Chk. # 14.

Name: ______________________ Block: __________

Pre-Calculus Midterm Exam Review 2018

Suggested list of review concepts which is not limited to the following:

Key

* p. 15 # 14 change

Unit 1 Functions

- Functions and their graphs: domain and range, increasing/decreasing, positive/negative/constant, odd/even
- Composition of Functions
- Transformations
- Inverse functions

Unit 2 Exponential and Logarithmic Functions

- Graphing exponential and log functions
- Properties of exponential and logarithmic graphs
- Re-writing exponential and logarithmic expressions
- Logarithmic Properties
- Solving exponential and logarithmic equations
- Growth and Decay/Interest/Newton's Law of Cooling

Unit 3 Rational Functions

- Factoring, including difference and sum of cubes
- Simplifying Rational Expressions
- Adding/Subtracting/Multiplying/Dividing Rational Expressions
- Solving Rational Equations
- Solving Rational Inequalities
- Graphing Rational Functions
- *Problem solving with Rational Functions – Average Cost, Max/Min, Area, Volume, Surface Area

Unit 4 Sequences and Series

- Arithmetic Sequences and Series
- Geometric Sequences and Series
- Applications

Review all reviews and assessments from Semester 1 as well.

You are allowed one side of a 3 x 5 index card!

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Unit 1 Functions

1) Using the standard form of a function $y = a\,f\left[b(x-c)\right] + d$

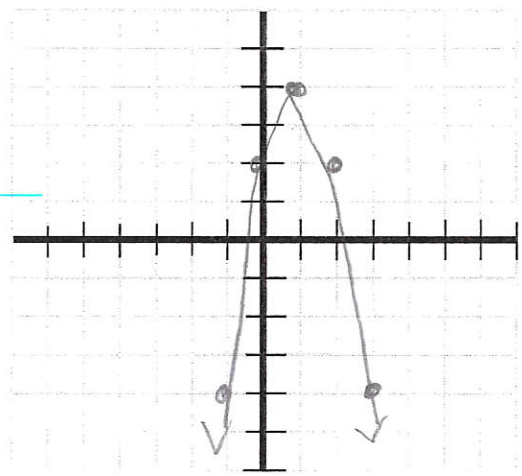
What does each variable control and how? Write in the correct order in which you'd perform the translations and/or indicate how each variable impacts the x & y values of the parent function:

b → $\frac{x}{b}$ if $b < 0$ → reflection over y axis
c → horizontal shift, c units right
a → vertical stretch; if $a < 0$, reflect over x
d → vertical shift, d units up

Directions: Name the parent function, and then describe the translation that will occur in words or as algebraic expressions. Graph! You may use an X/Y table.

2) $f(x) = -2(x-1)^2 + 4$

b = 1
c = 1 RT x+1
a : (x+1, ay) stetch & reflect over x
d (x+1, -2y +4) up 4



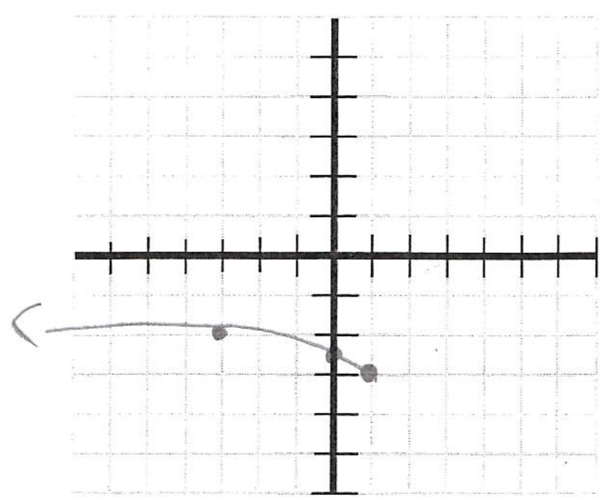
$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

x'	y'
-1	-4
0	2
1	4
2	2
3	-4

3) $f(x) = \frac{1}{2}\sqrt{-(x-1)} - 3$ $x \le 1$

(-x, y)
(-x + 1, y)
(-x + 1, ½y - 3)



x	$y = \sqrt{x}$
0	0
1	1
4	2
9	3

x'	y'
1	-3
0	-2.5
-3	-2
-8	-−1.5

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Directions: Now, word backwards. I'll give you the translation, you write the function.

