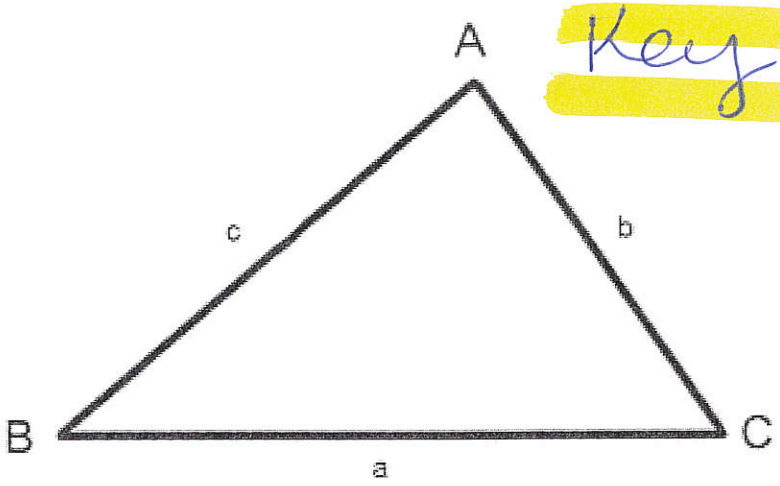
# Pre-Calc U 8 Lesson
# Law of Cosines

Key




Law of Sines works for ASA and SAA

Law of Sines, checking for the Ambiguous Case works for SSA

**So what about SAS and SSS?**

Law of Cosines! Derived from the Pythagorean Theorem

$$c^2 = a^2 + b^2 - 2abCOSC$$

1. What do we know in the triangle below? Do we have SSS, SAS, SSA, ASA, AAS? SAS
2. Law of Sines or Law of Cosines? LOC

Determine the missing side in the triangle below.

$c^2 = (4.8)^2 + (2.7)^2 - 2(4.8)(2.7)\cos 115$

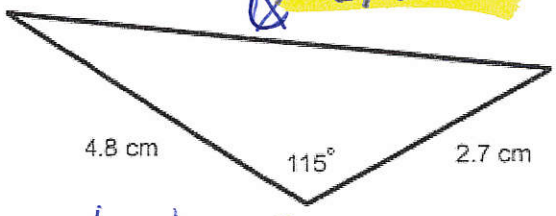
* Write out formula.

Enter expression into calc

$c^2 = 41.2843$ → Keep to 4 decimal places

$c = 6.4253$

& 6.42




Does answer make sense? Opposite lgst Δ

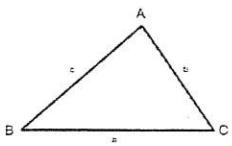
How is the unknown side in the Law of Cosines related to the given angle? opposite it

How is the given angle related to the two given sides? included Δ

# Pre-Calc U 8 Lesson 3
# Law of Cosines

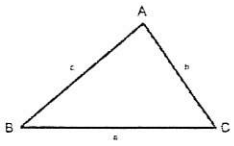
Given triangle ABC, decide whether the Law of Sines or the Law of Cosines should be used to begin solving the triangle.

a) a, b, C



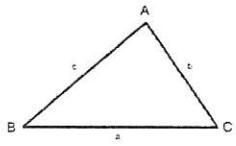
SAS → LOC

b) A, B, c



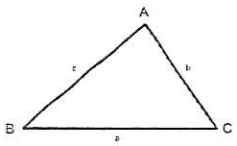
ASA → LOS

c) A, C, c



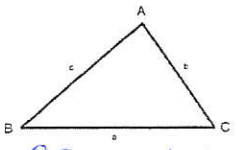
AAS → LOS

d) a, b, A



SSA → Sin; Amb. Case

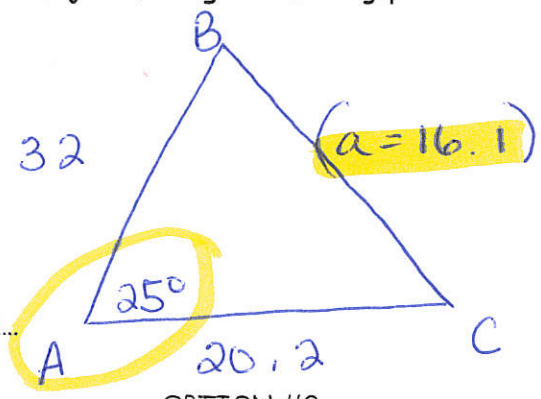
e) a, b, c



SSS → LOC

**What if we're being asked to SOLVE a triangle instead of just finding one missing piece?**

Ex 1: Solve $\triangle ABC$ if b = 20.2, c = 32, and $\measuredangle A = 25°$




1. Sketch and label
2. What side-angle pattern do you see?
   SAS →
3. Which rule should be used first?
   LOC
4. Which angle should we solve for first? HMMMM...

OPTION #1

OPTION #2

1st find side a to create a base ratio.

$a^2 = 20.2^2 + 32^2 - 2(20.2)(32)\cos 25 \approx 16.1$

① Half class → ∠C then ∠B

$\frac{\sin 25}{16.1} = \frac{\sin C}{32}$

∠C = 57.1 }
∠B = 97.9 } Does this make sense? NO

Why did this happen?

∠C - obtuse → CALC doesn't give you obtuse ∠ w/ LOS

How could we avoid this?

