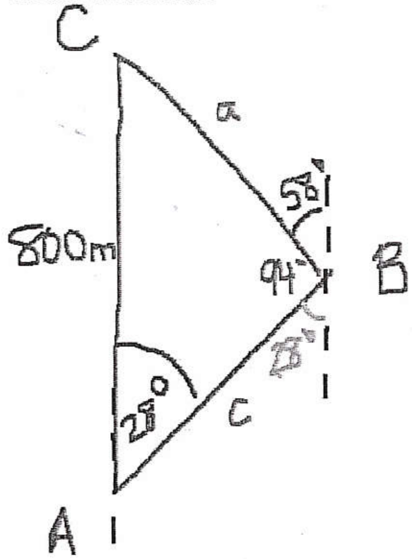
Pre-Calculus
LAW OF SINES/COSINES – word problems
REVIEW - Examples

Name ______________________

Example 7: An Application of the Law of Sines
On a small lake, a child swam from point A to point B at a bearing of N 28° E. The child then swam to point C at a bearing of N 58° W. Point C is 800 meters due north of point A. How many total meters did the child swim? $= a + c$

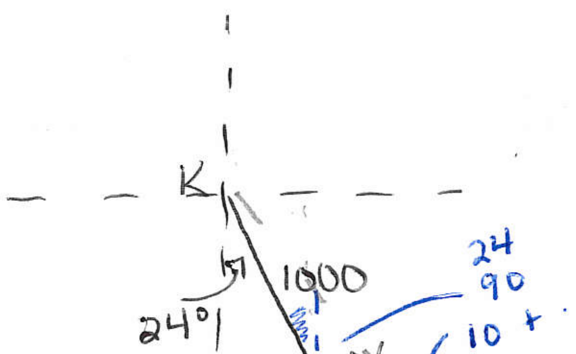


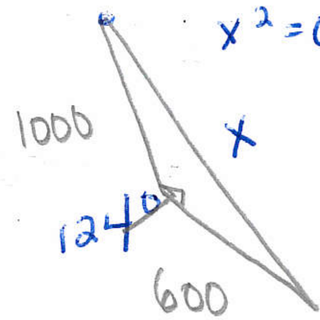
$$\frac{a}{\sin 28^\circ} = \frac{800}{\sin 94^\circ}$$

$a = 376.5$ m.

$\measuredangle B = 180 - (58^\circ + 28^\circ)$
$\measuredangle B = 94^\circ$

1) A plane leaves Kittredgeville at a bearing of S24°E and flies at a speed of 400 mph for 2.5 hours. Over Norwood Town, the plane turns at a bearing of S80°E and continues for another 1.5 hours. SAS
a) Draw a picture of this situation.
b) How far is the plane from the starting point?

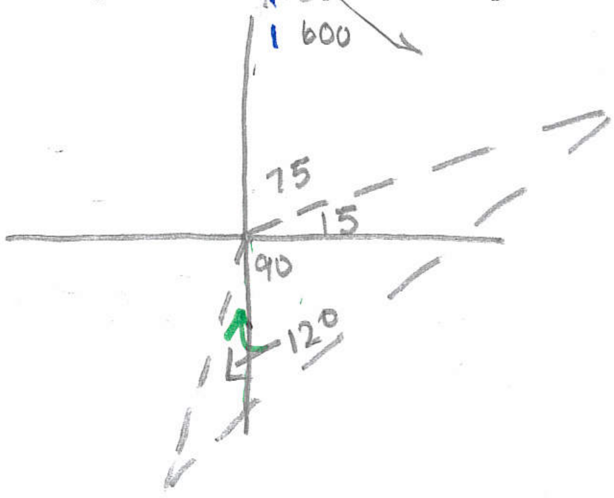


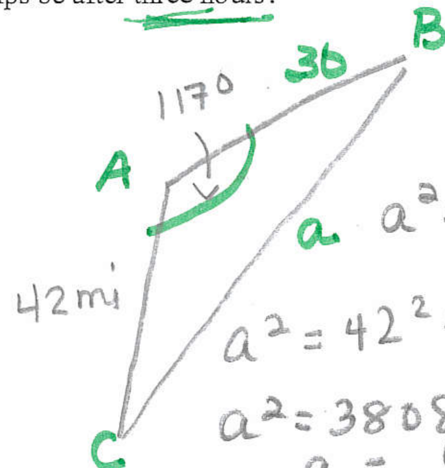


$x^2 = 600^2 + 1000^2 - 2(600)(1000)\cos 124$

$x \approx 1425$ miles.

2. Two ships leave a harbor at the same time. One ship travels at a bearing of S12°W at 14 mph. The other ship travels on a bearing of N75°E at 10 mph. How far apart will the ships be after three hours?





