MORE Applications of Rational Functions

1) The concentration C of a certain drug in a patient's bloodstream t minutes after injection is given by

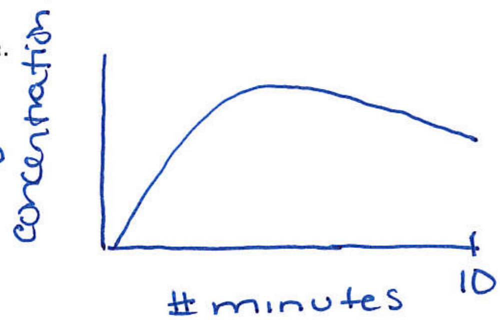
$$C(t) = \frac{30t}{(t^2+35)}$$

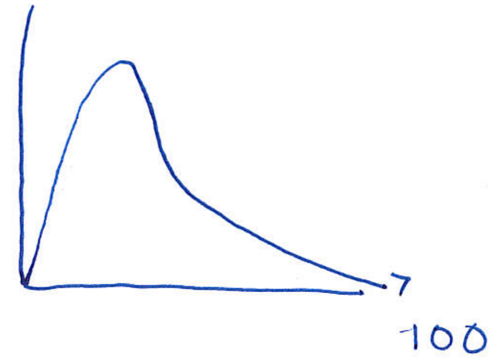
a) Graph this function.

window

x max ~~10~~ 100

y max 4





b) Determine the time at which the concentration is the highest.

Calc max: (5.92, 2.54)

The concentration is highest 5.92 minutes after injection.

c) What happens to the concentration of the drug as the time increases?

As time increases, the concentration approaches zero.

d) What is (are) the asymptote(s) for this function. Explain the meaning of the asymptote(s).

H.A. BOBO $y = 0$

The concentration approaches zero.

V.A. none (imaginary)

e) What is the domain and range of this situation? Explain.

Domain: set of x's $[0, \infty)$

y's $[0, 5.92]$

2) The ***average cost*** function is defined as: $$\overline{C}(x) = \frac{C(x)}{x}$$

The cost function C (in thousands of dollars) for printing x textbooks (in thousands of units) was found to be $C(x) = 0.25x^3 + 0.58x^2 + 9x + 78.5$

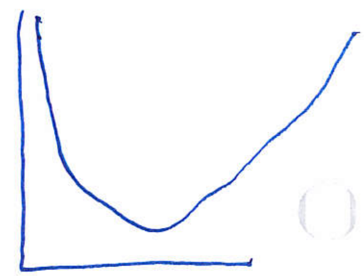
a) Find the ***average cost*** function for printing the textbooks.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{.25x^3 + .58x^2 + 9x + 78.5}{x}$$

b) Graph the ***average cost*** function.

cost

$Y_1 = (\qquad\qquad) \div x$



Average cost = cost for one.

window xmin 0 ymin 0 # books
xmax 30 ymax 100 in thousands

c) What is the average cost of printing 12 thousand textbooks per week?

12,000 → x = 12 Calc Value x = 12

58.501667

so $ 58.50 per book

d) Find the number of textbooks that should be printed to minimize average cost. What is the minimum average cost?

Calc min 5.0344203, 33.84897

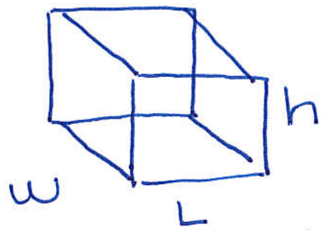
$ 33.85/book

when 5035 books are produced.

3) The United States Post Office has contacted you to design a closed box with a square base that has a volume of 5,000 cubic inches.

a) Draw a picture of the situation

$V = w \cdot L \cdot h$



$w = L$

so
$V = w^2 h$

$5000 = w^2 h \quad \therefore h = \dfrac{5000}{w^2}$

b) Does the height have to be the same dimension as a side of the square base? No.

c) Find a function for the surface area of the box

$SA_{box} = 2LW + 2WH + 2LH$

So, since $w = L$

$= 2w^2 + 2wh + 2wh$

$= 2w^2 + 4wh$

d) What are the dimensions of the box that minimize the surface area?

