

Sec 5.1 Verifying Trigonometric Identities

Technique 1 - Changing to Sines and Cosines:

Ex 1: Verify $\csc x - \cos x \cot x = \sin x$

$$\frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x} = \sin x$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} = \sin x \checkmark$$

Ex 2: Verify $(\csc x + \cot x)(1 - \cos x) = \sin x$

$$\csc x - \csc x \cos x + \cot x - \cot x \cos x$$

$$\frac{1}{\sin x} - \frac{\cos x}{\sin} + \frac{\cos x}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x} = \sin x \checkmark$$

Technique 2 - Separating a Single Term Quotient into Two Terms

Ex 3: $\frac{1 + \sin x - \sin^2 x}{\cos x} = \cos x + \tan x$

$$\frac{1 - \sin^2 x}{\cos x} + \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x} + \frac{\sin x}{\cos x}$$

$$\cos x + \tan x = \cos x + \tan x \checkmark$$

Ex 4: $\frac{1 - \sin \theta}{\cos \theta} = \sec \theta - \tan \theta$

$$\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$$

$$\sec \theta - \tan \theta = \sec \theta - \tan \theta \checkmark$$

Technique 3 - Multiplying Numerator and Denominator by the Same Factor:

Ex 5: Verify $\frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta}$

$$\frac{\cos \theta}{1 + \sin \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta}$$

$$\frac{\cos \theta - \cos \theta \sin \theta}{1 - \sin^2 \theta}$$

$$\frac{\cos \theta (1 - \sin \theta)}{\cos^2 \theta}$$

$$\frac{1 - \sin \theta}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta} \checkmark$$

Ex 6: Verify $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

$$\frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$\frac{\sin x (1 + \cos x)}{1 - \cos^2 x}$$

$$\frac{\sin x (1 + \cos x)}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin x} = \frac{1 + \cos x}{\sin x} \checkmark$$

Technique 4 - Rewrite over a Common Denominator:

Ex 7: Verify $\frac{1 + \sin x}{\sin x} + \frac{\cot x - \cos x}{\cos x} = 2 \csc x$

$$\frac{\cos x (1 + \sin x) + \sin x (\cot x - \cos x)}{\sin x \cos x}$$

$$\frac{\cos x + \sin x \cos x + \cos x - \sin x \cos x}{\sin x \cos x}$$

$$\frac{2 \cos x}{\sin x \cos x} = \frac{2}{\sin x} = 2 \csc x$$

Ex 8: Verify $\frac{\sin \theta + \cos \theta}{\cos \theta} + \frac{\cos \theta - \sin \theta}{\sin \theta} = \csc \theta \sec \theta$

$$\frac{\sin \theta (\sin \theta + \cos \theta) + \cos \theta (\cos \theta - \sin \theta)}{\sin \theta \cos \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{1}{\sin \theta \cos \theta} = \csc \theta \sec \theta$$

Technique 5 - Using Factoring:

Ex 9: Verify $\sin x - \sin x \cos^2 x = \sin^3 x$

$$\sin x (1 - \cos^2 x)$$

$$\sin x (\sin^2 x)$$

$$\sin^3 x = \sin^3 x \checkmark$$

Ex 10: Verify $\frac{\sin^2 x - 1}{\tan x \sin x - \tan x} = \cot x + \cos x$

$$\frac{(\sin x + 1)(\sin x - 1)}{\tan x (\sin x - 1)}$$

$$\frac{\sin x}{\tan x} + \frac{1}{\tan x}$$

$$\frac{\sin x \cdot \cos x}{1 \cdot \sin x} + \cot x$$

$$\cos x + \cot x$$