4) $f(x)=|x|$

Compress the x values by dividing the x values by 2. → $b = 2$

Horizontal shift 3 left. $c = -3$

Stretch the y values by factor of 5. $a = 5$

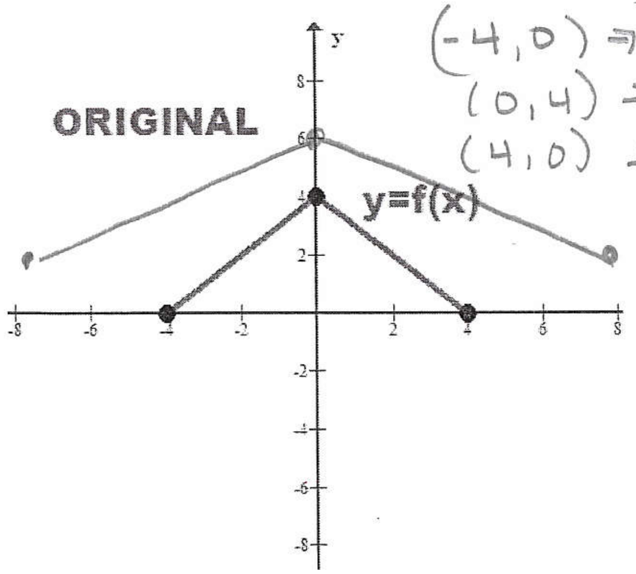
Reflect over the x. $d = -6$

Translate 6 units down. $f(x) = -5|2(x+3)| - 6$

5) Think of f(x) as the parent function. Use the graph of f(x) to graph g(x)

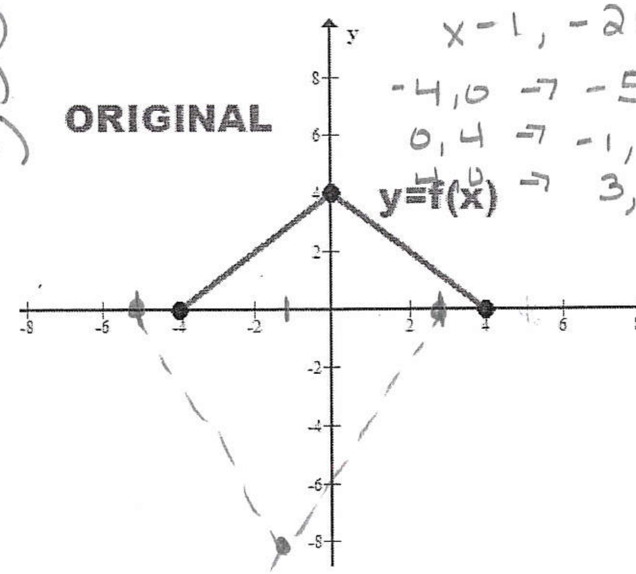
a) $g(x) = f\left(\frac{1}{2}x\right)+2$ → $\left(2x, y+2\right)$ $\frac{x}{b}$

$(-4,0) \rightarrow (-8,2)$
$(0,4) \rightarrow (0,6)$
$(4,0) \rightarrow (8,2)$



b) h(x) = -2f (x + 1) $a = -2$ $c = -1$

$x-1, -2y$
$-4,0 \rightarrow -5,0$
$0,4 \rightarrow -1,-8$
$4,0 \rightarrow 3,0$



We will not have any b's other than -1.

Given the parent function and translation(s), write the function.

6) $f(x)=\sqrt{x}$

Reflect over the y axis. → b is neg

Vertical stretch by a factor of 2. $a = 2$

Translated 3 units down. $d = -3$ $f(x) = 2\sqrt{-(x)} - 3$

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To find the equation of the inverse I. . .

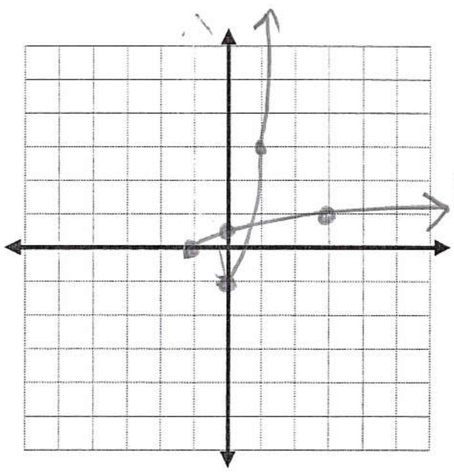
1. Interchange x and y
2. Solve for y
3. Rename y as $f^{-1}(x)$

For the inverse to be a function, the original graph must pass the HLT.
If it does not, we must restrict the domain/range.

7a) Is this function one-to-one? $f(x) = 4x^2 - 1$

One to one if passes HLT → no, parabola ∴ restrict domain so that it passes HLT

graph as $x \geq 0$



b) Generate the equation of the inverse of this function, restricting the domain so that the inverse is a function. Graph the original function and its inverse, clearly labeling both.

$y = 4x^2 - 1$

$x = 4y^2 - 1$

$x + 1 = 4y^2$

$\frac{x+1}{4} = y^2$

$y = \pm\sqrt{\frac{x+1}{4}}$

$f^{-1}(x) = +\frac{1}{2}\sqrt{x+1}$

$x \geq -1$

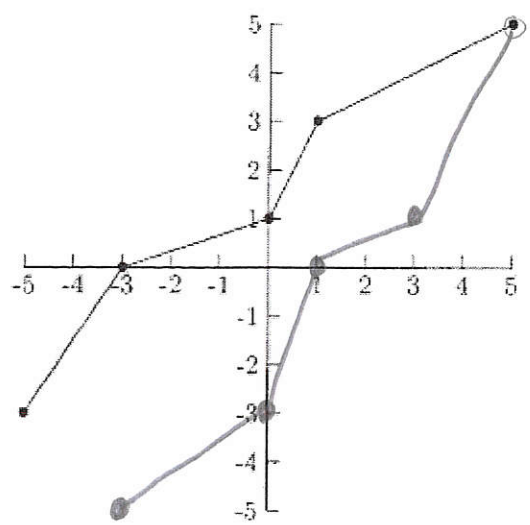
8a) Sketch the graph of the inverse of the function graphed below.

