CP Pre-Calculus

Algebra Review #1 Systems

Date: Key

Solve each system of equations for x and y. Check your work.

Methods: Substitution, elimination, graphing

1) $2 \cdot \left[-x+3y=0\right]$
$2x+6y=12$

$\boxed{(3,1)}$

$-2x+6y=0$

$12y=12$

$\boxed{y=1}$

$2x+6=12$

$2x=6$

$\boxed{x=3}$

3) $3x-9y=0$
$3\left[-x+3y=-3\right]$

$\to -3x+9y=-9$

$0=-9$

false

$\therefore$ no solution

5) $3\left[\frac{2}{3}m-\frac{5}{3}n=-\frac{1}{3}\right]$
$18\left[\frac{5}{9}m+\frac{7}{6}n=1\right]$

clear fractions

create opps.

$-5\left[2m-5n=-1\right]$

$10m+21n=18$

$\to -10m+25n=5$

$46n=23$

$\boxed{n=\frac{1}{2}}$

$2m-\frac{5}{2}=-1$ $\quad 2m=\frac{3}{2}$ $\quad \boxed{m=\frac{3}{4}}$

2) $\begin{cases} y=3x-12 \\ x-2y=14 \end{cases}$ ← solved for y → substitution

$x-2(3x-12)=14$

$x-6x+24=14$

$-5x+24=14$

$-5x=-10$

$x=2$ $\quad y=3(2)-12$

$6-12$

$y=-6$

$\boxed{(2,-6)}$

4) $6\left[\frac{5}{2}x+\frac{1}{3}y=13\right]$
$2\left[\frac{1}{2}x-y=-7\right]$

$15x+2y=78$

$x-2y=-14$

$16x=64$

$\boxed{x=4}$

$4-2y=-14$

$-2y=-18$

$\boxed{y=9}$

$\boxed{(4,9)}$

6) $6x+4y=2$
$-2\left[3x+2y=1\right]$

$-6x-4y=-2$

$0=0$

True

infinitely many solutions

Let d = # dimes q = # quarters

7) A boy has 14 coins in his pocket, all of which are dimes and quarters. If the total value of his change is $2.75, how many dimes and quarters does he have?

Quantity statement $d + q = 14$

Value statement $10d + 25q = 275$

$-10d - 10q = -140$

$15q = 135$

$q = 9$

9 quarters, 5 dimes

7b) You are selling candy for a fundraiser. Chocolate covered raisins and caramel apples each cost $6 per pound. You sold six more pounds of raisins than apples. If you sold $120, how many of each type did you sell? *Be careful! Be sure to make equations. . . if you sold 6 more pounds of raisins than apples, what might an equation look like that would make those values EQUAL?*

let r = # pounds raisins

a = # pounds apples

7 pounds apples
13 pounds raisins

\$: $6a + 6r = 120$

#: $a + 6 = r$

in order to make equal, you'd need to add 6 to apples.

substitution

$6a + 6(a+6) = 120$

$6a + 6a + 36 = 120$

$12a = 84$

$\boxed{a = 7}$

7c) You are in charge of allocating funds for the UConn basketball teams and need to order new sneakers. The players can choose from the new Air Jordan's costing $100 or the new Reebok's costing $125. If 50 pairs are ordered and the total cost is $5,850, how many of each type of sneaker were ordered?

let J = # pairs Jordan's

R = # pairs Reeboks

Quanity (#) → $J + R = 50$

Value ($) → $100J + 125R = 5850$

$-100J - 100R = -5000$

$25R = 850$

$R = 34$ pairs

∴ $J = 16$ pairs.

7d) The total cost of 15 gallons of regular unleaded gasoline and 10 gallons of premium gasoline is \$35.50. Premium costs \$0.20 more per gallon than regular unleaded. What is the cost per gallon of each type of gasoline?

$15r + 10p = 35.50$

$p = r + .20$

regular = 1.34

premium = 1.54

$15r + 10(r + .20) = 35.50$

$25r + 2 = 35.50$

$25r = 33.50$

$r = 1.34$

7e) How many ounces of 20% hydrochloric acid solution and 70% hydrochloric acid solution must be mixed to obtain 20 ounces of 50% hydrochloric acid solution?

