Precalculus **Name** Key

8.1 Notes: Sequences and Series-*Day 1*

A Sequence is a function whose DOMAIN is a set of consecutive integers. If a domain is NOT SPECIFIED it is understood that the domain starts with 1. The values in the RANGE are called the terms of the sequence.

Domain	1	2	3	...	...	...	n
Range	a_1	a_2	a_3				a_n

A finite sequence has a limited number of terms. An example would be: 1, 2, 4, 8, 16

a) How many terms are in this sequence? 5

b) What is a_3? 4

c) Write a rule for finding the nth term. $a_n = 2^{(n-1)}$

n	1	2	3	4	5
a_n	1	2	4	8	16

×2 ×2 ×2 ×2

repeated mult.

An infinite sequence continues without stopping. The set of natural numbers is an example of an infinite sequence. What are the natural numbers? 1, 2, 3, 4, 5, ...

a) What is a_5? 5

Instead of using function notation, sequences are usually written using subscript notation.

Write the first five terms of the sequence.

1. $a_n = 2n+1$

$a_1 = 2(1)+1 = 3$

$a_2 = 2(2)+1 = 5$

$a_3 = 2(3)+1 = 7$

$a_4 = 2(4)+1 = 9$

$a_5 = 2(5)+1 = 11$

3, 5, 7, 9, 11

Write the first five terms of the sequence.

2. $a_n = 2-(-1)^n$

$a_1 = 2-(-1)^1$ 3

$a_2 = 2-(-1)^2$ 1

$a_3 = 2-(-1)^3$ 3

$a_4 = 2-(-1)^4$ 1

$a_5 = 2-(-1)^5$ 3

3, 1, 3, 1, 3

Find the 3rd, 4th and 5th term of the sequence.

3. $a_n = \dfrac{2+(-1)^n}{n}$

$a_3 = \dfrac{2+(-1)^3}{3} = \dfrac{1}{3}$

$a_4 = \dfrac{2+(-1)^4}{4} = \dfrac{3}{4}$

$a_5 = \dfrac{2+(-1)^5}{5} = \dfrac{1}{5}$

$\frac{1}{3}, \frac{3}{4}, \frac{1}{5}$

Write an expression for the apparent n^{th} term of the sequence. (Assume *n* begins with 1).

n = 1 2 3 4

4. a_n = 2, 4, 6, 8... * infinite

What is the rule? $a_n = 2n$

What is a_7? $a_7 = 2(7) = 14$

n = 1 2 3 4

5. a_n = 1, 3, 5, 7 * finite

What is the rule? $a_n = 2n-1$

What is a_8? Does not exist... only 4 terms

Write an expression for the apparent n^{th} term of the sequence. (Assume n begins with 1).

$n = 1 \quad 2 \quad 3 \quad 4$

6. $a_n =$ 1, 4, 9, 16

$a_n = n^2$

$n = 1 \quad 2 \quad 3 \quad 4$

7. $a_n =$ 2, 5, 10, 17, ...

* each term is 1 more than n^2

$a_n = n^2 + 1$

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5$

8. $a_n =$ −1, 2, 7, 14, 23...

* each term is 2 less than n^2

$a_n = n^2 - 2$

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5$

9. $a_n =$ 1, 2, −7, 14, −23...

$(-1)^n$ alternates signs!

$a_n = (-1)^n \cdot (n^2 - 2)$

Challenge

When a sequence is built using **PREVIOUS TERMS** the sequence is said to be defined Recursively.

Fill in the missing terms:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_n
1	2	6	24	120	720	5040	*

×2 ×3 ×4 ×5 ×6 ×7

To find the rule:

1 less than current term

$a_2 = a_1 \cdot 2$ $\quad a_3 = a_2 \cdot 3$ $\quad a_4 = a_3 \cdot 4$ $\quad a_n = a_{n-1} \cdot n$

10. Consider the sequence 1, 1, 2, 3, 5, 8, 13, 21...

1 2 3 4 5 6 7 8

Describe the pattern in words.

* Starting with the 3rd term... each term is found by adding the 2 previous terms...

Write a recursive formula to define this sequence.

$a_3 = a_2 + a_1$

$a_4 = a_3 + a_2$

$a_n = a_{n-1} + a_{n-2}$ where $n \geq 3$

What is this very famous sequence of numbers called? Fibonacci Sequence

11. Write the first five terms of the sequence.

$a_{k+1} = \frac{1}{2} a_k$; $\quad a_1 = 32$

$a_2 = \frac{1}{2}(32) = 16$ $\quad$ note $K=1$

$a_3 = \frac{1}{2}(16) = 8$ $\quad K=2$

$a_4 = \frac{1}{2}(8) = 4$ $\quad K=3$

$a_5 = \frac{1}{2}(4) = 2$ $\quad K=4$

32, 16, 8, 4, 2

Write an expression for the apparent n^{th} term of the sequence.