TEST FOR ONE-TO –ONE: HLT

x	y
-5	-3
-3	0
0	1
1	3
5	5

Inverse

x^{-1}	y^{-1}
-3	-5
0	-3
1	0
3	1
5	5



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linear! $f(x) = \frac{-3x}{2} + \frac{5}{2}$

8b) Find the inverse of $f(x) = \frac{5-3x}{2}$. Is the function one-to-one? Is the inverse a function?

Yes → Linear ; yes!

$y = \frac{5-3x}{2}$

$x = \frac{5-3y}{2}$

$2x = 5 - 3y$

$2x - 5 = -3y$

$\frac{+2x-5}{-3} = y$

$y = -\frac{2}{3}x + \frac{5}{3}$

$f^{-1}(x) = -\frac{2}{3}x + \frac{5}{3}$

Use the composition of functions to verify that functions are inverses by showing that f(g(x)) = x and g(f(x)) = x

Find $f(g(x))$ *and* $g(f(x))$ and verify whether the pair of functions given below are inverses of each other **using function composition**.

9) $f(x) = 6x + 7$ and $g(x) = \frac{x-7}{6}$.

$f(g(x)) = 6\left(\frac{x-7}{6}\right) + 7$

$= x - 7 + 7$

$= x$ ☺

Yes

$g(f(x)) = \frac{6x+7-7}{6}$

$= \frac{6x}{6} = x$ ☺

10) $f(x) = 1 - x^3$

$g(x) = \sqrt[3]{1-x}$

$f(g(x)) = 1 - \left(\sqrt[3]{1-x}\right)^3$

$= 1 - (1-x)$

$= 1 - 1 + x$

$= x$ ✓

$g(f(x)) = \sqrt[3]{1-(1-x^3)}$

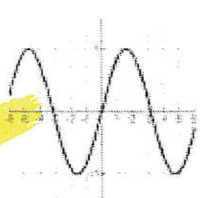
$= \sqrt[3]{x^3}$

$= x$ ✓

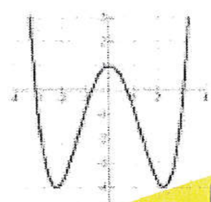
yes

11) **ODD OR EVEN – What do I look for?**

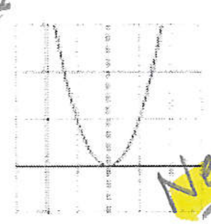
Even → sym. about y axis



a. odd



b. EVEN

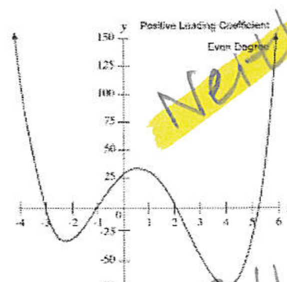


c. Neither

odd → flip, flop

d. $f(x) = 1 - 2x^5$ Neither

e.



Neither

* $7 = 7x^0$

* EOS odd → all terms odd degrees vs Even → all terms even deg.

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12) Graph. State the domain, range, and interval requested.

