PC REVIEW – SOLVING LOG EQS and EXPONENTIAL EQUATIONS USING LOGS APPLICATIONS

Key
2016

A. Solving Equations containing LOG or LN

Strategy 1: CONDENSE. Use if LOGM = LOGN, then M = N

A. $\log x - \log 5 = \log 2 - \log(x-3)$

$\log \frac{x}{5} = \log \frac{2}{x-3}$

$\frac{x}{5} = \frac{2}{x-3}$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\boxed{x = 5}$ $\cancel{x = -2}$

B. $2 \log_a 3 + \log_a (x-4) = \log_a (x+8)$

$\log_a 9(x-4) = \log_a (x+8)$

$9x - 36 = x + 8$

$8x = 44$

$x = \frac{44}{8}$

$\boxed{x = 5.5}$ ✓

Strategy 2: Use the definition of logarithm: $\log_b a = p$ iff $b^p = a$

C. $\log_2(2x+2) + \log_2(x-3) - \log_2(x-5) = 5$

$\log_2 \frac{(2x+2)(x-3)}{(x-5)} = 5$

b a p

$\frac{32}{1} = \frac{(2x^2-4x-6)}{(x-5)}$

$32x - 160 = 2x^2 - 4x - 6$

$16x - 80 = x^2 - 2x - 3$

$0 = x^2 - 18x + 77$

$0 = (x-11)(x-7)$

$\boxed{x = 11 \quad x = 7}$ ✓ ✓

D. $\log_2(-x) = 3 - \log_2(2-x)$

$\log_2(-x) + \log_2(2-x) = 3$

$\log_2 (-x)(2-x) = 3$

$2^3 = -2x + x^2$

$8 = x^2 - 2x$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x = 4 \quad x = -2$

Check

$x = 4$ makes argument negative.

$x = -2$

D2. $6(\ln 3x) = 36$

$\ln 3x = 6$

$e^6 = 3x$

$x = \frac{e^6}{3}$

$x = 134.4763$

B. SOLVING EXPONENTIAL EQUATIONS

x ln 125

$3x\log 5 = (x^2-15)\log 5$

$3x\log 5 = x^2\log 5 - 15\log 5$

Strategy 1: CREATE LIKE BASES

E. $125^x = 5^{x^2-15}$

Skip

$5^{3x} = 5^{x^2-15}$

$3x = x^2 - 15$

$0 = x^2 - 3x - 15$

need QF

$\cancel{0=(x-5)(x+2)}$

$\boxed{\cancel{x=5} \quad \cancel{x=-2}}$

Doesn't factor

Doesn't factor

F. $3^{2x-1} = \frac{1}{9}$

$3^{2x-1} = 3^{-2}$

$2x-1 = -2$

$2x = -1$

$\boxed{x = -1/2}$

Strategy 2: Isolate the base to the exponent. Take the LN or LOG of both sides. Apply the properties of logs to solve.

G. $140 = 7e^{3k} + 28$

Isolate!

$-28 \qquad -28$

$112 = 7e^{3k}$

$\frac{112}{7} = e^{3k}$

$16 = e^{3k}$

$\ln 16 = \ln e^{3k}$

$\ln 16 = 3k$

$k = \ln 16 \div 3$

$k = .9242$

H. $3^{4x-7} = 4^{2x+3}$

LN LN

$(4x-7)\ln 3 = (2x+3)\ln 4$

DISTRIBUTE!

$4x\ln 3 - 7\ln 3 = 2x\ln 4 + 3\ln 4$

collect x-terms on 1 side!

$4x\ln 3 - 2x\ln 4 = 3\ln 4 + 7\ln 3$

Factor out x

$\dfrac{x(4\ln 3 - 2\ln 4)}{4\ln 3 - 2\ln 4} = \dfrac{3\ln 4 + 7\ln 3}{4\ln 3 - 2\ln 4}$

$\boxed{x = 7.3060}$

Strategy 3: FACTOR. Zero Product Property. Then use strategy 2.

I. $e^{4x} + 6e^{2x} + 5 = 0$

like $x^4 + 6x^2 + 5 = 0$

$(x^2+5)(x^2+1) = 0$

so $(e^{2x}+5)(e^{2x}+1) = 0$

$e^{2x} = -5 \qquad e^{2x} = -1$

$2x = \ln(-5) \qquad 2x = \ln(-1)$

can't take ln of neg #!

a > 0 ∴ no soln.