*** Think: Create on value statement and one quantity statement. . .

Let x = # ounces 20% HA Let y = # ounces 70% HA

Quantity: $x + y = 20 \rightarrow x + y = 20$

Value: $.20x + .70y = .50(20) \rightarrow .20x + .70y = 10$

$-.20x - .20y = -4$

$.50y = 6$

$y = 12$

12 ounces 70% HA

8 ounces 20% HA

Solving Three Variable Systems of Equations. **You may use POLYSIMULT APP!**

Polysmlt solver eq3 eq, 3 var.

You can solve three variable systems word problems by following these steps:

1. Read the problem and underline (or highlight) the question.
2. Define the variables!
3. Read over the problem again. Anytime you see the word representing your variable, substitute your variable into the equation!
4. **Remember**: Three variables means there will be *three* equations!

Example 3: You have 17 coins in pennies, nickels, and dimes in your pocket. The value of the coins is \$0.47. There are four times the number of pennies as nickels. How many of each type of coin do you have?

x represents: # pennies 12 y represents: #nickels 3 z represents: # dimes 2

****** **Think of how you could create an equation from the info that there are four times the number of pennies as nickels. What would you have to do to the number of nickels in order to create an equation?**

quantity → EQ1 → $p + n + d = 17$

EQ1 → $\boxed{x + y + z = 17}$

value → $\boxed{1x + 5y + 10z = 47}$

4 times as many pennies as nickels → EQUATE $\boxed{p = 4n}$ so $\boxed{x = 4y}$

$\boxed{1x - 4y + 0z = 0}$

Example 4: For a party, you are cooking a large amount of stew that has meat, potatoes, and carrots. The meat costs \$6 per pound, the potatoes cost \$3 per pound, and the carrots cost \$1 per pound. You spend \$48.50 on 13.5 pounds of food. **You buy twice as many carrots as potatoes. How much of each ingredient did you buy?**

x represents: # pounds meat (6 pounds)   y represents: # pounds potatoes (2.5)   z represents: # pounds carrots (5 pounds)

→ $1x + 1y + 1z = 13.5$

\$ → $6x + 3y + 1z = 48.50$

$0 - 2y + 1z = 0$

polysmlt.

Twice as many carrots as potatoes → ∴ multiply potatoes by 2. $z = 2y$

You Try! You work at a fruit stand that sells apples for \$2 per pound, oranges for \$5 per pound, and bananas for \$3 per pound. Yesterday you sold 60 pounds of fruit and made \$180. **You sold 10 more pounds of apples than bananas.**

a. Write a system of equations representing the information above.

$a + r + b = 60$

$2a + 5r + 3b = 180$

$a = b + 10 \rightarrow \boxed{1a - 0r - b = 10}$

1	1	1	60
2	5	3	180
1	0	-1	10

b. How many pounds of each kind of fruit did you sell yesterday?

$x_1 = 28$ = pounds apples

$x_2 = 14$ = pounds oranges

$x_3 = 18$ = pounds bananas

c. What kind of fruit did you sell the most?

apples.

Challenge

A worker received a \$10,000 bonus and decided to split it among three different accounts. He placed part in a savings account paying 4.5% per year, twice as much in government bonds paying 5%, and the rest in a mutual fund that returned 4%. His income from these investments after one year was \$455. How much did the worker place in each account?

let x = \$ in 4.5% acct — \$2200

\$4400 ← y = \$ in 5% acct

\$3400 ← z = \$ in 4% acct

$x + y + z = 10{,}000$

$.045x + .05y + .04z = 455$

$y = 2x \rightarrow -2x + y + 0z = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 10{,}000 \\ .045 & .05 & .04 & 455 \\ -2 & 1 & 0 & 0 \end{bmatrix}$$

C. Solve the system

$y = x^2 + 4$

$3x + y = 8$

options : GC. $Y_1 = x^2 + 4$

$Y_2 = -3x + 8$

calc intersect.

option: algebraic substitution

$3x + x^2 + 4 = 8$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$\boxed{x = -4 \quad x = 1}$

Pre-Calc Review #2 QUADRATICS

Is there a GCF? - Always check this 1st!

Difference of 2 perfect squares?

$a^2 - b^2 = (a+b)(a-b)$

Ex

$4x^2 - 25 = (2x+5)(2x-5)$

Perfect Trinomial Squares?