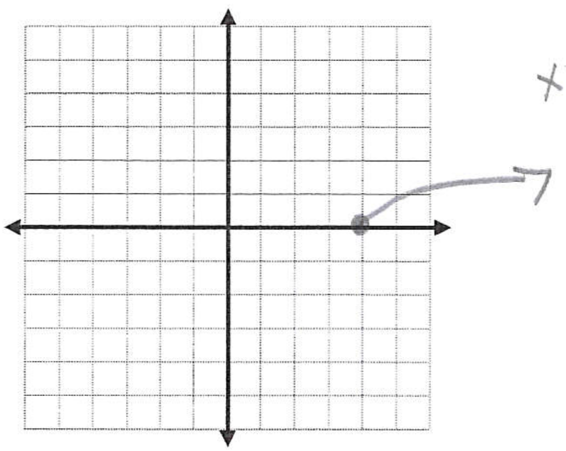
a) $f(p)=\sqrt{p-4}$

D: $p \geq 4$ $[4,\infty)$

R: $f(p) \geq 0$ $[0,\infty)$

Incr: $(4,\infty)$

x values



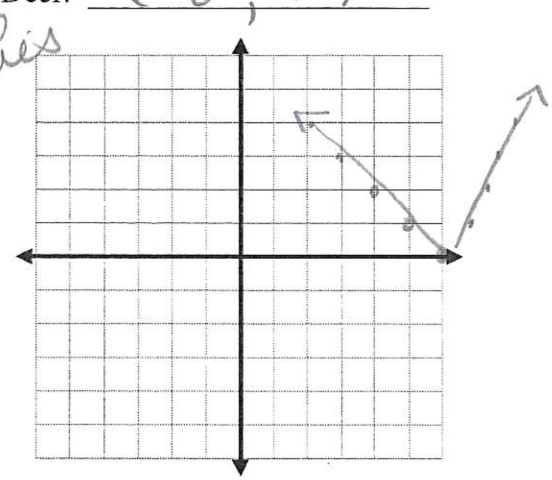
b) $f(x)=|x-6|$

D: $x \geq 6$ $[-\infty,\infty)$

R: $y \geq 0$ $[0,\infty)$

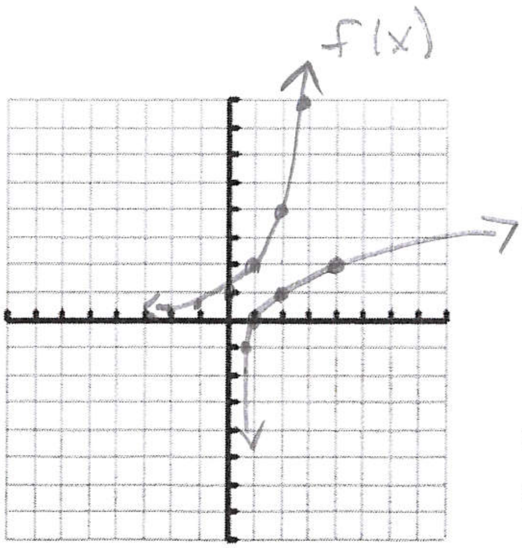
Decr: $(-\infty,6)$

x values



c) Sketch and label the graph of $f(x)=2^x$

x	f(x)
-2	1/4
-1	1/2
0	1
1	2
2	4



d) Sketch the inverse of the above function.

Inverse

LOG$_2$ FN

x	f(x)
1/4	-2
1/2	-1
1	0

2, 1

4, 2

e) Explain why (0,1) is a point on the graph of every function of the form $f(x)=b^x$

$1=b^0$

any non-zero # raised to the zero power is 1

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Unit 2 Logarithmic and Exponential Functions. NO CALC
RULES & Definitions:

13) Evaluate each logarithm.

a) $\log_2 \frac{1}{8}$ $= -3$

b) $\log_7 7$ $= 1$

c) $\log_{\frac{1}{5}} 125$ $= -3$

d) $\log_{11} 1$ $= 0$

e) "log base 5 of 1" = 0

$\log_5 1$

f) $\log_6 \sqrt{6} =$ $\frac{1}{2}$

14) . Use the properties of logarithms to evaluate each expression.

a) $\log_3 27 - \log_3 9$

$\log_3 \frac{27}{9}$

$\log_3 3$ $= 1$

b) $2\log_2 64 + \log_2 2$

$2\log_2 2^6 + \log_2 2$

$2(6) + 1$

$= 13$

c) $-\log_4 \frac{1}{16} - \log_4 64$

$\log_4 \frac{16}{64}$

$= \log_4 \frac{1}{4}$

$= -1$

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Solving Strategies Review: THE ARGUMENT CAN NEVER BE NEGATIVE!

Exponential Equations - Can you create like bases and then equate exponents?
Log, Log, Log. . . then condense and use the log property of equality which states:
Log, log, constant. . . collect log terms. Condense. Then use the definition of a logarithm.
Exponential Equations – Isolate the base. Re-write in log form and solve!
Exponential Equations – take the log of both sides, then use properties to solve.
Does it look like a quadratic? Factor. Solve using logs as needed.

15) Solve each equation.