$14 \cdot 3 = 42$ mi
$10 \cdot 3 = 30$ mi

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 42^2 + 30^2 - 2 \cdot 42 \cdot 30\cos 117$

$a^2 = 3808$
$a = 61.7$ miles.

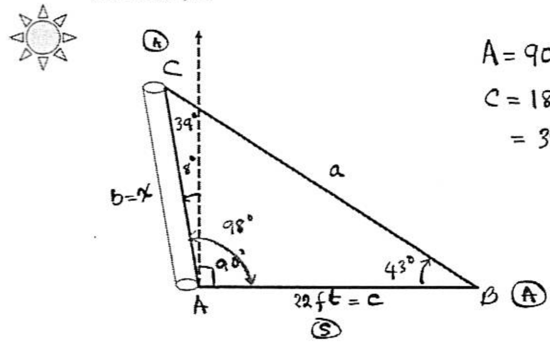
3. Kris is taking a walk along a straight road heading East. He decides to leave the road, so he walks on a path with a bearing of N 55°E. After walking for 450 meters, he turns right and heads back towards the road, ending up 676 meters from where he left the road.

a) What angle did Kris turn to head back toward the road?

b) How many meters did Kris walk?

Angles of Elevation and Depression:

(2) A poll tilts towards the sun at an 8° angle from the vertical as it casts a 22-ft shadow. The angle of elevation from the shadow to the top of the pole is 43°. How tall is the poll?



$A = 90 + 8 = 98°$

$C = 180° - (98 + 43)$

$= 39°$

find b

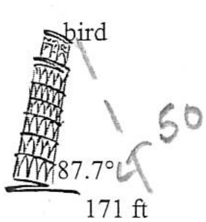
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

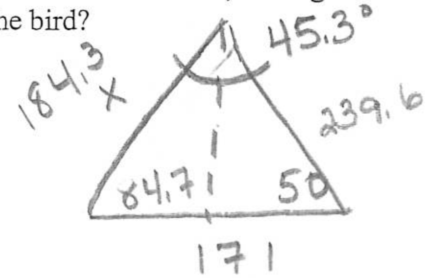
$$\sin 43 \cdot \frac{b}{\sin 43} = \frac{22 \sin 43}{\sin 39}$$

$$b = \frac{22 \sin 43}{\sin 39} \approx 23.841$$

The pole is 23.841 ft tall.

4. Closed to tourists since 1990, the Leaning Tower of Pisa in Italy leans at an angle of about 84.7°. The figure shows that 171 feet from the base of the tower, the angle of elevation to the top is 50°. If a bird is sitting on the very top of the tower, how high is the bird?





$$① \frac{\sin 45.3}{171} = \frac{\sin 50}{X}$$

$$X = \frac{171 \sin 50}{\sin 45.3} = 184.3$$

Rt. Δ.

$$\sin 84.7 = \frac{a}{184.3}$$

$184.3 \sin 84.7 = a$ $\quad a = 183.5$ ft.

5) Ally is in the smaller building looking out of a window. She sights the top of the taller building at an angle of elevation of 40°. She sights the bottom of the taller building with an angle of depression of 20°. ~~If the building is 200~~ feet tall, how high up is she in the smaller building? How far apart are the two buildings?

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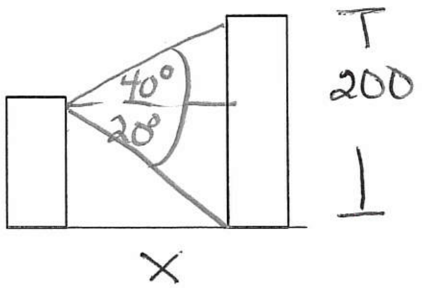
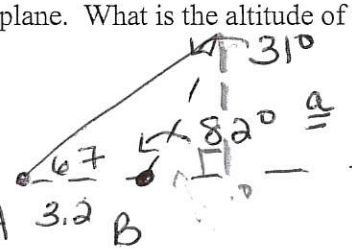
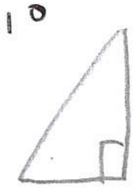


6. Two radar stations are tracking the same plane. The angle of elevation from Station A to the plane is 67°, the angle of elevation to the plane from Station B is 82°. Station A is 3.2 miles from Station B. Find the distances from each station to the plane. What is the altitude of the plane?