$a_n = 32\left(\frac{1}{2}\right)^{(n-1)}$

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8.1 Notes: Sequences and Series-*Day 2*

If n is a positive integer, n FACTORIAL is defined as: $n! = n \bullet (n-1) \bullet (n-2) \bullet \ldots \bullet 3 \bullet 2 \bullet 1$.

As a special case, **zero factorial** is defined as: $0! = 1$.

$n! = n(n-1)!$ if $n = 1 \rightarrow$ $1! = 1(1-1)!$, $1 = 1(0)!$, $1 = 0!$

1. Evaluate. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $= 5040$	**2.** Simplify the factorial expression. $\dfrac{9!}{3!7!}$ $\dfrac{9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{3 \cdot 2 \cdot 1 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$ $\dfrac{72}{6} = 12$ OR: $\dfrac{9 \cdot 8 \cdot \cancel{7!}}{3 \cdot 2 \cdot 1 \cdot \cancel{7!}} = 12$
3. Write the first 5 terms of the sequence. $a_n = \dfrac{2^n}{n!}$ $a_1 = \dfrac{2^1}{1!} = \dfrac{2}{1} = 2$ $a_2 = \dfrac{2^2}{2!} = \dfrac{4}{2 \cdot 1} = 2$ $a_3 = \dfrac{2^3}{3!} = \dfrac{8}{3 \cdot 2 \cdot 1} = \dfrac{4}{3}$ $a_4 = \dfrac{2^4}{4!} = \dfrac{16}{4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{2}{3}$ $a_5 = \dfrac{2^5}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{4}{15}$ 2, 2, 4/3, 2/3, 4/15	**4.** Simplify the factorial expression. $\dfrac{(n+1)!}{n!}$ $\dfrac{(n+1)(\cancel{n})(\cancel{n-1})(\cancel{n-2})\ldots}{(\cancel{n})(\cancel{n-1})(\cancel{n-2})\ldots}$ $n+1$

A Series is the sum of the terms in a sequence. A series can be written with Summation notation where the sum of the first n terms of a sequence is represented by $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$, where i is called the index of summation, n is the upper limit and **1** is the lower limit.

Find the sum.

5. $\sum_{i=1}^{4}(4i+1)$ $4(1)+1 = 5$ $4(2)+1 = 9$ $4(3)+1 = 13$ $4(4)+1 = 17$ $5+9+13+17 = 44$ How many terms are in this series? 4	**6.** $\sum_{k=2}^{5}(2+k^3)$ $2+(2)^3 = 10$ $2+(3)^3 = 29$ $2+(4)^3 = 66$ $2+(5)^3 = 127$ $10+29+66+127 =$ 232 How many terms are in this series? 4	**7.** $\sum_{n=0}^{8}\left(\dfrac{1}{n!}\right)$ $\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}$ $1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}+\frac{1}{5040}+\frac{1}{40320}$ $\approx 2.71828777 \approx e$ How many terms are in this series? 9
To find the number of terms in a series: (upper − lower) + 1		

A finite series is the sum of the first n terms of the sequence, which is also called a partial sum.

An infinite series is the sum of all the terms of the sequence.

Find the sum.

8. $\sum_{k=1}^{3}\left(\frac{3}{10^k}\right)$ $= \frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3}$ $= .3 + .03 + .003$ $= .333$ or $\frac{333}{1000}$ **third partial sum*	**9.** $\sum_{k=1}^{\infty}\left(\frac{3}{10^k}\right)$ $\frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3} + \frac{3}{10^4} + \frac{3}{10^5} + \ldots$ $.3 + .03 + .003 + .0003 + .00003 \ldots$ $= .33333\ldots$ $.\overline{3}$ or $\frac{1}{3}$
10. $\sum_{k=1}^{3} 5\left(\frac{1}{10^k}\right)$ $5\left(\frac{1}{10^1}\right) + 5\left(\frac{1}{10^2}\right) + 5\left(\frac{1}{10^3}\right)$ $5(.1) + 5(.01) + 5(.001)$ $.5 + .05 + .005$ $.555$ $\quad \frac{555}{1000} = \frac{111}{200}$ **third partial sum*	**11.** $\sum_{k=1}^{\infty} 5\left(\frac{1}{10^k}\right)$ $5\left(\frac{1}{10^1}\right) + 5\left(\frac{1}{10^2}\right) + 5\left(\frac{1}{10^3}\right) + 5\left(\frac{1}{10^4}\right) + \ldots$ $.5 + .05 + .005 + .0005 + \ldots$ $.55555\ldots$ $.\overline{5}$ or $5/9$

