Key

Pre-Calc UNIT 5 Lesson #7 Trigonometric Functions of Any Angle page 2

More Practice: Evaluate.

1) Let P = (-3,-5) be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

$\sin\theta = \frac{y}{r} = \frac{-5\sqrt{34}}{34}$

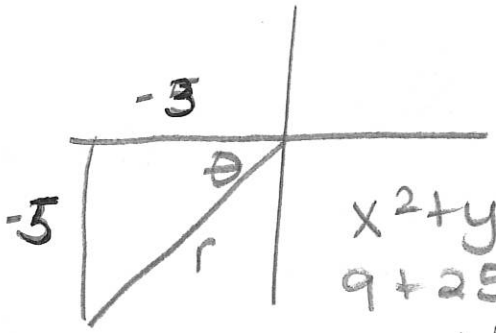
$\cos\theta = \frac{-3\sqrt{34}}{34}$

$\tan\theta = \frac{3}{5}$

$\csc\theta = -\frac{\sqrt{34}}{5}$

$\sec\theta = -\frac{\sqrt{34}}{3}$

$\cot\theta = \frac{5}{3}$



$x^2+y^2=r^2$

$9+25=r^2$

$r=\pm\sqrt{34}$

$r=\sqrt{34}$

2) Let T = $(-3\sqrt{6},-5)$ be a point on the terminal side of θ. Find each of the six trigonometric functions of θ.

$\sin\theta = \frac{-5\sqrt{79}/79}{-3\sqrt{6}/\sqrt{79}} = \frac{-3\sqrt{474}}{79}$

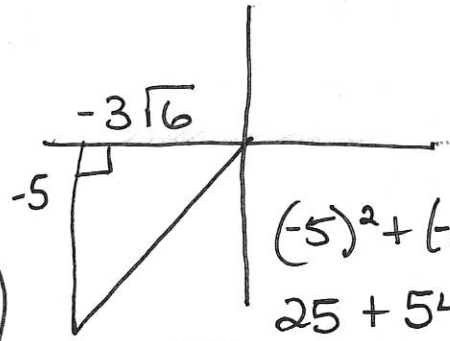
$\cos\theta = \frac{-3\sqrt{6}/\sqrt{79}}{}$

$\tan\theta = \frac{5}{-3\sqrt{6}} = \frac{5\sqrt{6}}{-18} = -5\sqrt{6}/18$

$\csc\theta = -\frac{\sqrt{79}}{5}$

$\sec\theta = -\frac{\sqrt{79}}{3\sqrt{6}}$

$\cot\theta = \frac{3\sqrt{6}}{5} = -\frac{\sqrt{474}}{3}$



$(-5)^2+(-3\sqrt{6})^2=r^2$

$25+54=r^2$

$79=r^2$

$r=\sqrt{79}$

3) Given $\tan\theta=-\frac{2}{3}$ and $\cos\theta>0$, find the remaining trigonometric functions of θ.

$\sin\theta = \frac{-2\sqrt{13}}{13}$

$\cos\theta = \frac{3\sqrt{13}}{13}$

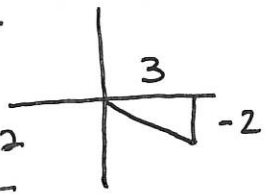
$\csc\theta = -\frac{\sqrt{13}}{2}$

$\sec\theta = \frac{\sqrt{13}}{3}$

$\cot\theta = \frac{-3}{2}$

$x=3$

$y=-2$



$9+4=r^2$

$r=\sqrt{13}$

4) Find the exact value of each of the remaining trigonometric functions of θ if $\cos\theta=\frac{3}{-5}$ and θ is in quadrant III.

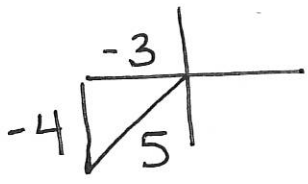
$\sin\theta = \frac{-4/5}{}$

$\tan\theta = \frac{4/3}{}$

$\csc\theta = \frac{-5/4}{}$

$\sec\theta = \frac{-5/3}{}$

$\cot\theta = \frac{3/4}{}$

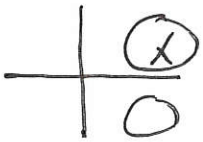


5) Let θ be in standard position. In which quadrant(s) can θ lie under the given conditions?

a) Sin $\theta < 0$ and tan $\theta > 0$ Q3

b) Sec $\theta > 0$ and sin $\theta > 0$ Q1

cos pos sin pos

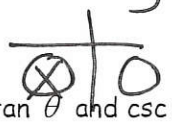


c) Cot $\theta < 0$ and tan $\theta < 0$ Q2, Q4

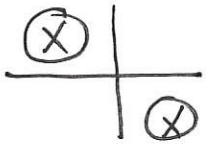
tan neg

d) Csc $\theta < 0$ and cot $\theta > 0$ Q3

sin neg tan pos

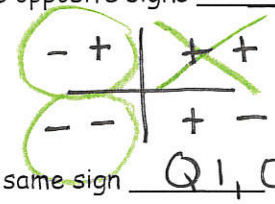


e) Sin θ and cos θ have opposite signs Q2, Q4

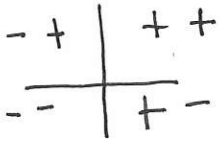


f) tan θ and csc θ have opposite signs Q2, Q3

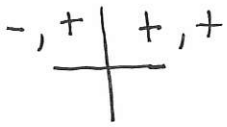
sin θ



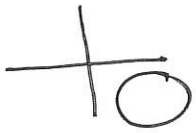
g) Sin θ and tan θ have the same sign Q1, 4



h) Cos θ and tan θ have the same sign Q1, Q2



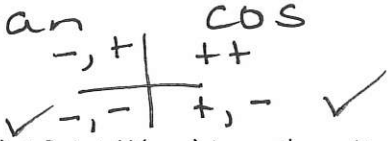
i) Sin θ is negative and cos θ is positive Q3



j) Sin θ, cos θ, tan θ all have the same sign Q1

k) Cot θ and sec θ have the opposite signs Q3, 4

tan cos



l) Sec θ and csc θ have the same sign Q1, Q3

cos θ sin θ

6) A Point M(x, y) is on the unit circle with the equation $x^2 + y^2 = 1$. Show that the following points are on the unit circle. Then find the exact values of sine β, cosine β and tangent β. (Hint: You can find the radius without drawing a picture)

a) M $\left(\frac{-3}{8}, \frac{\sqrt{55}}{8}\right)$

$\left(\frac{-3}{8}\right)^2 + \left(\frac{\sqrt{55}}{8}\right)^2 \stackrel{?}{=} 1$

$\frac{9}{64} + \frac{55}{64} = 1$ ✓

$r = 1$

Sin β = $\frac{\sqrt{55}}{8}$ Cos β= $\frac{-3}{8}$

Tan β = $\frac{\sqrt{55}}{8} \cdot \frac{8}{-3} = \frac{-\sqrt{55}}{3}$

b) $M\left(\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$

$\left(\left(\frac{\sqrt{7}}{4}\right)^2 + \left(\frac{-3}{4}\right)^2\right) = 1$

$\frac{7}{16} + \frac{9}{16} = 1$ ✓

Sin β = $\frac{-3}{4}$ Cos β= $\frac{\sqrt{7}}{4}$

Tan β = $\frac{-3}{4} \cdot \frac{4}{\sqrt{7}} = \frac{-3\sqrt{7}}{7}$