$180 - (82+67) = 180 - (149) = 31°$

ASA → LOS



$\frac{\sin C}{c} = \frac{\sin A}{a}$

altitude =

$\frac{\sin 31}{32} = \frac{\sin 82}{b}$

$\frac{32 \sin 82}{\sin 31} = b$

b = 61.5 miles

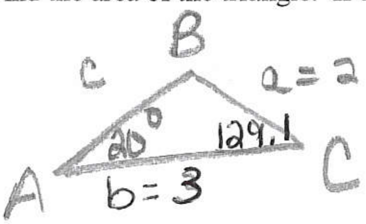
$\frac{\sin 31}{3.2} = \frac{\sin 67}{a}$

$\frac{3.2 \sin 67}{\sin 31} = a$

a = 5.7 miles From Station B

Solve the triangle

7) A triangle has the given information: a = 2, b = 3, A = 20°.
Find the area of the triangle. If two triangles are possible, find the area of both triangles.



① Use LOS

$\frac{\sin 20°}{2} = \frac{\sin B}{3}$

$\frac{3 \sin 20°}{2} = \sin B$

$.5130 = \sin B$

$\boxed{30.9° = B_1}$ $\boxed{C_1 = 129.1°}$

$A_{\Delta_1} = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin 129.1$

$= 3 \sin 129.1$

If a second Δ exists, then

$B_2 = 180 - 30.9 = 149.1$

$$\begin{array}{r} 149.1 \\ +20 \\ \hline 169.1 \end{array}$$

Yes, 2 Δs exist.

$\boxed{B_2 = 149.1}$

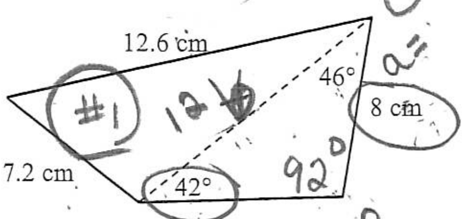
$\boxed{C_2 = 10.9}$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \frac{1}{2} ab \sin C$

AREA:

8. Find the area of the quadrilateral.



$\Delta 1$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 42}{8} = \frac{\sin 92}{b}$

$\frac{8 \sin 92}{\sin 42} = b$

$12 = b$

$\Delta_1 \quad s = \frac{1}{2}(7.2 + 12.6 + 12)$

$= \frac{1}{2}(31.8)$

$S = 15.9$	$b = 12.6$
$a = 7.2$	$c = 12.6$

$A = \sqrt{15.9(15.9-7.2)(15.9-12)(15.9-12.6)}$

$\boxed{A_1 = 42.2 \text{ cm}^2}$

$A_2 = \frac{1}{2}(8)(12)\sin 46°$

$= 48 \sin 46°$

$\boxed{A_2 \approx 34.5 \text{ cm}^2}$

$\boxed{A_{quad} = 76.7 \text{ cm}^2}$

9.

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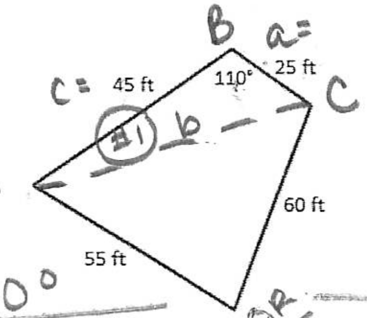
Problem: Jill, a surveyor, needs to approximate the **area** of a piece of land. She walks the perimeter of the land and measures the side distances and one angle, as shown below. What is the area of the piece of land?



SAS ⇒ LOC

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 25^2 + 45^2 - 2(25)(45)\cos 110°$

$b^2 = 3419.5$

$b = 58.5$

OR use Heron's

$A_{\Delta 1} = \frac{1}{2} ac \sin B$

$= \frac{1}{2}(25)(45)\sin 110°$

$\boxed{A_{\Delta 1} = 528.3 \text{ ft}^2}$

OR

$A_{\Delta_2} =$

$s = \frac{1}{2}(55 + 60 + 58.5)$

$S = 86.75$

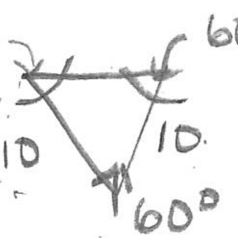
$A_2 = \sqrt{86.75(86.75-60)(86.75-55)(86.75-58.5)}$

$A_{\Delta 2} = 1442.7$

$A_{\Delta 1} = 528.3$

$\boxed{1970 \text{ ft}^2}$

10. Find the area of a regular hexagon that is inscribed in a circle with a radius of 10 feet.



$360 \div 6 = 60$

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 60$

$= 50 \cdot \frac{\sqrt{3}}{2}$

$= 25\sqrt{3}$

≈ 43.3

$\times 6$

$\boxed{259.8 \text{ ft}^2}$

If we had penta

$360 \div 5 = 72$

$180 - 72$

$= 108 \div 2 = 54$