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8.2 Notes: Arithmetic Sequences and Partial Sums

Arithmetic Sequences, also known as a discrete linear function, is a sequence for which consecutive terms have a **common difference, *d*.**

Determine whether or not the sequence is arithmetic. If it is, find the common difference.

1. 5, 8, 11, 14, 17,... 3 3 3 3 yes - arithmetic $d=3$	**2.** 1, 4, 9, 16, 25,... 3 5 7 9 Not arithmetic
3. $1, \frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \ldots$ Yes arithmetic $\frac{6}{6}, \frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \ldots$ $d=\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$	**4.** $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ Not Arithmetic $\frac{12}{12}, \frac{6}{12}, \frac{4}{12}, \frac{3}{12}, \ldots$ $-\frac{6}{12}$ $-\frac{2}{12}$ $-\frac{1}{12}$

Writing an explicit formula/rule for an arithmetic sequence a_n.

Fill in the missing terms from the sequence:

n	1	2	3	4	5	6	7	8
a_n	4	7	10	13	16	19	22	25

+3 +3 +3 +3

$d=3$

Expanded:

$a_2 = 4+3$

$a_3 = 4+3+3$

$a_4 = 4+3+3+3$

Condensed:

$a_2 = a_1 + 3(1)$

$a_3 = a_1 + 3(2)$

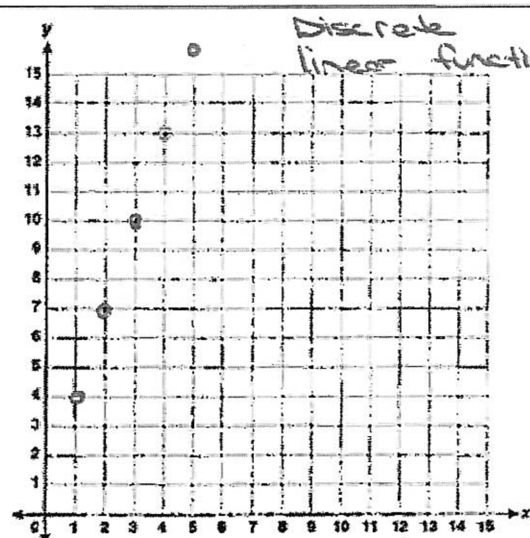
$a_4 = a_1 + 3(3)$

*Repeat addition is multiplication!

Arithmetic Explicit Rule: $\boxed{a_n = a_1 + d(n-1)}$

first term common difference

Discrete linear function



* d relates to the constant rate of change → slope!

Write an explicit rule for the given sequence. Then answer any additional questions. Assume $n \ge 1$.

5. 5, 12, 19, 26,... $d=7$

$a_n = a_1 + d(n-1)$

$\boxed{a_n = 5 + 7(n-1)}$

$a_n = 5 + 7n - 7$

$\boxed{a_n = 7n - 2}$

6. Find an explicit formula for a_n for the arithmetic sequence with the following terms: $a_3 = 19$ and $a_5 = 27$.

* Not consecutive!

Find d using the constant rate of change: slope.

$d = \frac{27-19}{5-3} = \frac{8}{2} = 4$

$a_n = a_1 + 4(n-1)$

$19 = a_1 + 4(3-1)$

$19 = a_1 + 8$

$11 = a_1$

$\boxed{a_n = 11 + 4(n-1)}$

$\boxed{a_n = 4n + 7}$

7. 29, 25, 21, 17, 13, 9,...

$d = -4$

$\boxed{a_n = 29 - 4(n-1)}$

$a_n = 29 - 4n + 4$

$\boxed{a_n = -4n + 33}$

8. 11, 5, −1, −7, −13, −19,...