J. $e^{2x} + 3e^x - 4 = 0$

$(e^x+4)(e^x-1) = 0$

$e^x = -4 \qquad e^x = 1$

$x = \ln -4 \qquad x = \ln 1$

$\phi \qquad \boxed{x = 0}$ ✓

It works!

PC REVIEW – SOLVING LOG EQS and EXPONENTIAL EQUATIONS USING LOGS APPLICATIONS

APPLICATION FORMULAS

Compound Interest	$A = P\left(1+\frac{r}{n}\right)^{nt}$	P is the initial investment, r is the annual interest rate, t is the time in years, and n is the number of times per year the interest will be compounded
Continuously Compounded Interest or Exponential Growth	$A = Pe^{rt}$	P is the initial investment, r is the annual interest rate, and t is the time in years
Exponential Growth or Decay	$y = ne^{kt}$ $A = A_0e^{kt}$	
Newton's Law of Cooling	$T = C + (T_0 - C)e^{kt}$	T is the temperature of a heated object at time t, C is the constant temperature of the surrounding medium, and k is a negative constant that is associated with the cooling object.

SOLVE by GRAPHING. Sketch and label your graph in the space provided.

1. The amount of pollutant, measured in parts per million, in Pine Lake can be modeled by the function
$A(t) = 14e^{-0.16t}$, where t is the number of years since a program began to clean up the lake. Approximately how long will it take for the amount of pollutant in Pine Lake to reach 7 parts per million?

$7 = 14e^{-.16t}$

$\frac{1}{2} = e^{-.16t}$

$\text{LN}\ \frac{1}{2} = -.16t$

$t = \text{LN}(1/2) \div -.16$

$t \approx 4.33$ years

SOLVE USING LOGARITHMS. SHOW ALL WORK.

2. Siya plans to invest $500 at 8.25% interest, compounded continuously. How long will it take for her money to triple?

$A = Pe^{rt}$

$A = 500e^{.0825t}$

$1500 = 500e^{.0825t}$

$3 = e^{.0825t}$

$\text{LN}\ 3 = .0825t$

$\ln 3 \div .0825 = t$

13.3 years $= t$

$\rightarrow \therefore$ k is negative

3. A $40,000 car depreciates at a constant rate of 12% per year. In how many years will the car be worth $12,000?

$$A = A_0 e^{kt}$$

$$12000 = 40000 e^{-.12t}$$

$$\frac{12}{40}$$

$$\ln\left(\frac{3}{10}\right) = -.12t$$

$$t = \ln(.3) \div .12$$

t = 10 years

4. Grant Investments Inc. promises to triple your money in 12 years. Assuming continuously compounding of interest, what rate of interest is needed?

Triple $\rightarrow$ 3 times

$\therefore A = 3 \quad A_0 = 1$

$$A = A_0 e^{kt}$$

$$3 = e^{12t}$$

$$\ln 3 = 12t$$

$$\frac{\ln 3}{12} = t \approx .0916$$

$\approx 9.16\%$

5. An organism of a certain type can grow from 30 to 195 organisms in 5 hours. Find *k* for the growth formula.

$$A_0 = A e^{kt}$$

$$\boxed{\begin{aligned} A_0 &= 30 \\ A &= 195 \\ t &= 5 \end{aligned}}$$

$$30 = 195 e^{5k}$$

$$\ln\left(\frac{30}{195}\right) = 5k$$

$$\ln\left(\frac{30}{195}\right) \div 5 = k$$

$$\boxed{k \approx -.3744}$$

6. A substance decomposes radioactively. Its half-life is 32 years. Find the constant k in the decay formula.

$t = 32$ $\quad A_0$ $\quad A = \frac{A_0}{2}$

$A = A_0 e^{kt}$

$\frac{1}{2} = e^{32k}$

$\ln\left(\frac{1}{2}\right) = 32k$

$\ln\left(\frac{1}{2}\right) \div 32 = k$

$\boxed{k = -.0217}$

6b. It is reported that 100 grams of the substance was spilled. Safety standards say that the environment is safe when there are 10 grams or less of the substance. How long will it take for the amount to reach safe levels?