a) $8^{2x-1} = 16^x$

$2^{3(2x-1)} = 2^{4x}$

$6x - 3 = 4x$

$2x = 3$

$\boxed{x = 3/2}$

b) $2 \bullet 3^x - 100 = 62$

$3^x = 81$

$\boxed{x = 4}$

c) $5^{\sqrt{x+8}} = 125^{\sqrt{x}}$

$5^{\sqrt{x+8}} = 5^{3\sqrt{x}}$

$\sqrt{x+8} = 3\sqrt{x}$

$x + 8 = 9x$

$8 = 8x$

$\boxed{1 = x}$ ✓

d) $\log_3 2x + \log_3 x - \log_3 9 = \log_3 8$

$\log_3 \frac{2x^2}{9} = \log_3 8$

$\frac{2x^2}{9} = \frac{8}{1}$

$2x^2 = 72$

$x^2 = 36$

$x = \pm\sqrt{36}$

$x = \pm 6$

Check $x \neq -6$

so $x = 6$

e) $\log_3(2x-5) = \log_3(7x+10) - 1$

$\log_3 (2x-5) - \log_3 (7x+10) = -1$

$\log_3 \frac{2x-5}{7x+10} = -1$

$3^{-1} = \frac{2x-5}{7x+10}$

$\frac{1}{3} = \frac{2x-5}{7x+10}$

$6x - 15 = 7x + 10$

$-25 = x \rightarrow$ arg neg

NO SOLN

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f) $\log 4x = 3$

$10^3 = 4x$

$1000 = 4x$

$\boxed{250 = x}$

g) $2\log x = -4$

$\log x^2 = -4$

$10^{-4} = x^2$

$\frac{1}{10000} = x^2$

$\pm \frac{1}{100} = x$

$\boxed{x = \frac{1}{100}}$

h) $3x^{\frac{5}{4}} = 96$

$x^{\frac{5}{4}} = 32$

$x = 32^{4/5}$

$x = 2^4$

$\boxed{x = 16}$

i) $12 = 7e^{2k}$

$\frac{12}{7} = e^{2k}$

$\text{LN}\left(\frac{12}{7}\right) = \text{LN}\, e^{2k}$

$\text{LN}\left(\frac{12}{7}\right) = 2k$

$\frac{1}{2}\text{LN}\left(\frac{12}{7}\right) = k$

$k \approx .2695$

j) $3^{x-1} = 24$

$\text{LN}\, 3^{x-1} = \text{LN}\, 24$

$(x-1)\text{Ln}3 = \text{LN}\, 24$

$x\text{Ln}3 - \text{Ln}3 = \text{Ln}24$

$x\text{Ln}3 = \text{Ln}24 + \text{Ln}3$

$x = \frac{\text{Ln}24 + \text{Ln}3}{\text{Ln}3}$

$x \approx 3.8928$

k) $4^{2x} = 3^{x+1}$

$2x\text{Ln}4 = (x+1)\text{Ln}3$

$2x\text{Ln}4 = x\text{Ln}3 + \text{LN}3$

$2x\text{Ln}4 - x\text{Ln}3 = +\text{LN}3$

$x(2\text{Ln}4 - \text{Ln}3) = +\text{Ln}3$

$x = \frac{+\text{Ln}3}{(2\text{Ln}4 - \text{Ln}3)}$

$x \approx +.6563$

l) $e^{2x} - e^x - 6 = 0$

$(e^x - 3)(e^x + 2) = 0$

$e^x = 3 \qquad e^x = -2 \rightarrow$ no soln.

$\downarrow$

$x\text{Lne} = \text{Ln}3$

$x = \text{Ln}3$

$x = 1.0986$

m) $\log_2(-x) = 3 - \log_2(2-x)$

$\log_2(-x) + \log_2(2-x) = 3$

$\log_2(-x)(2-x) = 3$

$\log_2(-2x + x^2) = 3$

$2^3 = -2x + x^2$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$\cancel{x = 4} \qquad x = -2$

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16) Solve each word problem. Formulas: $A = P(1+\frac{r}{n})^{nt}$ $A = Pe^{rt}$

a) How long will it take an investment of $300 to triple if the interest rate is 9.5% per year, compounded continuously?

$A = Pe^{rt}$

$900 = 300e^{.095t}$

$3 = e^{.095t}$

$\ln 3 = .095t$

$\frac{\ln 3}{.095} = t$

11.56 years

b) An investment was made at 9.5% interest compounded quarterly. How long will it take for the investment to double in value?

$n = 4$

$r = .095$

$A = P(1+\frac{r}{n})^{nt}$

$2 = 1(1+\frac{.095}{4})^{4t}$

$2 = 1.02375^{4t}$

$\ln 2 = 4t \ln 1.02375$

$\ln 2 / (4 \ln 1.02375) = t$

7.4 years

c) A new computer that cost $1,600 has a depreciated value of $900 after 2 years. Find the value of the computer after 3 years if it depreciates exponentially.

Options #1

$y = ne^{kt}$

$\frac{900}{1600} = \frac{1600e^{2k}}{1600}$

$\frac{9}{16} = e^{2k}$

$\ln(\frac{9}{16}) = 2k$

$\frac{\ln(\frac{9}{16})}{2} = k$

$-.2877 = k$

$y = 1600e^{-.2877t}$

$y = 1600e^{-.2877(3)}$

$y = \$674.96$

OR ~~gce ca~~

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Exponential Growth or Decay	$y = ne^{kt}$ $A = A_0e^{kt}$

d) Bacteria usually reproduce by a process known as binary fission. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal condition, some bacteria can reproduce every 20 minutes. Find the constant k for the growth of these types of bacteria under ideal conditions and write the growth equation.

$A = e^{.0347t}$ (In minutes)

$A_0 = 1$

$A = 2$

$t = 20$ mins.

① Find K (time mins)

$2 = 1e^{20K}$

$LN\,2 = 20K$

$\frac{LN\,2}{20} = K$

$\boxed{K \approx .0347}$

e) Carbon 14 has a half life of 2515 years. If the amount of Carbon 14 after 1000 years is 5 grams, what was the original quantity of Carbon 14?

$A = A_0e^{kt}$

$\frac{1}{2} = e^{2515K}$

$LN\left(\frac{1}{2}\right) = 2515K$

$\frac{Ln\,.5}{2515} = K$

$K \approx -.0002756$

Model

$A = A_0e^{-.0002756t}$

Use model created to solve

$t = 1000$ yrs.

$A = 5$

$A_0 = ?$

$5 = A_0e^{(-.0002756 \cdot 1000)}$

$5 = A_0e^{-.2756}$

$5 = .7591A_0$

$\frac{5}{.7591} = A_0$

$A_0 = 8.634$ grams.

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Newton's Law of Cooling	$T = C + (T_0 - C)e^{kt}$	T is the temperature of a heated object at time t, C is the constant temperature of the surrounding medium, and k is a negative constant that is associated with the cooling object.

f) A cake removed from an oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F. Find a model for the temperature of the cake, T, after t minutes.

i. What is the temperature of the cake after 40 minutes?