$d = -6$

$\boxed{a_n = 11 - 6(n-1)}$

$a_n = 11 - 6n + 6$

$\boxed{a_n = -6n + 17}$

9. Find the first five terms of the arithmetic sequence where $a_8 = 25$ and $a_{12} = 41$.

$d = \frac{41-25}{12-8} = \frac{16}{4} = 4$

$\boxed{a_n = -3 + 4(n-1)}$

$a_n = a_1 + 4(n-1)$

$25 = a_1 + 4(8-1)$

$25 = a_1 + 28 \qquad a_1 = -3$

$\boxed{-3, 1, 5, 9, 13}$

10. Find the 10^{th} term of the arithmetic sequence whose first two terms are 8 and 20.

$d = 12$

$\boxed{a_n = 8 + 12(n-1)}$

$a_{10} = 8 + 12(10-1)$

$a_{10} = 8 + 12(9) \qquad \boxed{a_{10} = 116}$

Arithmetic Series

Find the sum of: $\quad 40+37+34+31+28+25+22$ $\quad 62$

$S_7 = \frac{7}{2}(40+22) = 217$

The ___sum___ of a finite arithmetic sequence with n terms (n^{th} partial sum) can be found by:

$S_n = \frac{n}{2}(a_1 + a_n)$ where n= ___number of terms___ a_1 = ___first term___ and a_n = ___last term___

* not necessarily the first position

Find the sum of the finite arithmetic sequence.

11. Sum of integers from 1 to 35.

$1+2+3+4+\ldots+35$

Arithmetic b/c $d = 1$

35 terms

$a_1 = 1$

$a_{35} = 35$

$S_{35} = \frac{35}{2}(1+35)$

$S_{35} = \boxed{630}$

12. Sum of odd integers from 1 to 57

$1+3+5+\ldots+57 \qquad d = 2$

How many terms?

$a_n = a_1 + d(n-1)$

$57 = 1 + 2(n-1)$

$56 = 2(n-1)$

$28 = n-1$

$29 = n$

$S_{29} = \frac{29}{2}(1+57)$

$S_{29} = \boxed{841}$

13. 50^{th} partial sum of the arithmetic sequence −6, −2, 2, 6,... $\quad d = 4$

$S_{50} = \frac{50}{2}(-6 + a_{50})$

missing

$a_n = a_1 + d(n-1)$

$a_{50} = -6 + 4(50-1)$

$a_{50} = 190$

$S_{50} = \frac{50}{2}(-6+190)$

$S_{50} = \boxed{4600}$

14. Determine the seating capacity of an auditorium with 30 rows of seats if there are 20 seats in the first row, 22 in the second, 24 in the third row, and so on. $\quad 20+22+24+26+\ldots+a_{30}$

$d = 2$

$a_{30} = 20 + 2(30-1)$

$a_{30} = 78$

$S_{30} = \frac{30}{2}(20+78)$

$= \boxed{1470 \text{ seats}}$

15. $\sum_{n=1}^{100}(2+3n)$

Arithmetic rule

$a_1 = 2 + 3(1) = 5$

$a_{100} = 2 + 3(100) = 302$

100 terms in the series.

$S_{100} = \frac{100}{2}(5+302)$

$S_{100} = \boxed{15350}$

16. $\sum_{n=21}^{100}(2+3n)$

Arithmetic rule

$a_{21} = 2 + 3(21) = 65$

$a_{100} = 2 + 3(100) = 302$

80 terms!

$S_{80} = \frac{80}{2}(65 + 302)$

$S_{80} = \boxed{14680}$

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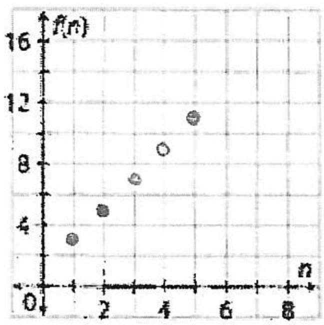
12.1 & 12.2 Practice Worksheet #2

Arithmetic & Geometric Sequences (Explicit Rules and Finding n^{th} terms)

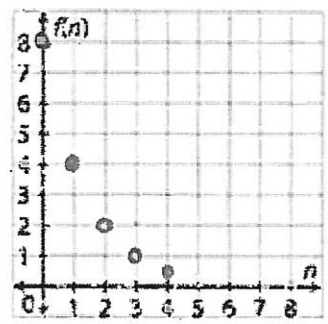
Use the explicit rule to write the first five terms for the sequence. Then graph the sequence.

1. $f(n) = 3 + 2(n-1), \quad 1 \le n \le 5$



$\boxed{3, 5, 7, 9, 11}$

2. $f(n) = 8 \cdot \left(\frac{1}{2}\right)^n, \quad 0 \le n \le 4$



$\boxed{8, 4, 2, 1, \frac{1}{2}}$

Write an explicit rule for each sequence.