$a^2 + 2ab + b^2$

$a^2 - 2ab + b^2$

$= (a \pm b)^2$

Ex.

$4x^2 + 20x + 25$

$= (2x+5)^2$

Ex. $16x^2 - 24x + 9$

$= (4x-3)^2$

4 term factor by grouping

$\boxed{x^3 + x^2} - 5x - 5$

$x^2(x+1) - 5(x+1)$

match

$= (x+1)(x^2-5)$

Factor each expression completely

8) $m^2 - 6m + 8$

$(m-4)(m-2)$

9) $16x^2 + 56x + 49$

$(4x+7)^2$

10) $x^2 - 3x - 10$

$(x-5)(x+2)$

11) $r^3 + 3r^2 - 54r$

$= r(r^2 + 3r - 54)$

$= r(r+9)(r-6)$

12) $4a^2 + a - 3$

$= (a+1)(4a-3)$

☺

13) free space!

14) $2t^3 + 32t^2 + 128t$

$= 2t(t^2 + 16t + 64)$

$= 2t(t+8)^2$

15) $4x^6 - 4x^2$

$= 4x^2(x^4 - 1)$

$= 4x^2(x^2+1)(x^2-1)$

16) $6n^2 - 11n - 2$

$= (n-2)(6n+1)$

☺

17) $-35 + 16y + 3y^2$

$= 3y^2 + 16y - 35$

$= (3y-5)(y+7)$

☺

18) $x^2 - 22x + 121$

$= (x-11)^2$

19) $16x^2 - 49$

$= (4x+7)(4x-7)$

Solve each quadratic equation. Leave answers in simplest radical form.

20) $5x^2 = 6 - 13x$ $x = 2/5 \quad x = -3$

$5x^2 + 13x - 6 = 0$

$(5x - 2)(x + 3) = 0$

$5x - 2 = 0 \qquad x + 3 = 0$

$x = 2/5 \qquad x = -3$

21) $2x^2 = 250$ $x = \pm 5\sqrt{5}$

$2x^2 = 250$

$x^2 = 125$

$x = \pm\sqrt{125}$

$= \pm\sqrt{25}\sqrt{5} = \pm 5\sqrt{5}$

22) $2x^2 + 3x = 1$ $\frac{-3 \pm \sqrt{17}}{4}$

$2x^2 + 3x - 1 = 0$

$(2x \quad 1)(x \quad 1) = 0$

DNF

∴ USE QF $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{9 + 8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$

23) $(2x - 5)(x + 1) = 2$ $x = \frac{3 \pm \sqrt{65}}{4}$ use QF.

not = 0

must multiply out!

$2x^2 + 2x - 5x - 5 = 2$

$\boxed{2x^2 - 3x - 7 = 0}$

$(2x \quad)(x \quad) = 0$

24) $(2x + 7)^2 = 25$ $\boxed{x = -1, -6}$

use $\pm\sqrt{}$

$2x + 7 = \pm\sqrt{25}$

$2x + 7 = 5 \qquad 2x + 7 = -5$

$2x = -2 \qquad 2x = -12$

$x = 1 \qquad x = -6$

25) $45x - 30x^2 + 5x^3 = 0$ $\boxed{x = 0, 3}$

$5x(x^2 - 6x + 9) = 0$

$5x(x - 3)^2 = 0$

$x = 0 \qquad x = 3$

26) $3x^2 - 27 = 0$

$3x^2 = 27$

$x^2 = 9$

$x = \pm\sqrt{9} = \boxed{\pm 3}$

27) $x^3 + 4x^2 - x - 4 = 0$ $\boxed{x = -4, -1, 1}$

$x^2(\underline{x+4}) - 1(\underline{x+4}) = 0$

$(x+4)(x^2-1) = 0$

keep going!

$(x+4)(x+1)(x-1) = 0$

$\boxed{x = -4, -1, 1}$

28) $4x^2(x - 10)(x + 4) = 0$

$\boxed{x = 0 \quad x = 10 \quad x = -4}$

Applications of Quadratics and Polynomials:

29. Calculators are sold to students for 20 dollars each. Three hundred students are willing to buy them at that price. For every 5 dollar increase in price, there are 30 fewer students willing to buy the calculator. What selling price will produce the maximum revenue and what will the maximum revenue be?