$T_0 = 210$
$c = 70$
$t = 30$
$T = 140$

Find K to create a model

$140 = 70 + (210 - 70)e^{30K}$

$140 = 70 + 140e^{30K}$

$\frac{70}{140} = \frac{140e^{30K}}{140}$

$\frac{1}{2} = e^{30K}$

$\ln\frac{1}{2} = 30K$

$\ln(\frac{1}{2}) \div 30 = K$

$K = -.0231$

Model $T = 70 + 140e^{-.0231t}$

After 40 min

$T = 70 + 140e^{-.0231(40)}$

use calc!

$T = (125.57°)$

ii. When will the temperature of the cake be 90°F?

$90 = 70 + 140e^{-.0231t}$

$20 = 140e^{-.0231t}$

$\frac{1}{7} = e^{-.0231t}$

$\ln\frac{1}{7} = -.0231t$

$\ln(\frac{1}{7}) \div -.0231 = t$

$84.24 = t$ mins

≈ 84 minutes.

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Unit 3 Rational Functions – review test problems!

Simplify each expression.

SOAP a b a^2 ab b^2

1. $\dfrac{\dfrac{p^2+7p}{3p}}{\dfrac{49-p^2}{3p-21}}$

$\dfrac{\cancel{p}(p+7)}{3\cancel{p}} \div \dfrac{(7+p)(7-p)}{3(p-7)}$

$\dfrac{p+7}{3} \cdot \dfrac{3(p-7)}{-1(p-7)(p+7)}$

$= -1$

2. $\dfrac{8x^3-27}{4x^2-9}$

$\dfrac{\cancel{(2x-3)}(4x^2+6x+9)}{\cancel{(2x-3)}(2x+3)}$

3. $\dfrac{3r}{2r-s}-\dfrac{2r}{2r+s}+\dfrac{2s^2}{4r^2-s^2}$

$\left(\dfrac{3r}{2r-s}\right)\left(\dfrac{2r+s}{2r+s}\right)-\dfrac{2r}{(2r+s)}\left(\dfrac{2r-s}{2r-s}\right)+\dfrac{2s^2}{4r^2-s^2}$

$=\dfrac{6r^2+3rs-4r^2+2rs+2s^2}{\text{den}}$

$=\dfrac{2r^2+5rs+2s^2}{(2r+s)(2r-s)}=\dfrac{(2r+s)(r+2s)}{(2r+s)(2r-s)}=\boxed{\dfrac{r+2s}{2r-s}}$

Solve each equation. Be sure to check your solutions.

4. $\dfrac{1}{x+1}+\dfrac{1}{x-1}=\dfrac{2}{x^2-1}$

$(x-1)+(x+1)=2$

$2x=2$

$x=1$

but $x\neq 1$

so no soln

5. $\dfrac{4}{x^2-2x-3}=\dfrac{-1}{x+1}-\dfrac{x}{3-x}$

$-1(x-3)$

$4=-1(x-3)+x(x+1)$

$4=-1x+3+x^2+x$

$0=x^2-1$

$0=(x+1)(x-1)$

~~$x=-1$~~

$x=1$

6. $\dfrac{5x+2}{x^2-4}=\dfrac{-5x}{2-x}+\dfrac{2}{x+2}$

$-(x-2)$

$5x+2=5x(x+2)+2(x-2)$

$5x+2=5x^2+10x+2x-4$

$0=5x^2+7x-6$

$0=(5x-3)(x+2)$

~~$x=-2$~~

$x=3/5$

7. $\dfrac{5}{x-5}=\dfrac{x}{x-5}-1$

$5=x-1(x-5)$

$5=x-x+5$

$5=5$

Inf solutions, $x\neq 5$

$(-\infty,5)\cup(5,\infty)$

Name: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Block: $\boxed{x = \frac{-1 \pm \sqrt{145}}{4}}$

$x = \frac{-1 \pm \sqrt{1-4(2)(-18)}}{4}$

8. $\frac{4}{x-2} - \frac{x+6}{x+1} = 1$

$4(x+1) - (x+6)(x-2) = (x+1)(x-2)$

$4x + 4 - [x^2 + 4x - 12] = x^2 - x - 2$

$4x + 4 - x^2 - 4x + 12 = x^2 - x - 2$

$-x^2 + 16 = x^2 - x - 2$

$0 = 2x^2 - x - 18$

$0 = (x \quad)(2x \quad)$ DNF

9. $\frac{1}{x+2} + \frac{1}{x-2} = \frac{3}{x+1}$

$(x-2)(x+1) + (x+2)(x+1)$

$= 3(x+2)(x-2)$

$x^2 - x - 2 + x^2 + 3x + 2$

$= 3x^2 - 12$

$2x^2 + 2x = 3x^2 - 12$

$0 = x^2 - 2x - 12$

DNF SO USE QF

$x = \frac{2 \pm \sqrt{4+48}}{2}$

$\boxed{x = \frac{2 \pm \sqrt{52}}{2}}$

Solve each inequality. Recall: 1.Set one side = to zero 2. Rewrite one side as one fraction. 3. Set den = 0 to find excluded values (open circle) 4. Set numerator = 0, solve. Use inequality symbol of the problem to determine if circle is open or closed. 5. Create number line. Plot points. Test points.6. Write solution using interval notation.

