Key $a^3-b^3=(a\ \ b)(a^2\ \ ab\ \ b^2)$
S O AP

Rational Expressions, Equations, and Inequalities Review
CP Pre-Calculus

Simplify each expression. Provide any restrictions on the domain where appropriate.

1. $\dfrac{\frac{p^2+7p}{3p}}{\frac{49-p^2}{3p-21}}$

$\dfrac{p(p+7)}{3p}\cdot\dfrac{3(p-7)}{(7-p)(7+p)}$

$-1(p-7)(p+7)$

$=\dfrac{1}{-1}=-1$

2. $\dfrac{8x^3-27}{4x^2-9}$

$=(2x)^3-3^3$
$=(2x-3)(4x^2+6x+9)$

3. $\dfrac{8x^2+14x+3}{5x^2-18x-8}\div\dfrac{4x^2+16x+15}{10x^2+29x+10}$

$\dfrac{(4x+1)(2x+3)}{(5x+2)(x-4)}\cdot\dfrac{(5x+2)(2x+5)}{(2x+3)(2x+5)}$

$=\dfrac{4x+1}{x-4}$

4. $\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}$

num

$\dfrac{3(x+3)}{(x+3)}-\dfrac{1}{(x+3)}$

$=\dfrac{3x+8}{x+3}$ = num

$\dfrac{3x+10}{x+3}$ = den

So $\dfrac{3x+8}{(x+3)}\cdot\dfrac{(x+3)}{3x+10}=\dfrac{3x+8}{3x+10}$

5. $\dfrac{3x}{x-y}+\dfrac{4x}{y-x}$

$-1(x-y)$

$\dfrac{3x}{(x-y)}-\dfrac{4x}{(x-y)}$

$=\dfrac{-1x}{x-y}$

6. $\dfrac{3r}{2r-s}-\dfrac{2r}{2r+s}+\dfrac{2s^2}{4r^2-s^2}$

$\left(\dfrac{2r+s}{2r+s}\right)$ $(2r-s)$ $(2r-s)$

LCD is $\boxed{(2r+s)(2r-s)}$

$\dfrac{6r^2+3rs-4r^2+2rs+2s^2}{\text{den}}$

$=\dfrac{2r^2+5rs+2s^2}{(2r+s)(2r-s)}$

$=\dfrac{(2r+s)(r+2s)}{(2r+s)(2r-s)}$

Either mult by LCD to eliminate fractions or rewrite w/ LCD, solve num.

Solve each equation. Be sure to check your solutions.

7. $\left(\frac{x-1}{x-1}\right)\left(\frac{1}{x+1}\right)+\left(\frac{1}{x-1}\right)\left(\frac{x+1}{x+1}\right)=\frac{2}{x^2-1}$

LCD $= (x+1)(x-1)$

$x-1+x+1=2$

$2x=2$

$x=1$

Excluded values

$x \neq 1, -1$

no soln

8. $\frac{4}{x^2-2x-3}=\frac{-1}{x+1}-\frac{x}{3-x}$

$(x-3)(x+1)$

$-1(x-3)$

$4=-1(x-3)+x(x+1)$

$4=-1x+3+x^2+x$

$4=x^2+3$

$1=x^2$

$x=\pm 1$

But $x \neq -1$

$x \neq 3$

so $x=1$

9. $\frac{5x+2}{x^2-4}=\frac{-5x}{2-x}+\frac{2}{x+2}$

$-1(x-2)$

$5x+2=5(x+2)+2(x-2)$

$5x+2=5x+10+2x-4$

$5x+2=7x+6$

$-4=2x$

$\cancel{-2=x}$

but $x \neq -2$

no soln

10. $(x-5)\left[\frac{5}{x-5}=\frac{x}{x-5}-1\right]$

$5=x-(x-5)$

$5=x-x+5$

$5=5$

solution:

all $\mathbb{R}, x \neq 5$

or $(-\infty,5)\cup(5,\infty)$

11. $\left(\frac{4}{x-2}-\frac{x+6}{x+1}=1\right)$

$4(x+1)-(x+6)(x-2)=(x-2)(x+1)$

$4x+4-[x^2+4x-12]=x^2-1x-2$

$\cancel{4x}+4-\cancel{x^2}-\cancel{4x}+12=x^2-1x-2$

$x \neq 2$
$x \neq -1$

$0=2x^2-1x-18$

$0=(2x\quad)(x\quad)$

QF

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{1\pm\sqrt{1-72}}{4}=\frac{1\pm\sqrt{-71}}{4}=\frac{1\pm 71i}{4}$

12. $\left[\frac{1}{x+2}+\frac{1}{x-2}=\frac{3}{x+1}\right]$

$(x+1)(x-2)+(x+2)(x+1)$

$=3(x+2)(x-2)$

$x^2-x-2+x^2+3x+2$

$=3x^2-12$

$2x^2+2x=3x^2-12$

$0=x^2-2x-12$

$(x\quad)(x-\quad)$

use QF.