Let x = # price increases.

Price	X	Quantity	=	Revenue
20		300		6000
25		270		
30		240		
35		210		
⋮				⋮

Xmin = 0
Xmax = 10
Ymin = 0
Ymax = 10,000

$(20+5x)\ (300-30x) = R$ calc. max → $x = \underline{3}$, $y = 7350$

Then sub that into correct ~~expression~~ expression

Selling price = $\$35$
$20+5x$

maximum revenue = $\$7350$
y value of calc max.

30. At a swim meet, Janet dives from a diving board. Her position above the water is represented by the equation $y = -16x^2 + 24x + 40$, where x represents the time in seconds and y represents the height above the water.

a) After how many seconds does Janet enter the water (Hint: the height above the water would equal zero.)

$0 = -16x^2 + 24x + 40$

Algebraic

$\boxed{2.5 \text{ secs}}$

$0 = -8(2x^2 - 3x - 5)$

$0 = (2x-5)(x+1)$

$0 = 2x - 5 \qquad 0 = x + 1$

$\boxed{2.5 = x} \qquad x = -1$

options GC

$y_1 = -16x^2 + 24x + 40$

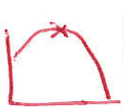
CALC ZERO

time { xmin = 0
xmax = 6
ymin = 0 or below to get calc zero
ymax = 50

b) What is the greatest height that Janet reaches in her dive? When does she reach that height?

Option 1: CALC MAX.
The y value will be the height.

Option 2: $x = \frac{-b}{2a}$ = x coord. of vertex.
Then sub it into eq. to find. y.

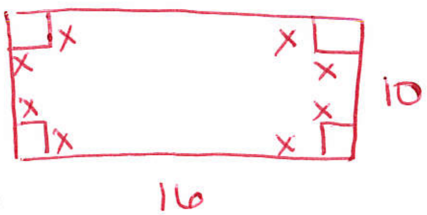
Calc max 

[.75, 49
sec   ft above water]

7

31. An open box is to be made from a 10 inch by 16 inch sheet of cardboard by cutting squares out of the four corners and folding up the sides. What dimensions of the box will yield the largest volume? What is the maximum volume of the box? $\boxed{\text{(12 in by 6 in by 2 in , 144 cubic in)}}$

Sketch & Label: V = LWH



$10-2x$

$V = (16-2x)(10-2x)(x)$

$x_{min} = 0$

$x_{max} = 5$ (no box!)

$y_{min} = 0$

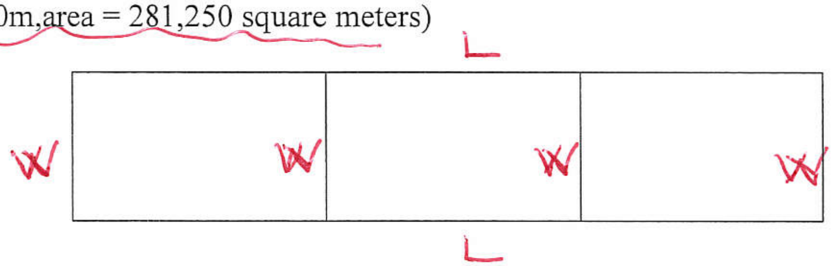
$y_{max} = ?$ 250 ? 300 ?

$16-2x$

Use calc max.

$x = 2$ max. Vol = 144 in^3

32. An ecologist is conducting a research project on breeding pheasants in captivity. She must first construct suitable pens. She wants a rectangular region with two additional fences as shown in the diagram. Find the maximum area that can be enclosed with 3000 meters of fencing. (w = 375m, l = 750m,area = 281,250 square meters)



A = LW

3000 = $4w + 2L$

$\frac{3000-4w}{2} = L$

Solve for one of the variables above. Then substitute for this variable into your Area formula. Find the maximum area.

$A = \left[\frac{3000-4w}{2}\right] \cdot w$

$A = (1500-w)(w)$

let $x \Rightarrow w$

$y \Rightarrow A$

$x_{min} = 0$

$x_{max} = 400$?

$y_{min} = 0$

$y_{max} = ?$ big