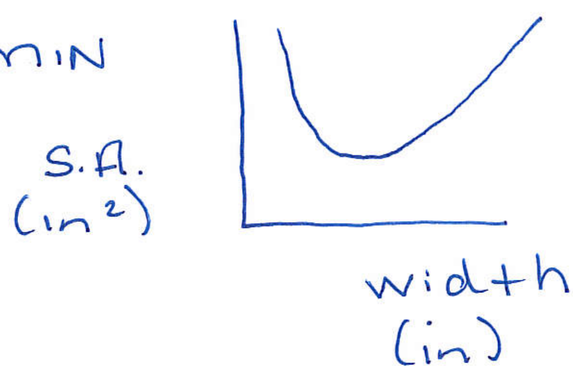
since $h = \dfrac{5000}{w^2}$ $\quad SA = 2w^2 + 4w \cdot \dfrac{5000}{w^2}$

Let x max $= 40$
y max $= 3000$

$SA = 2w^2 + \dfrac{20{,}000}{w}$

e) Why would the post office want to minimize the surface area of the box? - $\boxed{\text{COST}}$

CALC MIN



min $x = 17.1$
$y = 1754.41$

So $w = 17.1$ in
$L = 17.1$ in

$h = \dfrac{5000}{(17.1)^2} = 17.3$ in

<u>17.1 in</u>

$17.1^3 = 5000$

$V = 5058 \ldots$

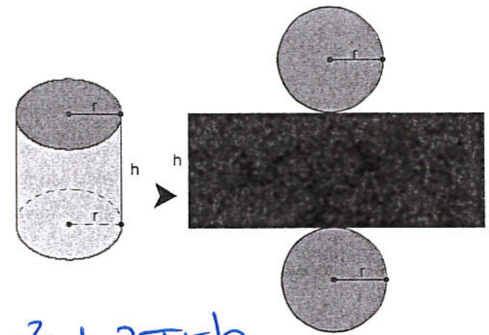
4) A company manufactures aluminum cans in the shape of a cylinder with a capacity of 250 cubic centimeters. The top and bottom of the can are made of an alloy that costs 0.04 cents per square centimeter. The sides of the can are made of material that costs 0.03cents per square centimeter.

a) Express the cost of material for the can as a function of the radius r of the can

$V = \pi r^2 h$

$250 = \pi r^2 h$

Solve for one variable



$\frac{250}{\pi r^2} = h$

$S.A_{cyl} = 2\pi r^2 + 2\pi r h$

$C = .04(2\pi r^2) + .03(2\pi r h)$

Replace h to create eq. in ~~one~~ 2 variables

$C = .08\pi r^2 + .06\cancel{\pi} r \cdot \left(\frac{250}{\cancel{\pi} r^2}\right)$

$\boxed{C = .08\pi r^2 + \frac{15}{r}}$

b) Find the least cost

X = radius

X min = 0

X max = 20

Y = cost

Ymin = 0

Ymax = 40

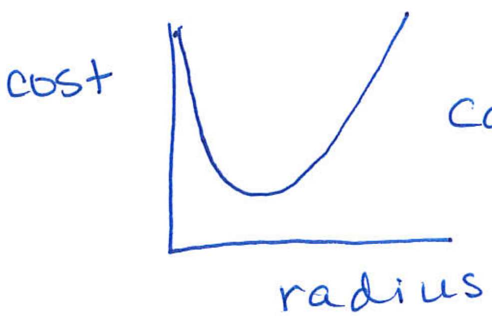
$C = \frac{.08\pi r^3 + 15}{r}$

BOTNO → no H.A.

