PC Review A

Name: Key

Unit 3: Graphs of Trigonometric Functions

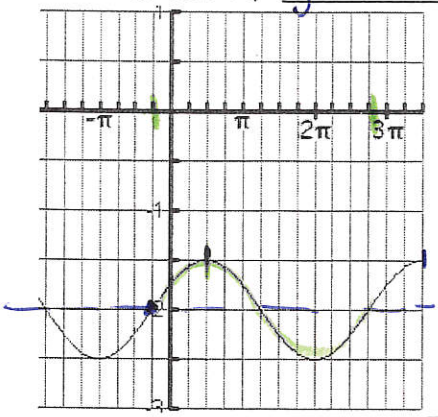
Date: _____ Period: ____

Write an equation of each graph.

1. sine equation $y = \frac{1}{2}\sin\frac{2}{3}\left(x+\frac{\pi}{4}\right)-2$

2. cosine equation _____

Cosine eq: $y = \frac{1}{2}\cos\frac{2}{3}\left(x-\frac{\pi}{2}\right)-2$

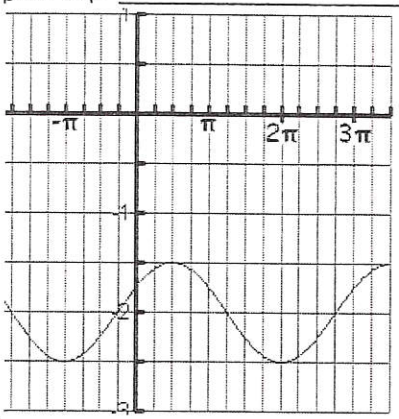


$per = 3\pi = \frac{2\pi}{b}$

$3\pi b = 2\pi$

$b = \frac{2}{3}$

Sine eq: _____



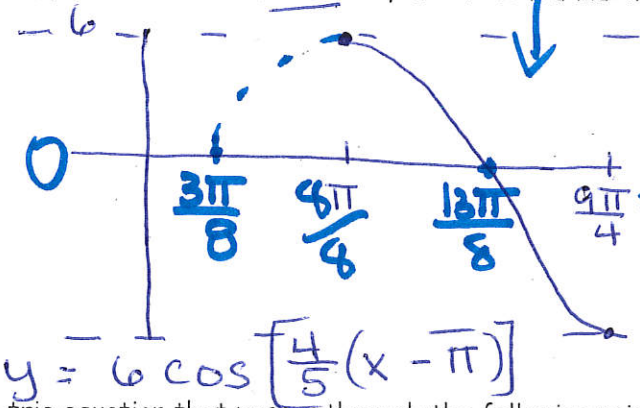
$k = -2$

$|a| = 1/2$

$h = \frac{\pi}{2}$

$y = 6\sin\left[\frac{4}{5}\left(x-\frac{3\pi}{8}\right)\right]$

3. Show all work to Write a sine and a cosine equation with a maximum at $(\pi, 6)$ and a minimum at $\left(\frac{9\pi}{4}, -6\right)$.



$k = 0$

$|a| = 6$

$b = \frac{4}{5}$

$h = \pi$

$\frac{1}{2}per = \frac{9\pi}{4} - \frac{4\pi}{4}$

$\frac{1}{2}per = \frac{5\pi}{4}$

$1\,per = \frac{10\pi}{4} = \frac{5\pi}{2}$

$y = 6\cos\left[\frac{4}{5}(x-\pi)\right]$

$\frac{2\pi}{b} = \frac{5\pi}{2}$ $\quad 5\pi b = 4\pi$

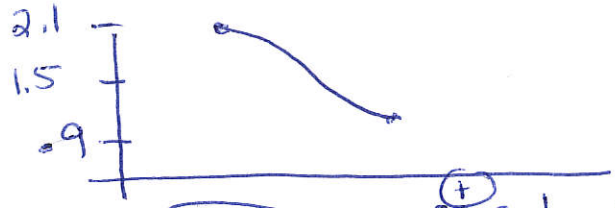
$b = \frac{4}{5}$

4. Find and graph a trig equation that passes through the following points BY HAND. Show all steps.

$per = .8$

x	1.5	1.7	1.9	2.1	2.3		
y	2.1	1.5	.9	1.5	2.1		

max min



cos + h = 1.5

$per = .8 = \frac{2\pi}{b}$

$.8b = 2\pi$

$b = 2\pi \div \frac{8}{10}$

$b = 2\pi \cdot \frac{10}{8} = \frac{5\pi}{2}$

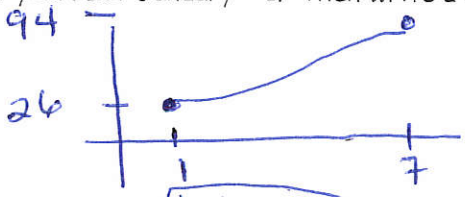
$k = \frac{3}{2} = 1.5 = k$

$|a| = .6$

$y = .6\cos\left[\frac{5\pi}{2}(x-1.5)\right] + 1.5$

5. The lowest average temperature in January in Barcelona, Spain was 26 degrees. The highest average temperature for the year was 94 degrees in July. Calculate the amplitude, phase shift, vertical shift, and period for the past year. Let January = 1. Then write a sinusoidal equation to represent the situation by hand.