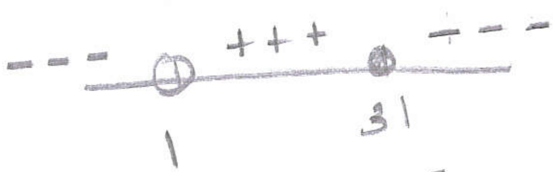
10. $1 + \frac{5}{a-1} \le \frac{7}{6}$

$\frac{5}{a-1} - \frac{1}{6} \le 0$

$\frac{30 - (a-1)}{6(a-1)} \le 0$

$\frac{31 - a}{6(a-1)} \le 0$

$a \ne 1 \quad a = 31$



$(-\infty, 1) \cup [31, \infty)$

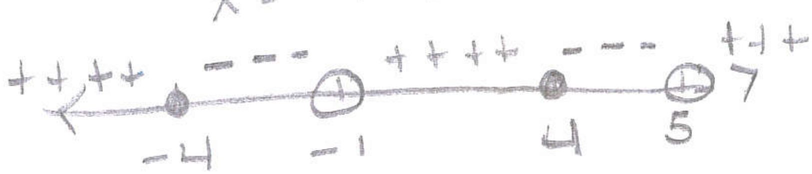
11. $\frac{x^2 - 16}{x^2 - 4x - 5} \ge 0$

$(x-5)(x+1) \ne 0$

$x \ne 5 \quad x \ne -1$

$(x+4)(x-4) = 0$

$x = -4 \quad x = 4$



$(-\infty, -4] \cup (-1, 4] \cup (5, \infty)$

Name: ______________________ Block: __________

12. $5+\frac{1}{x}>\frac{16}{x}$

$5 - \frac{15}{x} > 0$

$\frac{5x-15}{x} > 0$ $\frac{5(x-3)}{x} > 0$

$x \neq 0$ $x = 3$ open

+++ ⊕ - - - ⊕ +++

0 3

$(-\infty, 0) \cup (3, \infty)$

13. $\frac{2a-5}{6}-\frac{a-5}{4}<\frac{3}{4}$

$\frac{2a-5}{6} - \frac{a-5}{4} - \frac{3}{4} < 0$

$\frac{4a - 10 - 3(a-5) - 3(3)}{12} < 0$

$4a - 10 - 3a + 15 - 19 < 0$

$\frac{a-14}{12} < 0$ $a - 14 < 0$

$a < 14$

or $(-\infty, 14)$

14. Identify any holes, vertical asymptotes, horizontal asymptotes, and slant asymptotes of each graph. If they do not exist write **none** in the space provided. Accurately graph each rational function.

a. $R(x)=\frac{x^2+3x+2}{(x+2)^2}$ $\frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}(x-3)(x+3)}$

Domain $x \neq -2, 3, -3$

Hole(s) $\left(-2, \frac{+1}{5}\right)$ Vertical Asymptotes $x = 3, x = -3$

X-intercepts $(-1, 0)$ Horizontal Asymptotes BOBO $y = 0$

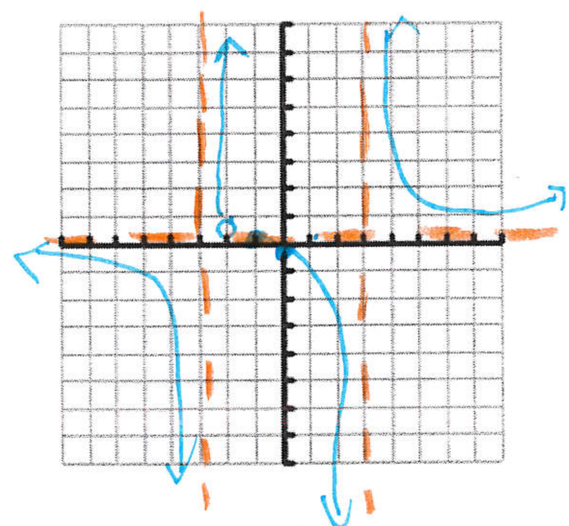
Y-intercepts $(0, -1/9)$ Slant Asymptotes —

