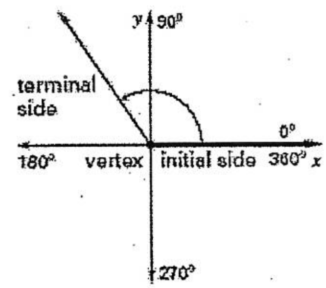
Precalculus Name Key

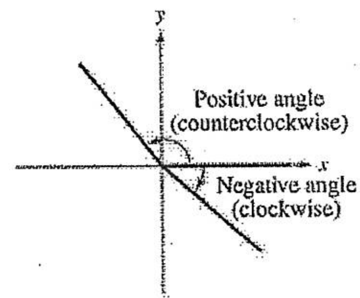
4.1 Notes: Radian and Degree Measure-*Day 1* *(Radians & Reference Angles)*

TRIGONOMETRY is the study of angles & triangles. We begin this unit with the basics of angles.

An angle is determined by rotating a ray about its endpoint. The starting position of the ray is the initial side of the angle, and the position after rotation is the terminal side. The endpoint of the ray is called the vertex of the angle. When the vertex of the angle is fixed at the origin of the coordinate plane with the initial side sitting on the positive x-axis, the angle is said to be in standard position.



Positive angles are generated by **counterclockwise** rotation and negative angles are generated by clockwise rotation. Angles that have the same initial and terminal sides are called coterminal angles. If the terminal side of an angle falls on the x-axis or the y-axis, then that angle is called a quadrantal angle.

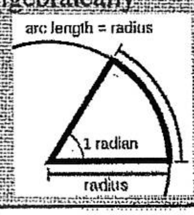


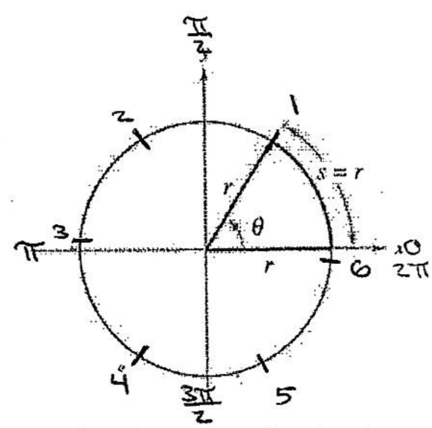
Definition of Radian

One radian (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically this means that

$$\theta = \frac{s}{r} = \frac{\text{arc length}}{\text{radius}} \qquad c = 2\pi r \qquad c = \frac{2\pi r}{r} \qquad c = 2\pi$$

where θ is measured in radians.





Arc length = radius when θ = 1 radian.

In other words… Radians are a way to measure angles in terms of the length of the radius. An *angle* of 1 radian results in an arc with a length equal to the radius of the circle.

The circumference of a circle is found using the formula: $2\pi r$

If $r = 1$ then the circumference is: 2π

1 revolution = 2π radians = 360 °

$\frac{1}{2}$ revolution = $\frac{2\pi}{2}$ = π radians = 180 °

$\frac{1}{4}$ revolution = $\frac{2\pi}{4}$ = $\frac{\pi}{2}$ radians = 90 °

$\frac{1}{6}$ revolution = $\frac{2\pi}{6}$ = $\frac{\pi}{3}$ radians = 60 °

$\frac{1}{8}$ revolution = $\frac{2\pi}{8}$ = $\frac{\pi}{4}$ radians = 45 °

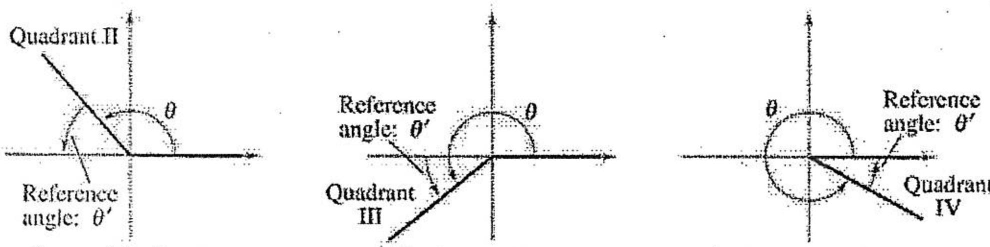
$\frac{1}{12}$ revolution = $\frac{2\pi}{12}$ = $\frac{\pi}{6}$ radians = 30 °

Reference Angles

The values of the trigonometric functions of angles greater than 90° (or less than 0°) can be determined from their values at the corresponding acute angles called **reference angles.**

Let θ be an angle in standard position. Its **reference angle** is the acute angle θ' formed by the terminal side of θ and the x-axis.