(1, 26)
(7, 94)



$y = -34\cos\left[\frac{\pi}{6}(x-1)\right]+60$

$k = \frac{94+26}{2} = 60$ $|a| = 34$ per = 12 so $b = \frac{\pi}{6}$ h Reg cos = 1

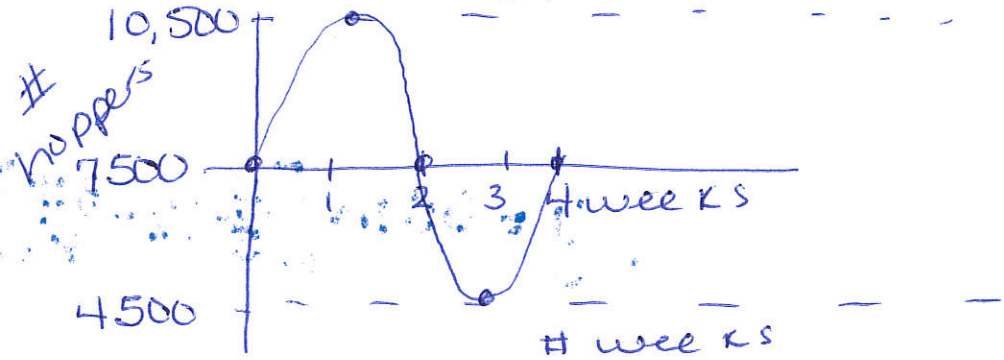
a. Use your model to predict the average temperature in March.

march = 3

$y = -34\cos\left[\frac{\pi}{6}(2)\right]+60 = -34\cos\frac{\pi}{3}+60 = -34\cdot\frac{1}{2}+60 = -17+60 = 43°$

6. The population estimate of grasshoppers after t weeks is given by P(t) = 7500+3000sin[(pi/2)(t)]:

a. Sketch and label the graph



$b = \frac{\pi}{2}$ so per = $2\pi\cdot\frac{2}{\pi} = 4$

b. What was the initial estimate of grasshoppers? 7500

c. Between what numbers does the grasshopper population vary? [4500, 10,500]

d. How many grasshoppers would you expect there to be 4 weeks into the population study?

7500

e. How many grasshoppers would you expect there to be 2 weeks into the study?

7500

f. What is the first time the population will reach 11,000 grasshoppers?

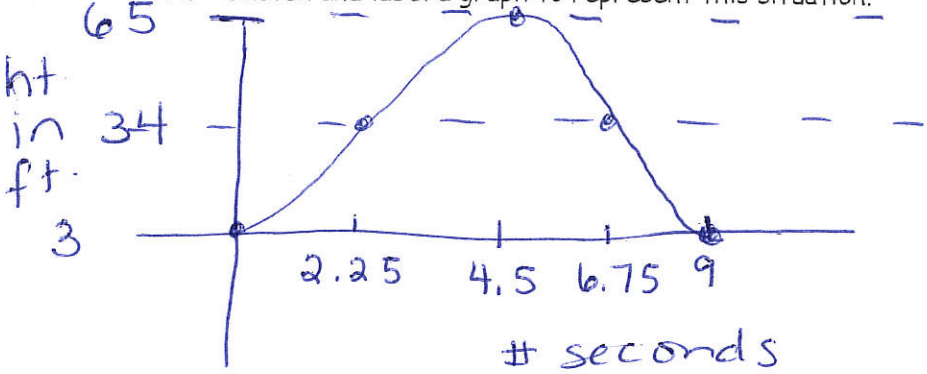
$y_2 = 11{,}000$ Never ☺

g. When is the first time the population will reach 9000 grasshoppers?

$y_2 = 9000$ 33 weeks or $\frac{1}{3}$ of the way through 1st week.

7. Riders board the ferris wheel at its lowest height of 3 feet off the ground. The amusement park advertises that riders will reach a height of 65 feet, and that they will complete 20 full rotations on the wheel during the 3 minute ride.

A. Sketch and label a graph to represent this situation.



$\frac{20 \text{ rev}}{180 \text{ sec}} = \frac{1 \text{ rev}}{x}$

$20x = 180$

$x = 9$ seconds

$k = \frac{3+65}{2} = 34$

$|a| = 31$

$\text{per} = 9 = \frac{2\pi}{b}$

$9b = 2\pi$

$b = \frac{2\pi}{9}$

B. Write a cosine model to represent this situation

$y = -31 \cos \frac{2\pi}{9}[x] + 34$

C. Write a corresponding sine model to represent this situation.

$y = 31 \sin\left[\frac{2\pi}{9}(x - 2.25)\right] + 34$

D. How many seconds after the ride starts will a rider first reach a height of 40 feet?

$y_2 = 40$

2.53 seconds

E. Your friends want to take a picture of you on the ferris wheel. They have calculated that they could get the best show when you are 10 feet off the ground. When should they take the picture?

