Notes Section 3.3 – Properties of Logarithms

Key

Common Logarithms: base 10

Ex: $\log 100 = \log_{10} 100 = 2$

Natural Logarithms: $\ln \Rightarrow \log_e$

Ex:

Properties for Expanding Logarithmic Expressions	
Product Rule	$\log_b(MN) =$ $\log_b m + \log_b N$
Quotient Rule	$\log_b\left(\frac{M}{N}\right) =$ $\frac{\log_b m}{\log_b N}$
Power Rule	$\log_b(M)^p =$ $p \log_b N$

Use the product rule to expand each logarithmic expression.

1. $\log_4(7 \cdot 5)$

$\log_4 7 + \log_4 5$

2. $\log(10x)$

$\log 10 + \log x$

$1 + \log x$

3. $\log_6(xy)$

$\log_6 x + \log_6 y$

Use the quotient rule to expand each logarithmic expression:

4. $\log_7\left(\frac{19}{x}\right)$

$\log_7 19 - \log_7 x$

5. $\ln\left(\frac{e^3}{7}\right)$

$\ln e^3 - \ln 7$

$\log_e e^3 - \ln 7$

$3 - \ln 7$

6. $\log_8\left(\frac{23}{x}\right)$

$\log_8 23 - \log_8 x$

Use the power rule to expand each logarithmic expression.

7. $\log_5 7^4$

$4 \log_5 7$

8. $\ln\sqrt{x}$

$\ln x^{\frac{1}{2}}$

$\frac{1}{2} \ln x$

9. $\log(4x)^5$

$5 \log(4x)$

$5 \log 4 + 5 \log x$

Use logarithmic properties to expand each expression as much as possible.

10. $\log_b(x^2\sqrt{y})$

$\log_b(x^2 y^{\frac{1}{2}})$

$\log_b x^2 + \log_b y^{\frac{1}{2}}$

$2\log_b x + \frac{1}{2}\log_b y$

11. $\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right)$

$\log_6 \frac{x^{\frac{1}{3}}}{36y^4}$

$\log_6 x^{\frac{1}{3}} - \log_6 36y^4$

$\frac{1}{3}\log_6 x - [\log_6 36 + 4\log_6 y]$

$\frac{1}{3}\log_6 x - [2 + 4\log_6 y]$

12. $\log_5\left(\frac{\sqrt{x}}{25y^3}\right)$

Write as a single logarithm.

condense

13. $\log_4 2 + \log_4 32$

$\log_4(2 \cdot 32)$

$\log_4 64$

$= 3$

14. $\log(4x-3) - \log x$

$\log\left(\frac{4x-3}{x}\right)$

15. $\frac{1}{3}\log x + 4\log(x-1)$

$= \frac{1}{3}\log_6 x - 4\log_6 y - 2$

$\log x^2 + \log(x-1)$

$\log\left[x^{\frac{1}{2}}(x-1)^4\right]$

* Rewrite

$- [\_\_ + \_\_]$

16. $3\ln(x+7) - \ln x$

$\ln\left[\frac{(x+7)^3}{x}\right]$

17. $4\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y$

$\log_b x^4 - [\log_b 6^2 + \log_b y^{\frac{1}{2}}]$

$\log_b\left[\frac{x^4}{36\sqrt{y}}\right]$

18. $2\log(x-3) - \log x$

$\log\left[\frac{(x-3)^2}{x}\right]$

17.

Change of Base Property

$$\log_b M = \frac{\log M}{\log b} \qquad \log_b M = \frac{\ln M}{\ln b}$$

CALCULATOR

Evaluate each expression using either common logarithms or natural logarithms.

