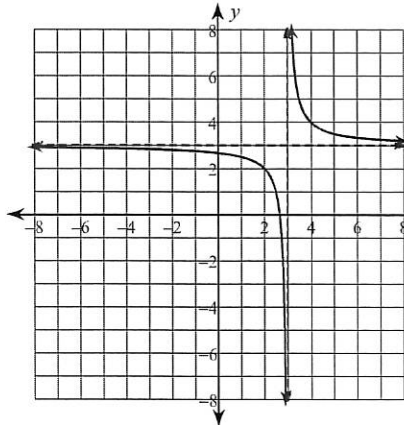


Graphs of Rational Functions

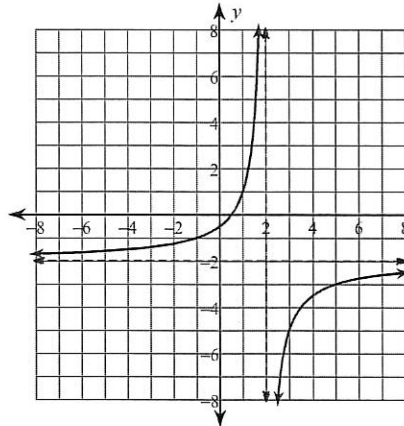
For each function, identify the points of discontinuity, holes, intercepts, horizontal asymptote, domain, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

1) $f(x) = \frac{1}{x-3} + 3$



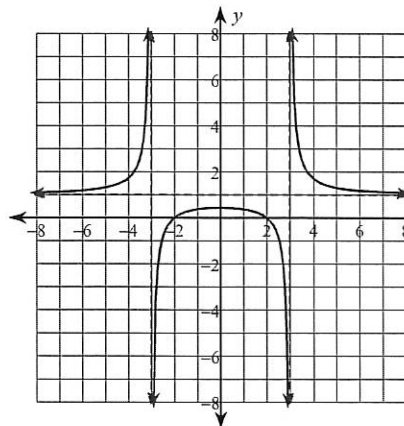
Discontinuities: 3
 Holes: None
 Horz. Asym.: $y = 3$
 x-intercepts: $\frac{8}{3}$, y-intercept: $\frac{8}{3}$
 Domain:
 All reals except 3
 Vert. Asym. behavior:
 $\lim_{x \rightarrow 3^-} f(x) = -\infty$, $\lim_{x \rightarrow 3^+} f(x) = \infty$
 End behavior asym.: $y = 3$

2) $f(x) = -\frac{3}{x-2} - 2$



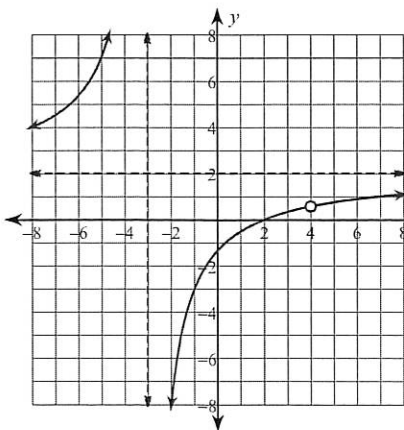
Discontinuities: 2
 Holes: None
 Horz. Asym.: $y = -2$
 x-intercepts: $\frac{1}{2}$, y-intercept: $-\frac{1}{2}$
 Domain:
 All reals except 2
 Vert. Asym. behavior:
 $\lim_{x \rightarrow 2^-} f(x) = \infty$, $\lim_{x \rightarrow 2^+} f(x) = -\infty$
 End behavior asym.: $y = -2$

3) $f(x) = \frac{x^2 - 4}{x^2 - 9}$



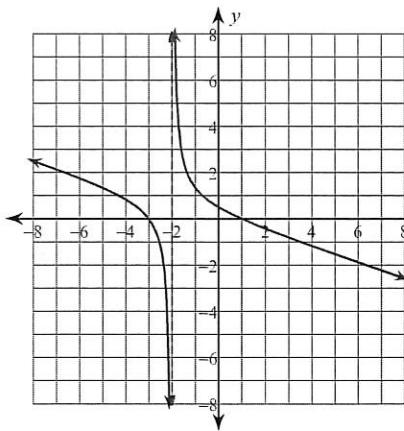
Discontinuities: 3, -3
 Holes: None
 Horz. Asym.: $y = 1$
 x-intercepts: 2, -2, y-intercept: $\frac{4}{9}$
 Domain:
 All reals except -3, 3
 Vert. Asym. behavior:
 $\lim_{x \rightarrow -3^-} f(x) = \infty$, $\lim_{x \rightarrow -3^+} f(x) = -\infty$
 $\lim_{x \rightarrow 3^-} f(x) = -\infty$, $\lim_{x \rightarrow 3^+} f(x) = \infty$
 End behavior asym.: $y = 1$

$$4) f(x) = \frac{2x^2 - 12x + 16}{x^2 - x - 12}$$



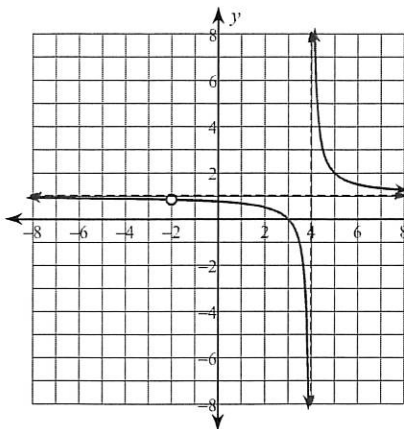
Discontinuities: $-3, 4$
 Holes: $x = 4$
 Horz. Asym.: $y = 2$
 x-intercepts: 2 , y-intercept: $-\frac{4}{3}$
 Domain:
 All reals except $-3, 4$
 Vert. Asym. behavior:
 $\lim_{x \rightarrow -3^-} f(x) = \infty$, $\lim_{x \rightarrow -3^+} f(x) = -\infty$
 End behavior asym.: $y = 2$

$$5) f(x) = \frac{x^2 + 2x - 3}{-3x - 6}$$



Discontinuities: -2
 Holes: None
 Horz. Asym.: None
 x-intercepts: $1, -3$, y-intercept: $\frac{1}{2}$
 Domain:
 All reals except -2
 Vert. Asym. behavior:
 $\lim_{x \rightarrow -2^-} f(x) = -\infty$, $\lim_{x \rightarrow -2^+} f(x) = \infty$
 End behavior asym.: $y = -\frac{x}{3}$

$$6) f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 8}$$



Discontinuities: $4, -2$
 Holes: $x = -2$
 Horz. Asym.: $y = 1$
 x-intercepts: 3 , y-intercept: $\frac{3}{4}$
 Domain:
 All reals except $4, -2$
 Vert. Asym. behavior:
 $\lim_{x \rightarrow -2^-} f(x) = -\infty$, $\lim_{x \rightarrow -2^+} f(x) = \infty$
 End behavior asym.: $y = 1$