* Note: Reference angles are always positive.



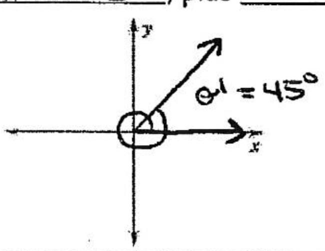
$\theta' = \pi - \theta$ (radians)	$\theta' = \theta - \pi$ (radians)	$\theta' = 2\pi - \theta$ (radians)
$\theta' = 180° - \theta$ (degrees)	$\theta' = \theta - 180°$ (degrees)	$\theta' = 360° - \theta$ (degrees)

EXAMPLE 1 - *Draw angles in standard position and finding reference angles*

Draw each angle in standard position. Then determine the reference angle (if it's not quadrantal).

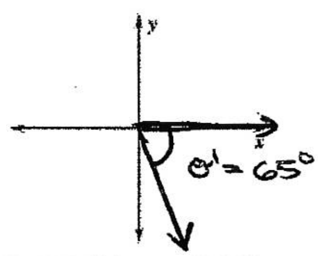
a. 405°

Because 405° is 45° more than 360°, the terminal side makes one whole revolution counter clockwise, plus 45° more.

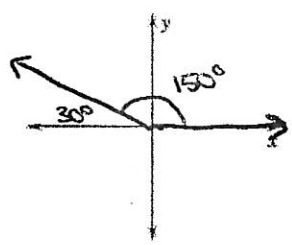


b. –65°

Because –65° is negative, the terminal side is clockwise from the positive x-axis.

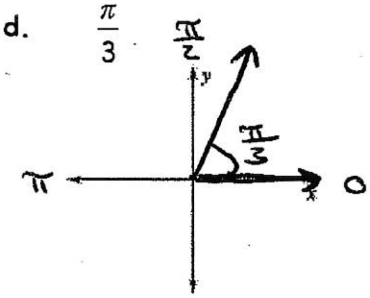


c. 150°



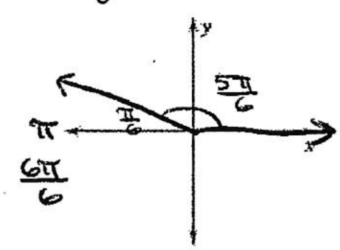
Reference Angle: 30°

d. $\frac{\pi}{3}$



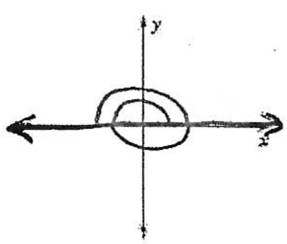
Reference Angle: $\frac{\pi}{3}$

e. $\frac{5\pi}{6}$



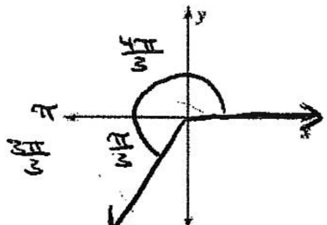
Reference Angle: $\frac{\pi}{6}$

f. 3π *Quadrantal



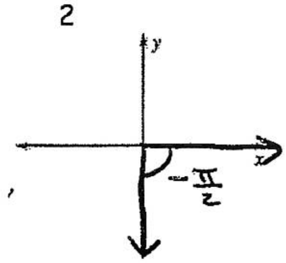
Reference Angle: N/A

g. $\frac{4\pi}{3}$



Reference Angle: $\frac{\pi}{3}$

h. $-\frac{\pi}{2}$

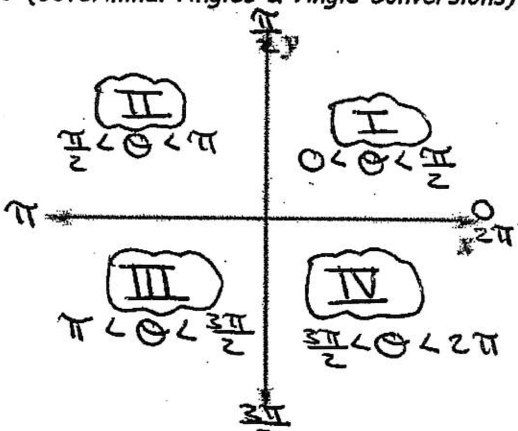


Reference Angle: N/A

Precalculus Name Key

4.1 Notes: Radian and Degree Measure-*Day 2* *(Coterminal Angles & Angle Conversions)*

Summarize the range of angle measures (in radians) for each Quadrant in the diagram to right.