19. $\log_5 140$

$\log_{10} \frac{\log 140}{\log 5}$

$\log_5 140 = \frac{\log 140}{\log 5} \approx 3.07$

20. $\log_7 6$

$\log_7 6 = \frac{\log 6}{\log 7}$

21. $\log_2 15$

$\frac{\log 15}{\log 2}$

HW: p. 407 #'s 1-25 (e.o.o) and 41-65 (e.o.o)

Honors Algebra 2 7.5 Notes
Apply Properties of Logarithms

Name Key

Evaluate the following logarithms:

1. $\log_3 3$ → 1	4. $\log_3 1$ → 0
2. $\log_5 5$ → 1	5. $\log_5 1$ → 0
3. $\log 10$ → 1	6. $\log 1$ → 0
$\log_b b =$ 1	$\log_b 1 =$ 0

Properties of Logarithms

Condensed Form:	Expanded Form:
$\log_b m \cdot n$	$\log_b m + \log_b n$
$\log_b \frac{m}{n}$	$\log_b m - \log_b n$
$\log_b m^p$	$p \cdot \log_b m$

Expand the following expressions:

7. $\log_3(6 \cdot 2)$ → $\log_3 6 + \log_3 2$

8. $\log_7 3x^2$ → $\log_7 3 + \log_7 x^2$ → $\log_7 3 + 2\log_7 x$

9. $\log_3 3x^4$ → $\log_3 3 + \log_3 x^4$ → $\log_3 3 + 4\log_3 x$ → $1 + 4\log_3 x$

10. $\log_6 \frac{5x^3}{y}$ → $\log_6 5x^3 - \log_6 y$ → $\log_6 5 + \log_6 x^3 - \log_6 y$ → $\log_6 5 + 3\log_6 x - \log_6 y$

11. $\log\sqrt[5]{2x} \rightarrow \log(2x)^{1/5}$ → $\frac{1}{5}\log(2x)$ → $\frac{1}{5}(\log 2 + \log x)$

12. $\log_2\sqrt{x} \rightarrow \log_2 x^{1/2}$ → $\frac{1}{2}(\log_2 x)$

Condense the following expressions:

13. $\log 9 + 3\log 2 - \log 3$

$\log 9 + \log 2^3 - \log 3$

$\log (9 \cdot 8 \div 3)$

$\log 24$

14. $\ln 4 + 3\ln 3 - \ln 12$

$\ln 4 + \ln 3^3 - \ln 12$

$\ln (4 \cdot 27 \div 12)$

$\ln 9$

15. $\frac{1}{2}\log_3 64 + \log_3 x$

$\log_3 64^{\frac{1}{2}} + \log_3 x$

$\log_3 8 + \log_3 x$

$\log_3 8x$

16. $2\ln 8 - \ln 4 - \ln 16$

$\ln 8^2 - \ln 4 - \ln 16$

$\ln 64 - \ln 4 - \ln 16$

$\ln (64 \div 4 \div 16)$

$\ln 1$

0

17. $4\log x - 6\log 2$

$\log x^4 - \log 2^6$

$\log (x^4 \div 64)$

$\log \frac{x^4}{64}$

18. $2(\log_3 12 - \log_3 3) + \frac{1}{3}\log_3 8$

$2(\log_3 (12 \div 3)) + \log_3 8^{1/3}$

$2\log_3 4 + \log_3 2$

$\log_3 4^2 + \log_3 2$

$\log_3 (16 \cdot 2)$

$\log_3 32$

For the following examples use $\log_3 2 = .6310$ and $\log_3 7 = 1.7712$ to evaluate the logarithm using the properties of logarithms. Do not use a calculator!

19. $\log_3 4$

$\log_3 (2^2)$

$2\log_3 2$

$2(.6310)$

1.2620

20. $\log_3 \frac{7}{2}$

$\log_3 7 - \log_3 2$

$1.7712 - .6310$

1.1402

21. $\log_3 108$

$\log_3 (2^2 \cdot 3^3)$

$\log_3 2^2 + \log_3 3^3$

$2(\log_3 2) + 3\log_3 3$

$2(.6310) + 3(1)$

$1.2620 + 3$

4.2620