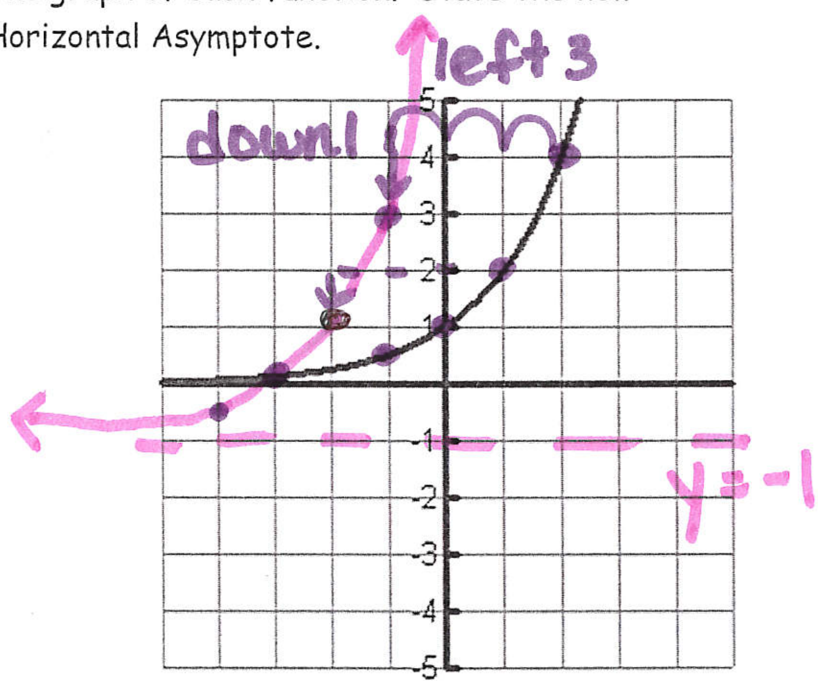
Key

Lesson 3 Warm Up – Graphing Exponential Functions

$$y = a(base)^{b(x-c)} + d$$

Use the parent function to create the graph of each function. State the new domain, range, and equation of the Horizontal Asymptote.

1. $y = 2^{x+3} - 1$



$a = 1$

$base = 2$

$b = 1$

$c = -3$ (left 3)

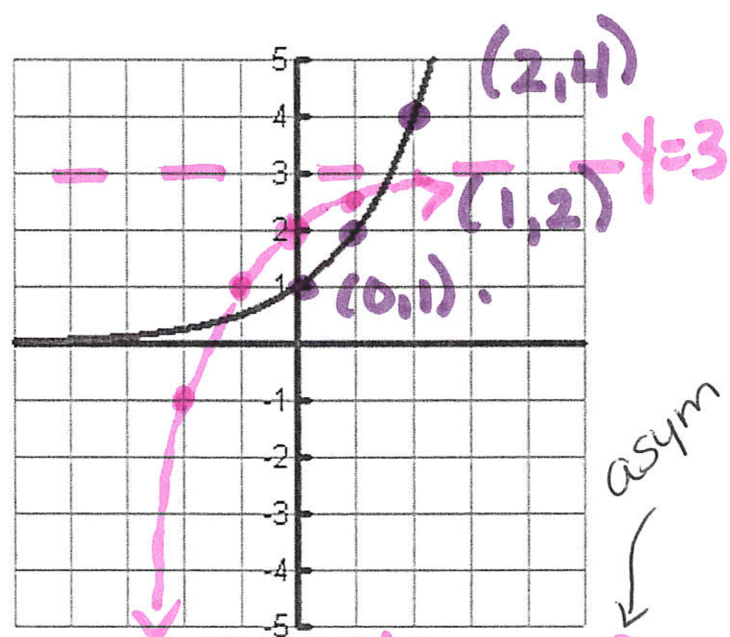
$d = -1$ (down 1)

Algebraically:

$\left(\frac{x}{b} + c,\ ay + d\right)$

$(x - 3,\ y - 1)$

2. $y = -2^{-x} + 3$



$a = -1 \rightarrow$ reflect over x

$base = 2$

$b = -1$ reflect over y

$c = 0$

$d = 3 \rightarrow$ up 3

$\left(\frac{x}{-1},\ -1y + 3\right)$

x	$y = 2^x$
-1	1/2
0	1
1	2
2	4

$\frac{x}{-1}$	$-1y + 3$
1	2.5
0	2
-1	1
-2	-1

Unit 2 Lesson #3 Logarithmic Functions

How do we graph the inverse of a function?

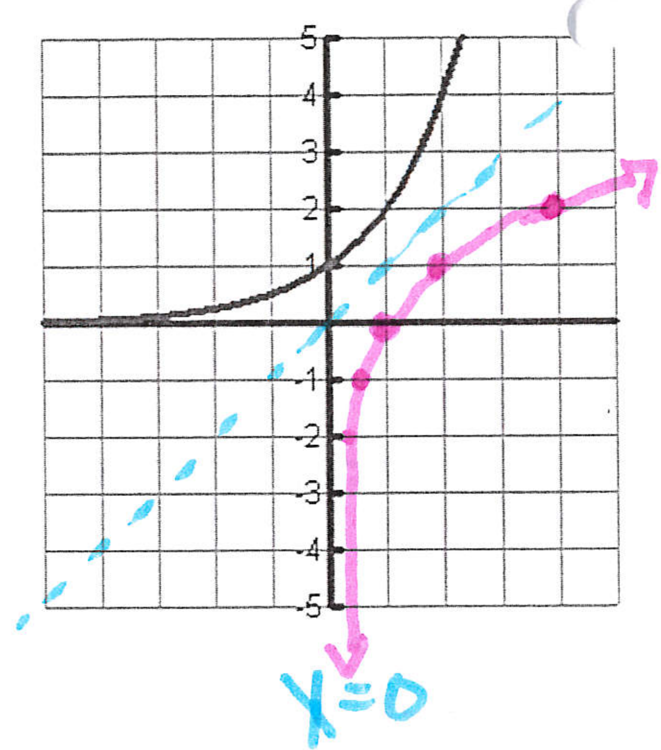
Interchange x & y

Draw the inverse of the graph of $y = 2^x$

Use a table and points to help you!

x	$y = 2^x$
-2	1/4
-1	1/2
0	1
1	2
2	4

x'	y'
1/4	-2
1/2	-1
1	0
2	1
4	2



Exponential Function Properties

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Asymptotes? What variable in the function model changes the asymptote? y=0

Before shifts, it always passes through the point (0, 1) since $\text{base}^0 = 1$.