V.A. $r \neq 0$

$r = 0$

(no can!)



calc min

(3.1, 7.25)

$\boxed{r = 3.1 \text{ in}}$, cost = $\boxed{\$7.25}$

$h = 250/\pi r^2 = \boxed{1.415 \text{ in}}$

5. The junior class is renting a laser tag facility with a capacity of 325 people. The cost of the facility is \$1200. The party must have 13 adult chaperones.

a. If every student who attends shares the facility cost equally, what function models the cost per student C with respect to the number of students, n, who attend? What is the domain of the function? How many students must attend to make the cost per student no more than \$7.50?

n = # students

$\bar{C}$ = cost per student

people < 325

n + 13 < 325

C < 7.50

$\bar{C}(n) = \frac{1200}{n}$

Domain

$0 < n \leq 312$

Domain

$(0, 312]$

use G.C. or solve

$\frac{1200}{n} \leq 7.50$

on G.C.

$y_1 = 1200/n$ $y_2 = 7.50$

$\boxed{\text{Atleast } 160}$

b. The class wants to promote the event by giving away 30 spots to students in a drawing. How does the model change? Now how many paying students must attend so the cost for each is no more than \$7.50?

$\boxed{160 - 312}$

$\frac{1200}{n} - 7.50 \leq 0$

$\frac{1200 - 7.50n}{n} \leq 0$

$n \neq 0$

$1200 - 7.50n = 0$

$-7.50n = 1200$

$n = 160$

Subtract 30 spots, In addition to the 13 for chaperones

$325 - (30 + 13) = 282$

∴ domain is reduced to $(0, 282]$

Still, atleast 160 students must attend, but no more than 282 may attend, thereby ~~l~~ing # of available spots.

XXX + + + + + + − − − −

0 160

n = 180 $\frac{1200 - 1350}{180}$ $\frac{neg}{pos}$ = neg

Test

n = 20 $\frac{1200 - 7.50(20)}{20} = \frac{1200 - 150}{20} = \frac{1050}{20} = 52.5.$

6. A rare species of insect was discovered in the rain forest of Costa Rica. Environmentalists transplant the insect into a protected area. The population of the insect t months after being transplanted if given by the function:

$$P(t)=\frac{45(1+.6t)}{(3+.02t)}$$

A. How many insects were transplanted? (15)

time 0 → t = 0

$P(t) = \frac{45}{3} = 15$ insects

B. When will there be 549 of these insects in the protected area? (100 months later)

$y_1 = 549 = \frac{45(1+.6t)}{(3+.02t)} = y_2$

Calc Intersect
or algebra

$549(3+.02t) = 45 + 27t$

$1647 + 10.98t = 45 + 27t$

$1602 = 16.02t$

$100 = t$

7. The concentration, C, in mg/dll, of a certain antibiotic in a patient's bloodstream is given by the function

Where t is the tine in hours after taking the antibiotic.

$$f(t)=\frac{50t}{t^2+25}$$

A. What is the concentration of the med in the bloodstream 4 hours after taking the med? (4.9 mg/dll)

$f(4) = \frac{50(4)}{16+25} = \frac{200}{41} \approx 4.878$ mg/dll.

B. In order for the antibiotic to be effective, 4 or more mg/dll must be present in the bloodstream. Therefore, when would you have to take the antibiotic again? (10 hrs)

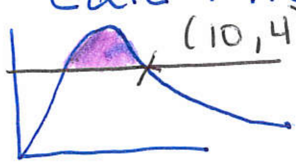
$y_1 = \frac{50t}{t^2+25}$

$y_2 = 4$

Calc intersect

(10, 4)

4



10 hrs later

or

$\frac{50t}{t^2+25} > 4$

$\frac{50t}{t^2+25} - 4 > 0$

$\frac{50t - 4t^2 - 100}{t^2+25} > 0$

$\frac{-4t^2 + 50t - 100}{\text{den}} > 0$

$\frac{2t^2 - 25t + 50}{t^2+25} < 0$

$(2t\quad)(t\quad)$