$A = A_0 e^{-.0217t}$

$10 = 100 e^{-.0217t}$

$\text{LN}\left(\frac{1}{10}\right) = -.0217t$

$\boxed{106 \text{ yrs} = t}$

7. A chemist, Dr. Ruge has developed a new chemical that will cure any person's lack of "common sense". The rate of reaction to produce the new chemical called Rugeoxide can be modeled by the equation below, where P is the amount of product formed (grams) and "t" is the time of the reaction (hours). Dr. Ruge has data that shows that 44.5 grams of Rugeoxide is formed in 5.3 hours. The doctor has calculated he needs 2352 grams to cure everyone at CHHS. How long will it take to produce this quantity to save the school?

$P = 5.8e^{kt}$ $\quad t = 5.3$ $\quad P = 44.5$

Step 1 use initial info to find k, create a model

$44.5 = 5.8e^{5.3k}$

$\ln\left(\frac{44.5}{5.8}\right) = 5.3k$

$\boxed{k = .3845}$ $\rightarrow$

Model: $P = 5.8e^{.3845t}$

$2352 = 5.8e^{.3845t}$

$\text{LN}\ \frac{2352}{5.8} = .3845t$

$\boxed{t = 15.62 \text{ hours}}$

8. Carbon 14 has a half life of 2515 years. If the amount of Carbon 14 after 1000 years is 5 grams, what was the original quantity of Carbon 14?

$A_0 = ?$

$t = 1000$

$A = 5$

$K = -.0002756$

$5 = A_0 e^{-.2756}$

$5 = .7591 A_0$

$A_0 = 5 \div .7591 \rightarrow 6.5867$ grams.

Step 1 – half life

$\frac{1}{2} = e^{2515K}$

Find K

$\ln \frac{1}{2} = 2515K$

$K = \ln \frac{1}{2} \div 2515$

$K = -.0002756$

9. A object is heated to 100 °C and is then allowed to cool in a room whose air temperature is 30 °C.

$T_0 = 100 \quad C = 30 \quad T = 80 \quad t = 5$

a) If the temperature of the object is 80 °C after 5 minutes, when will its temperature be 50 °C?

① find K.

$T = C + (T_0 - C)e^{Kt}$

$80 = 30 + (100 - 30)e^{5K}$

$50 = 70e^{5K}$

$\ln\left(\frac{5}{7}\right) = 5K$

$\boxed{K = -.0673}$

model $T = 30 + 70e^{-.0673t}$

$50 = 30 + 70e^{-.0673t}$

$\ln(2/7) \div -.0673 = t$

$\boxed{t \approx 18.6 \text{ yrs}}$

b) Determine the elapsed time before the temperature of the object is 35 °C.

$35 = 30 + 70e^{-.0673t}$

$\ln\left(\frac{5}{70}\right) = -.0673t$

$\boxed{t = 39.2 \text{ minutes}}$

10. The homicide unit arrives at the crime scene to find a body. When they first arrive, it is 10 a.m. and the temperature of the body is 90 degrees. The thermostat shows that the temperature in the room has been steady for the past 12 hours, at 70 degrees. The medical examiner takes another temperature reading in the same location, 1 hour later, at 11 a.m. The temperature at this time is 86 degrees. A witness claims to have seen the victim alive at 8 a.m. that morning. Could the witness be telling the truth? Clearly show all math work done to answer this question ON an SEPARATE SHEET OF PAPER@. Your work has to be admissible in court

10.

10 a.m $\rightarrow$ 90° $\therefore$ $T_0 = 90°$

$C = 70°$

let $t = 0$ represent 10 a.m.

$t = 1$

$T = 86$

$$T = C + (T_0 - C)e^{kt}$$

$$86 = 70 + (90 - 70)e^{k}$$

$$16 = 20e^{k}$$

$$\frac{16}{20} = e^{k}$$

$$\ln\left(\frac{4}{5}\right) = k$$

$$k = -.2231$$

Model - has a T and t

$$T = 70 + 20e^{-.2231t}$$

8 a.m. $\rightarrow$ $t = -2$

$$T = 70 + 20e^{-.2231(-2)}$$

$$T = 101.25°$$

$\therefore$ It is possible that the victim was seen at 8 a.m $\rightarrow$ with a 101.25° fever..