Domain: ____________

Range: ____________

$x = 4 \rightarrow \frac{5}{7}$

$x = -4 \rightarrow \frac{-}{(-)(-)}$



b. $F(x)=\frac{2x-6}{x}$ $x \neq 0$ $\frac{2(x-3)}{x}$

Domain $x \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

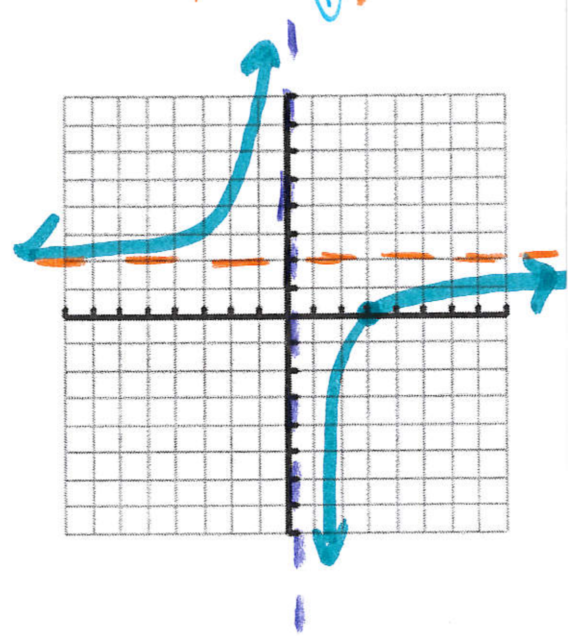
Hole(s) — Vertical Asymptotes $x = 0$

X-intercepts $(3, 0)$ Horizontal Asymptotes $y = 2$

Y-intercepts none Slant Asymptotes ——

Domain: ____________

Range: $(-\infty, 2) \cup (2, \infty)$

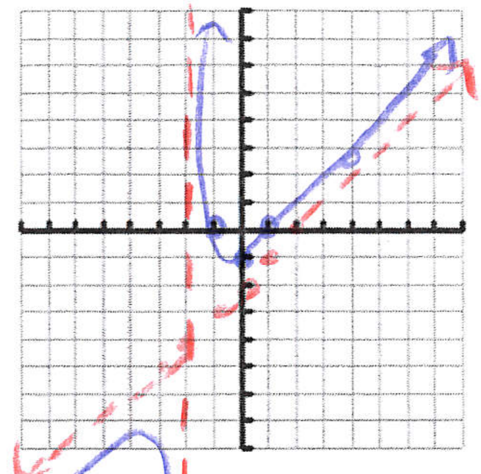


Name: ______________________________ Block: ______________

hole @ $x=4$

$y = \frac{(4+1)(4-1)}{(4+2)} = \frac{5(3)}{6} = 5/2$

c. $G(x) = \frac{(x^2-1)(x-4)}{x^2-2x-8}$ $\quad \frac{(x+1)(x-1)\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$



Domain $x \neq 4, -2$

Hole(s) $(4, 2.5)$ Vertical Asymptotes $x = -2$

X-intercepts $(-1, 0)$ $(1, 0)$ Horizontal Asymptotes BOTNO

Y-intercepts $(0, -1/2)$ Slant Asymptotes $y = x - 2$

Domain: ______________________________

omit

Range: $(-\infty$ ______________________________

$G(x) = \frac{x^2-1}{x+2}$

d. Under what conditions would the horizontal asymptote have the equation y = 0?

BOBO → Bigger on bottom!

$f(x) = \frac{x+2}{x^2+3x+2}$

$$\begin{array}{r} x - 2 \\ x+2 \overline{) x^2 + 0x - 1} \\ \underline{x^2 + 2x} \\ -2x - 1 \\ \underline{-2x - 4} \\ 3 \end{array}$$

APPLICATIONS OF RATIONALS

1. You have created a new type of jelly bean that you would like to market to Harry Potter World. They have requested that the packages contain 12 cubic centimeters of jelly beans, and that they be packaged in cylindrical containers. You decide to explore two situations.

   A. What dimensions would minimize the amount of packaging used?

Constraint: $V = \pi r^2 h = 12$

Function to be optimized: $SA: 2\pi r^2 + 2\pi r h$

Real world domain: $r > 0, h > 0$

Work:

$\pi r^2 h = 12$

$h = \frac{12}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r\left(\frac{12}{\pi r^2}\right)$

$SA = 2\pi r^2 + \frac{24}{r}$

$x = r$

$y = SA$

$y = 2\pi x^2 + \frac{24}{x}$

Name: ____ Block: ____________

B. You learn that the packaging material for the base and top costs $.10 per square cm, while the cost for the lateral area is $.07 per square cm. If you want to minimize your packaging costs, What dimensions should you use for the container?

$$C = .10(2\pi x^2) + .07\left(\frac{24}{x}\right)$$

$$C = .2\pi x^2 + \frac{1.68}{x}$$

CALC MIN

(1.1, 2.29)

r, $

radius = 1.1 cm

$h = \frac{12}{\pi r^2} = 12 \div (\pi 1.1^2) = 3.16$ cm

cost = $ 2.29

2. The fixed cost of production is $120,000 per month. The cost per unit is $100

A. Write an equation to represent the average cost of producing n units/

$$\overline{C}(x) = \frac{120{,}000 + 100x}{x}$$

B. What is the average cost of producing 100 units?

$$\overline{C}(100) = \frac{120{,}000 + 10{,}000}{100} = \frac{130{,}000}{100} = \$1300 \text{ per unit}$$

C. What is the average cost of producing 100,000 units?

$$\overline{C}(100{,}000) = \frac{120{,}000 + 10{,}000{,}000}{100{,}000} = \frac{10{,}120{,}000}{100{,}000}$$

$101.20 per unit

D. What is the horizontal asymptote of the average cost function?

$y = 100$

E. What does the horizontal asymptote represent?

The minimum average cost → No matter how many units are produced, the average cost can never hit/fall below $100 per unit.

Series and Sequences Review Packet was distributed separately!