② ½ class ∠B then ∠C

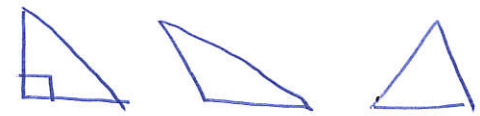
$\frac{\sin 25}{16.1} = \frac{\sin B}{20.2}$

∠B = 32.02 }
∠C = 122.98 } Does this make sense?

SO (see bottom of page 640 as needed)

**Important:** Always find the smaller ∠ first w/ Law of Sines to avoid Ambiguous Case (it will be acute)

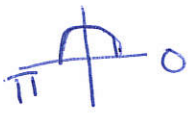
# Pre-Calc U 8 Lesson 3
# Law of Cosines



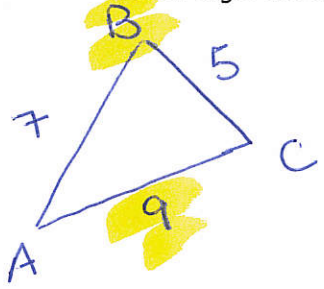

Ex 2: Solve $\triangle ABC$ if a = 5, b = 9, and c = 7.

Which angle should we solve for first this time?

SSS → need to find an ∠. $\cos^{-1}$ can work in $Q_1$ and $Q_2$




So $\cos^{-1}$ can give you an obtuse ∠. ∴ Start with finding LARGEST ∠ → opp. longest side, (Find B 1st) using LOC.




$9^2 = 5^2 + 7^2 - 2(5)(7)\cos B$ Careful!

$81 = 25 + 49 - 70\cos B$

$81 = 74 - 70\cos B \rightarrow 7 = -70\cos B$

$-.10 = \cos B$ (∴ ∠ will be obtuse)

$B = 95.74$

} can then find A or C using LOS → both acute

**Important:** With SSS, first use LOC to solve for LARGEST ANGLE, since LOC can give you an ∠ > 90. Then use LOS to find others in any order.

**Heron's Formula:** find the area of a triangle based on only the 3 given sides

$s = \frac{a+b+c}{2}$ S = semiperimeter

$A = \sqrt{s(s-a)\times(s-b)\times(s-c)}$

$\frac{\sin 95.74}{9} = \frac{\sin A}{5} = \frac{\sin C}{7}$

$\angle A = 33.56°$

$\angle C = 50.7°$

makes sense? ✓

Ex: Find the area of the given triangle below. Round your answer to the nearest hundredth.

a. In △ABC, a = 47, b = 53, and c = 44

$S = \frac{20 + 30 + 40}{2} = 45$

$A_\Delta = \sqrt{45(45-20)(45-30)(45-40}$

$A_\Delta = 290.47\ ft^2$

Area △

$A_\Delta = \frac{1}{2}bh$

$A_\Delta = \frac{1}{2}bc\sin A$

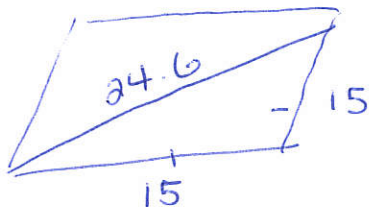
$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

# Pre-Calc U 8 Lesson 3
## Law of Cosines

$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)\times(s-b)\times(s-c)}$$

Ex: The side of a rhombus is 15 cm long, and the length of its longer diagonal is 24.6 cm. Find the area of the rhombus.




$S = 27.3$

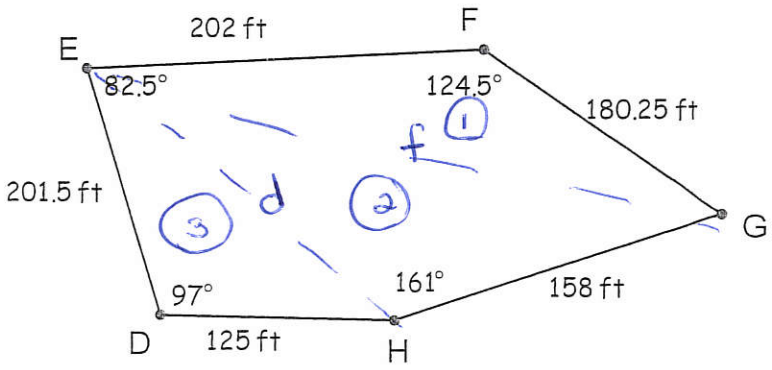
$A = 211.2 cm^2$

$$S = \frac{15+15+24.6}{2} = \frac{54.6}{2} = 27.3$$

$$A_\Delta = \sqrt{27.3(27.3-15)(27.3-15)(27.3-24.6}$$ $\times 2\Delta s$

$= 211.2 cm^2$

Ex: Find the area of the given polygon. Round your answers to the nearest hundredth.




$d^2 = 201.5^2 + 125^2 - 2(201.5)(125)\cos 97$

$d = 249.73$

$f^2 = 202^2 + 180.25^2 - 2(202)(180.25)\cos 124.5$

$f = 338.44$

$\Delta 1$

$A_{\Delta 1} = \frac{1}{2}(202)(180.25)\sin 124.5$

$= 15003.42 ft^2$

$\Delta 3 = \frac{1}{2}(201.5)(125)\sin 97$

$= 12499.88 ft^2$

$\Delta 2 = \sqrt{s(s-a)(s-b)(s-c)}$

$s = 373.085$ $= 18,518.58 ft^2$

Total

$46,021.88 ft^2$