The Inverse Function of an Exponential Function is called a LOGARITHMIC FUNCTION,

$$\log_b x = y$$

Logarithmic Function Properties

Domain: Interchange with range of inverse → $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptotes? x = 0

Before shifts, it always passes through the point (1, 0) (interchanged

Examine the following equations. Then, complete the blank below!

$b^y = x$ $\quad \log_b x = y$

1. $8^{\frac{1}{3}} = 2$ $\quad \log_8 2 = \frac{1}{3}$

2. $5^3 = 125$ $\quad \log_5 125 = 3$

3. $8^{-1} = 1/8$ $\quad \log_8 \frac{1}{8} = -1$

4. $6^2 = 36$ $\quad \log_6 36 = 2$

5. $3^{-3} = \frac{1}{27}$ $\quad \log_3 (1/27)$

6. $(\frac{1}{2})^{-3} = 8$ $\quad \log_{\frac{1}{2}} 8 = -3$

$\log_b x = y$ if and only if (iff) $b^y = x$

$(x > 0 \text{ and } b > 0,\ b \neq 1)$

log base answer = exp if base exp = ans

In Words:

Exponential Form	Logarithmic Form
$b^y = x$	$y = \log_b x$

Write each equation in the equivalent exponential form

a) $2 = \log_5 x$

$5^2 = x$

b) $\log_3 7 = y$ ← exp

ans

↑ base

$3^y = 7$

c) $\log_{10} \sqrt{10} = 1/2$

$10^{\frac{1}{2}} = \sqrt{10}$

Write each equation in its equivalent logarithmic form

a) $2^3 = 8$ ← ans.

← exp

base

$\log_b x = y$

$\log_2 8 = 3$

b) $\sqrt{9} = 3$

$9^{\frac{1}{2}} = 3$

$\log_9 3 = \frac{1}{2}$

c) $(\frac{1}{3})^x = 27$

$\log_{\frac{1}{3}} 27 = X$

Evaluating Logarithms. Evaluate/Simplify. You may set each expression equal to x and solve for x.

a) $\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$x = 4$

b) $\log_3 9 = x$

$3^x = 9$

$x = 2$

c) $\log_{\frac{1}{25}} 5$

$\left(\frac{1}{25}\right)^x = 5$

$5^{-2x} = 5^1$

$x = -1/2$

d) $\log_5 1$

$5^x = 1$

$5^x = 5^0$

$x = 0$

e) $\log_{36} \frac{1}{6}$

$36^x = \frac{1}{6}$

$6^{2x} = 6^{-1}$

$x = -1/2$

f) $\log_3 3$

$3^x = 3$

$x = 1$

Inverse Properties of Logarithms. If the functions are inverses, their compositions are each equal to x. So,

Let f(x) = b^x and g(x) = $\log_b x$

f(g(x)) g(f(x)) =

$f(\log_b x)$ $g(b^x)$

$b^{\log_b x} \Rightarrow x \Leftarrow \log_b b^x$

since inverses, these = x!

$\log_b b^x = x$ and $b^{\log_b x} = x$

Evaluate/simplify:

a) $\log_5 5^3$

3

b) $6^{\log_6 (2x+5)}$

$2x + 5$

$\log_{base} answ = exp$

Practice A – Definition of Logs

Write each equation in its equivalent exponential form.

1) $6 = \log_2 64$

$2^6 = 64$

2) $2 = \log_9 x$

$9^2 = x$

so $x = 81$

3) $3 = \log_b 27$

$b^3 = 27$

so $b = 3$

Write each equation in its equivalent logarithmic form.

4) $5^4 = 625$ ← exp ← ans.

base → $\log_5 625 = 4$

5) $5^{-3} = \frac{1}{125}$

$\log_5 \frac{1}{125} = -3$

6) $8^y = 300$

$\log_8 300 = y$

Evaluate each expression <u>without</u> using a calculator.

7) $\log_7 49$ $= 2$

8) $\log_2 64$ $=$

$2^x = 64$

$2^x = 4^3$

$2^x = 2^6$ $x = 6$

9) $\log_3 27$

3

10) $\log_6 \frac{1}{6}$

-1

11) $\log_2 \frac{1}{8}$

-3

12) $\log_3 \frac{1}{9}$

-2

13) $\log_{\frac{1}{81}} 9$

$\left(\frac{1}{81}\right)^x = 9$

$9^{-2x} = 9^1$

$x = -1/2$

14) $\log_5 5^7$

7

15) $7^{\log_7 23}$

23

Hmmmm. . . could you rewrite any part of the following using rational exponents? Can you rewrite using a common base?

16) $\log_6 6\sqrt{6}$

$\log_6 6^1 \cdot 6^{\frac{1}{2}}$

$\log_6 6^{\frac{3}{2}}$

$\frac{3}{2}$

17) $\log_8 4$

$8^x = 4$

rewrite using like bases

$2^{3x} = 2^2$

$3x = 2$

$x = 2/3$