3. 4, 7, 10, 13, 16, 19, 22

Arithmetic $d = 3$

$\boxed{f(n) = 4 + 3(n-1),\ 1 \le n \le 7}$

or

$\boxed{f(n) = 4 + 3n,\ 0 \le n \le 6}$

4.

n	1	2	3	4	5	6
f(n)	11	5	−1	−7	−13	−19

Arithmetic $d = -6$

$\boxed{f(n) = 11 - 6(n-1),\ 1 \le n \le 6}$

5.

n	0	1	2	3	4	…
f(n)	100	10	1	0.1	0.01	…

Geometric $r = \frac{1}{10}$ or $r = 0.1$

$\boxed{f(n) = 100\left(\frac{1}{10}\right)^n,\ n \ge 0}$

6. $\frac{1}{4}$, 1, 4, 16, 64, …

Geometric $r = 4$

$\boxed{f(n) = \frac{1}{4}(4)^{n-1},\ n \ge 1}$

or

$\boxed{f(n) = \frac{1}{4}(4)^n,\ n \ge 0}$

7. 9, −6, 4, $-\frac{8}{3}$, …

Geometric $r = -\frac{2}{3}$

$\boxed{f(n) = 9\left(-\frac{2}{3}\right)^{n-1},\ n \ge 1}$

or

$\boxed{f(n) = 9\left(-\frac{2}{3}\right)^n,\ n \ge 0}$

8.

n	0	1	2	3	4
f(n)	−6	1	8	15	22

Arithmetic $d = 7$

$\boxed{f(n) = -6 + 7n,\ 0 \le n \le 4}$

Write an explicit rule and then find the indicated term of the sequence.

9. $\frac{1}{2}, 4, \frac{15}{2}, 11, \frac{29}{2}, \ldots$

Find the 82nd term.

Arithmetic

$d = 3.5$ or $\frac{7}{2}$

$\boxed{f(n) = \frac{1}{2} + \frac{7}{2}(n-1),\ n \ge 1}$

$\boxed{\text{82nd term is 284}}$

$f(82) = \frac{1}{2} + \frac{7}{2}(82-1)$

$= \frac{1}{2} + \frac{7}{2}(81)$

$= 284$

or $f(n) = \frac{1}{2} + \frac{7}{2}n,\ n \ge 0$

$f(81) = \frac{1}{2} + \frac{7}{2}(81)$

OR: $d = \frac{129-87}{12-6} = 7$

10. The sixth term of an arithmetic sequence is 87 and the twelfth term is 129. Find the 120th term.

$f(n) = f(1) + d(n-1),\ n \ge 1$

$f(6) = 87$

$f(12) = 129$

$\boxed{f(n) = 52 + 7(n-1),\ n \ge 1}$

$87 = f(1) + d(6-1)$

$129 = f(1) + d(12-1)$

$87 = f(1) + 5d$

$129 = f(1) + 11d$

$87 - 5d = f(1)$

$129 = 87 - 5d + 11d$

$42 = 6d$

$7 = d$

$f(120) = 52 + 7(120-1)$

$= 885$

$87 - 5(7) = f(1)$

$52 = f(1)$

$\boxed{\text{120th term is 885}}$

11. The second term of a geometric sequence -18 and the fifth term is $\frac{2}{3}$. Find the sixth term.

$f(n) = f(1) \cdot r^{n-1},\ n \ge 1$

$\frac{2}{3} = -18r^3$

$f(2) = -18$

$f(5) = \frac{2}{3}$

$-\frac{1}{27} = r^3$

$-18 = f(1) \cdot r^{2-1}$

$\frac{2}{3} = f(1) \cdot r^{5-1}$

$-\frac{1}{3} = r$

$-18 = f(1) \cdot r$

$\frac{2}{3} = f(1) \cdot r^4$

$\boxed{f(n) = -54\left(-\frac{1}{3}\right)^{n-1},\ n \ge 1}$

$\frac{-18}{r} = f(1)$

$\frac{2}{3} = \frac{-18}{r} \cdot r^4$

$f(1) = \frac{-18}{-1/3} = -54$

$\boxed{\text{6th term is } -\frac{2}{9}}$

12. 3, −9, 27, −81, 243, ...

Find the 20th term.

Geometric

$r = -3$

$\boxed{f(n) = 3(-3)^{n-1},\ n \ge 1}$

$f(20) = 3(-3)^{20-1}$

$= -3{,}486{,}784{,}401$

$\boxed{\text{20th term is } -3{,}486{,}784{,}401}$