$y_2 = 10$

On way up: .98 seconds into the ride

On way down: 8 seconds into the ride

F. What is the diameter of the wheel?

$65 - 3 = 62$ feet

9.

Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time t = 0 years. A minimum number, 200 foxes, existed when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

a) Write an equation expressing the number of foxes as a function of time t.

$a = -300$

cos

$h = 2.9$

$\frac{1}{2}$ per $= 5.1 - 2.9$

$= 2.2$

per $= 4.4 = \frac{2\pi}{b}$ $\quad 4.4b = 2\pi$

$b = \frac{20\pi}{44} = \frac{5\pi}{11}$

$y = -300 \cos\left[\frac{5\pi}{11}(x - 2.9)\right] + 500$

b) Predict the population 7 years after records started being kept.

let $x = 7$

$y = -300 \cos\left[\frac{5\pi}{11}(4.1)\right] + 500$

$= 227$ foxes

c) Foxes are declared to be an endangered species when their population drops below 300. Between what two nonnegative values of t were foxes first endangered.

GC

$y_2 = 300$

Between 2.3 and 3.49

No Calculators! SIMPLIFY!

11. $\sin\left(2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\sin\frac{2\pi}{3}$

$\frac{\sqrt{3}}{2}$

12. $\sin\left(\arctan(-1)\right)$

$\sin\frac{-\pi}{4}$

$-\frac{\sqrt{2}}{2}$

13. $\cos\left(\frac{5\pi}{3} - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

$\cos\left(\frac{5\pi}{3} - \frac{\pi}{6}\right)$

$\cos\left(\frac{10\pi}{6} - \frac{\pi}{6}\right)$

$\cos\frac{3\pi}{2} = 0$

14. $\cos\left(\cos^{-1}1 + \sin^{-1}\left(-\frac{1}{2}\right)\right)$

$\cos\left(0 + \frac{-\pi}{6}\right)$

$\cos\frac{-\pi}{6}$

$\frac{\sqrt{3}}{2}$

15. $\tan\left(\frac{11\pi}{4} + \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

$\tan\left(\frac{11\pi}{4} + \frac{-\pi}{4}\right)$

$\tan\frac{5\pi}{2}$

$\tan\frac{\pi}{2}$ = undef.

16. $\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{2}\right)$

$\sin\left(\frac{\pi}{6} + \frac{3\pi}{6}\right)$

$\sin\frac{2\pi}{3}$

$= \frac{\sqrt{3}}{2}$

17. $\cos\left(2\tan^{-1}\sqrt{3}\right)$

$\cos\frac{2\pi}{3}$

$= -\frac{1}{2}$

18. $\cos\left(\arctan\left(\frac{-\sqrt{3}}{3}\right) - \arcsin\left(\frac{-\sqrt{3}}{2}\right)\right)$

$\cos\frac{-\pi}{6} - \frac{-\pi}{3}$

$= \cos\frac{-\pi}{6} + \frac{2\pi}{6}$

$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

19. $\sin\left(\tan^{-1}\sqrt{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

$\sin\left(\frac{\pi}{3} - \frac{-\pi}{6}\right)$

$\sin\left(\frac{2\pi}{6} + \frac{\pi}{6}\right)$

$\sin\frac{\pi}{2} = 1$

10. The average monthly temperature in Savannah is given in the table below.

Month t	Jan. 1	Feb. 2	Mar. 3	Apr. 4	May 5	June 6	July 7	Aug. 8	Sept. 9	Oct. 10	Nov. 11	Dec. 12
Temperature (Fahrenheit)	48.9	51.8	59.2	66	73.5	79.1	81.8	81	76.6	67.3	59.1	51.7

A. Use your calculator to generate a scatterplot and asine model to represent the data. Round to the nearest hundredth.

$y = 16.85 \sin(.49x - 1.92) + 65.33$

B. Use your model to predict the temperature in October. Explain or show your method.

Oct = 10 Calc value x = 10

68°

C. Rewrite in y = asinb(x-h) + k form

$y = 16.85 \sin[.49(x - 3.92)] + 65.33$

D. Rewrite your model as a cosine model.

k = 65.33 Use calc max or min

7.124

$y = 16.85 \cos[.49(x - 7.124)] + 65.33$

E. Generate a model by hand. Then compare the model you created by hand with that created on your calculator. SHOW ALL WORK:

min (1, 48.9)

max (7, 81.8)

$k = \frac{48.9 + 81.8}{2} = 65.35 = k$

$|a| = 81.8 - 65.35 = 16.45$

$\frac{1}{2}$ per = 6

per = 12 $= \frac{2\pi}{b}$

$12b = 2\pi$ $b = \pi/6$

$y = 16.45 \cos\left[\frac{\pi}{6}(x - 7)\right] + 65.35$

neg cos a = −16.45

pos cos a = 16.45

close ∴ h = 7