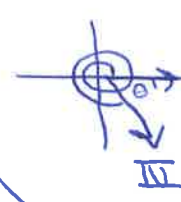
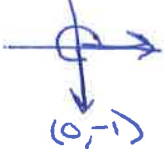
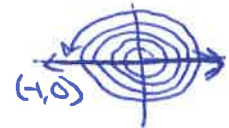
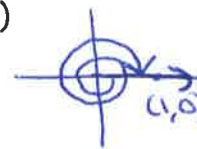
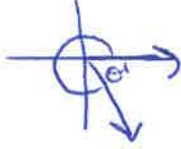
Ref L: 30°

$$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$



$$\sin(-390^\circ) = -\frac{1}{2}$$

Evaluate the following trigonometric functions.

<p>27. $\tan 300^\circ$</p> <p>Ref L: 60° $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$</p>  <p>$\tan 300^\circ = -\sqrt{3}$</p>	<p>28. $\cot\left(\frac{15\pi}{4}\right)$</p> <p>Ref L: $\frac{\pi}{4}$ $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$</p>  <p>$\cot\left(\frac{15\pi}{4}\right) = -1$</p>	<p>29. $\csc\frac{3\pi}{2}$</p>  <p>$\csc\frac{3\pi}{2} = -1$</p>
<p>30. $\cos 9\pi$</p>  <p>$\cos 9\pi = -1$</p>	<p>31. $\sin(-720^\circ)$</p>  <p>$\sin(-720^\circ) = 0$</p>	<p>32. $\sec\frac{5\pi}{3}$</p> <p>Ref L: $\frac{\pi}{3}$ $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$</p>  <p>$\sec\frac{5\pi}{3} = 2$</p>

Use trigonometric identities to transform one side of the equation into the other ($0 < \theta < 2\pi$).

<p>33. $\tan\theta \cos\theta = \sin\theta$</p> $\frac{\cancel{\sin\theta} \cdot \cancel{\cos\theta}}{\cancel{\cos\theta}} \cdot \frac{1}{1} = \sin\theta$ $\sin\theta = \sin\theta$ <p style="text-align: center;">✓</p>	<p>34. $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$</p> $1 - \cancel{\cos\theta} + \cancel{\cos\theta} - \cos^2\theta = \sin^2\theta$ $1 - \cos^2\theta = \sin^2\theta$ $\sin^2\theta = \sin^2\theta$ <p style="text-align: center;">✓</p>
<p>35. $\frac{\cot\theta \tan\theta}{\sin\theta} = \csc\theta$</p> $\frac{1 \cdot \cancel{\tan\theta}}{\cancel{\tan\theta} \cdot \sin\theta} = \csc\theta$ $\frac{1}{\sin\theta} = \csc\theta$ $\csc\theta = \csc\theta$ <p style="text-align: center;">✓</p>	<p>36. $\frac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x$</p> $\frac{1 + \sec^2 x}{\sec^2 x} = 1 + \cos^2 x$ $\frac{1}{\sec^2 x} + \frac{\sec^2 x}{\sec^2 x} = 1 + \cos^2 x$ $\cos^2 x + 1 = 1 + \cos^2 x$ $1 + \cos^2 x = 1 + \cos^2 x$ <p style="text-align: center;">✓</p>