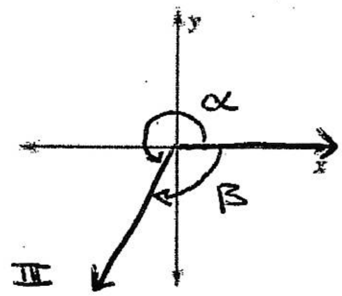


NOTE: If an angle is measured in degrees, you will see the degree symbol. Often, an angle measured in radians will have no units.

Angles are typically labeled in diagrams with Greek letters θ (theta), α (alpha), β (beta), etc.

We use these Greek letters as the variable to represent angles in diagrams, in formulas and in calculations.

In the diagram at the right, draw the terminal side of an angle in Quadrant III. Label the positive angle α and the negative angle β.



Notice how α and β have the same terminal side when drawn in standard position...

Remember...these angles are called coterminal angles.

EXAMPLE 2 - *Find coterminal angles* Answers may vary.

Find one positive and one negative angle that is coterminal with... * Infinite coterminal angles.

a. $\frac{5\pi}{6}$	b. $\frac{-3\pi}{4}$	c. $\frac{11\pi}{3}$
To find a positive angle with the same terminal side as $\frac{5\pi}{6}$, add one revolution of a circle or 2π. $\frac{5\pi}{6}+2\pi=\frac{5\pi}{6}+\frac{12\pi}{6}=\frac{17\pi}{6}$	$-\frac{3\pi}{4}+2\pi$ $-\frac{3\pi}{4}+\frac{8\pi}{4}=\frac{5\pi}{4}$ $\frac{5\pi}{4}+\frac{8\pi}{4}=\frac{13\pi}{4}$	$\frac{11\pi}{3}+2\pi$ $\frac{11\pi}{3}+\frac{6\pi}{3}=\frac{17\pi}{3}$ $\frac{11\pi}{3}-\frac{6\pi}{3}=\frac{5\pi}{3}$ * Subtract again!
To find a negative angle with the same terminal side as $\frac{5\pi}{6}$, subtract one revolution of a circle or 2π. $\frac{5\pi}{6}-2\pi=\frac{5\pi}{6}-\frac{12\pi}{6}=\frac{-7\pi}{6}$	$-\frac{3\pi}{4}-\frac{8\pi}{4}=\frac{-11\pi}{4}$ $-\frac{11\pi}{4}-\frac{8\pi}{4}=\frac{-19\pi}{4}$	$\frac{5\pi}{3}-\frac{6\pi}{3}=-\frac{\pi}{3}$ $-\frac{\pi}{3}-\frac{6\pi}{3}=\frac{-7\pi}{3}$

Finding coterminal angles measured in degrees is much simpler - just add or subtract 360°.

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\frac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship π rad $= 180°$. (See Figure 4.14.)

$$\frac{180°}{180} = \frac{\pi \text{ rad}}{180} \qquad \frac{\pi \text{ rad}}{\pi} = \frac{180°}{\pi}$$

$$1° = \frac{\pi \text{ rad}}{180°} \qquad 1 \text{ rad} = \frac{180°}{\pi}$$

$$1° \approx 0.0175 \text{ rad} \qquad 1 \text{ rad} \approx 57.296°$$

EXAMPLE 3 - *Convert between degree and radian units of measure*

Convert each angle measure into the "other" unit. Leave radian measures in terms of π.

a. $\theta = 75°$ $75°\left(\frac{\pi}{180°}\right)$ $\frac{75°\pi}{180°}$ $\frac{75\pi}{180}$ $\frac{5\pi}{12}$	b. $\theta = 320°$ $320°\left(\frac{\pi}{180°}\right)$ $\frac{320°\pi}{180°}$ $\frac{320\pi}{180}$ $\frac{16\pi}{9}$	c. $\theta = -45°$ $-45°\left(\frac{\pi}{180°}\right)$ $\frac{-45°\pi}{180°}$ $\frac{-45\pi}{180}$ $-\frac{\pi}{4}$
d. $\theta = \frac{5\pi}{3}$ $\frac{5\cancel{\pi}}{\cancel{3}}\left(\frac{\overset{60°}{\cancel{180°}}}{\cancel{\pi}}\right)$ *cross Reduce $300°$	e. $\theta = \frac{-13\pi}{4}$ $\frac{-13\cancel{\pi}}{\cancel{4}}\left(\frac{\overset{45°}{\cancel{180°}}}{\cancel{\pi}}\right)$ $-585°$	f. $\theta = 3$ ↑ Radian measure (almost π) $3\left(\frac{180°}{\pi}\right)$ $\frac{540°}{\pi}$ $\approx 171.887°$