Imaginary

$(-\infty, -4] \cup (-1, 4] \cup (5, \infty)$

Solve each inequality.

13. $1+\dfrac{5}{a-1} \le \dfrac{7}{6}$

$1 - \dfrac{7}{6} + \dfrac{5}{a-1} \le 0$

$-\dfrac{1}{6} + \dfrac{5}{a-1} \le 0$

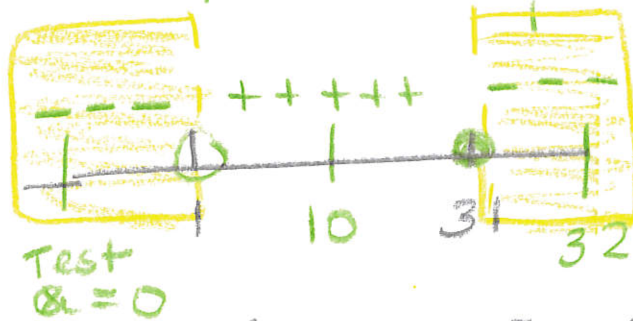
$\dfrac{5(6)}{6(a-1)} - \dfrac{1(a-1)}{6(a-1)} \le 0$

$\dfrac{30-a+1}{6(a-1)} \le 0$

$\dfrac{-a+31}{6(a-1)} \le 0$

den: $a \neq 1$

num: $a = 31$

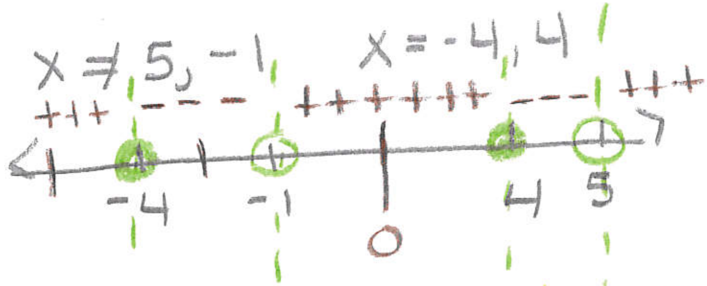


$(-\infty, 1) \cup [31, \infty)$

14. $\dfrac{x^2-16}{x^2-4x-5} \ge 0$

$\dfrac{(x+4)(x-4)}{(x-5)(x+1)} \ge 0$

$x \neq 5, -1$, $x = -4, 4$



15. $5+\dfrac{1}{x} > \dfrac{16}{x}$

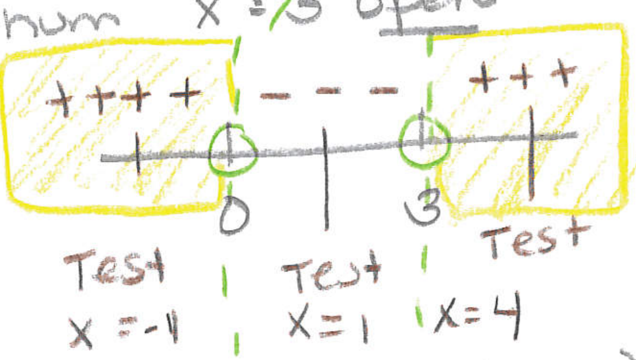
$5 + \dfrac{1}{x} - \dfrac{16}{x} > 0$

$5 - \dfrac{15}{x} > 0$

$\dfrac{5x-15}{x} > 0$

den: $x \neq 0$

num $x = 3$ open



$(-\infty, 0) \cup (3, \infty)$

16. $\dfrac{2a-5}{6} - \dfrac{a-5}{4} < \dfrac{3}{4}$

$\dfrac{2a-5}{6} - \dfrac{(a-4)}{4} - \dfrac{3}{4} < 0$

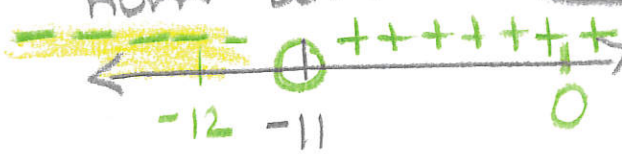
$\dfrac{2}{2}\left(\dfrac{2a-5}{6}\right) - \left(\dfrac{a-7}{4}\right)\dfrac{3}{3} < 0$

$\dfrac{4a-10-3a+21}{12} < 0$

$\dfrac{a+11}{12} < 0$

den $\neq 0$ ✓

num $a+11=0$ $a=-11$



$(-\infty, -11)$