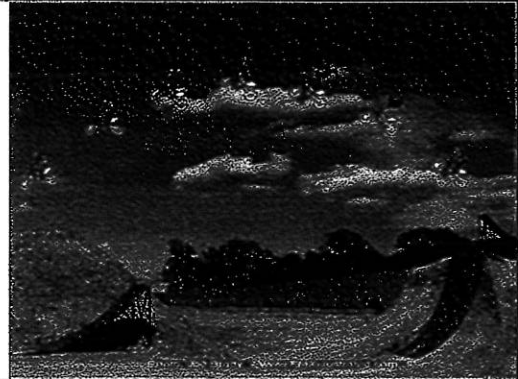


Vertical Motion Models

A *projectile* is an object that is propelled into the air but has no power to keep itself in the air. A thrown ball is a projectile, but an airplane is not. The height  $h$  (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where  $t$  is the time (in seconds),  $v$  is the initial vertical velocity (in feet per second), and  $s$  is the initial height (in feet).



For each problem scenario, write a vertical motion equation, sketch a graph of the projectile's motion, and solve appropriately.

1. A startled armadillo jumps straight into the air with an initial vertical velocity of 14 feet per second. After how many seconds does it land on the ground?

$$v = 14$$

$$s = 0$$

→ Solve for  $t$  when  $h = 0$

$$h = -16t^2 + 14t + 0$$

$$0 = -16t^2 + 14t$$

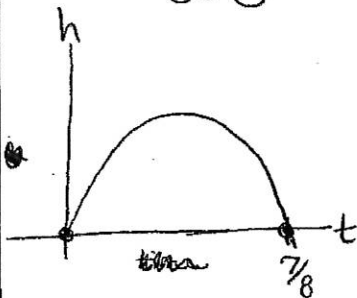
$$0 = -2t(8t - 7)$$

$$-2t = 0 \quad 8t - 7 = 0$$

$$t = 0 \quad 8t = 7$$

$$t = \frac{7}{8}$$

$\frac{7}{8}$  second



2. An athlete throws a discus from an initial height of 6 feet and with an initial vertical velocity of 46 feet per second. Write an equation that gives the height in feet of the discus as a function of time (in seconds) since it left the athlete's hand. After how many seconds does the discus hit the ground? → solve for  $t$  when  $h = 0$

$$s = 6$$

$$v = 46$$

$$h = -16t^2 + 46t + 6$$

$$0 = -16t^2 + 46t + 6$$

$$0 = -2(8t^2 - 23t - 3)$$

$$0 = (8t^2 + t)(24t - 3)$$

$$0 = t(8t + 1) - 3(8t + 1)$$

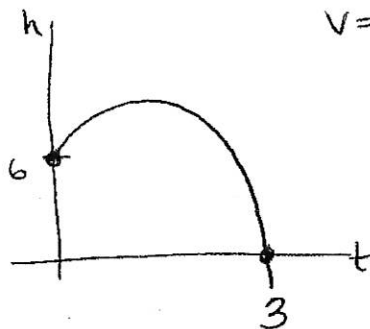
$$0 = (t - 3)(8t + 1)$$

$$t - 3 = 0 \quad 8t + 1 = 0$$

$$t = 3 \quad 8t = -1$$

$$\frac{ac = -24}{b = -23}$$

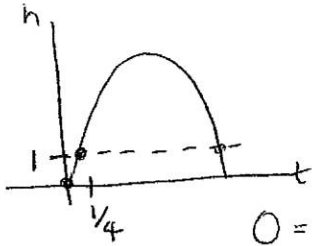
$$\frac{1, -24}{-23}$$



{ 3 seconds } ~~t = -1/8~~



3. A grasshopper jumps straight up from the ground with an initial vertical velocity of 8 feet per second. After how many seconds will the grasshopper be 1 foot off of the ground?



$$s=0$$

$$v=8$$

$$h = -16t^2 + 8t + 0$$

$$1 = -16t^2 + 8t$$

$$0 = -16t^2 + 8t - 1$$

$$0 = -1(16t^2 - 8t + 1)$$

$$0 = (16t^2 - 4t)(4t + 1)$$

$$0 = 4t(4t - 1) - 1(4t - 1)$$

$$\frac{ac = 16}{b = -8}$$

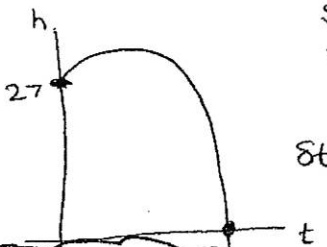
$$-4, 4$$

$$0 = (4t - 1)(4t - 1)$$

$$4t - 1 = 0$$

$$t = \frac{1}{4} \text{ sec}$$

4. A squirrel is 27 feet up in a tree and tosses a nut out of the tree with an initial vertical velocity of 6 feet per second. The squirrel climbs down the tree in 2 seconds. Does it reach the ground before the nut?



$$s = 27$$

$$v = 6$$

$$h = -16t^2 + 6t + 27$$

$$0 = -16t^2 + 6t + 27$$

$$0 = -1(16t^2 - 6t - 27)$$

$$0 = (16t^2 - 24t) + (18t - 27)$$

$$0 = 8t(2t - 3) + 9(2t - 3)$$

$$0 = (8t + 9)(2t - 3)$$

$$ac = -432$$

$$b = -6$$

$$-24, 18$$

$$8t + 9 = 0$$

$$8t = -9$$

$$t = -\frac{9}{8}$$

$$2t - 3 = 0$$

$$t = \frac{3}{2}$$

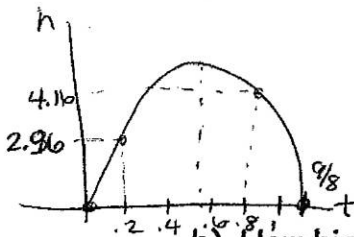
The nut reaches the ground first

5. A kangaroo can jump with an initial vertical velocity of 18 feet per second.

$$v = 18$$

$$s = 0$$

- a) When will the kangaroo land on the ground?



$$h = -16t^2 + 18t + 0$$

$$0 = -16t^2 + 18t$$

$$0 = -2t(8t - 9)$$

$$-2t = 0$$

$$t = 0$$

$$8t - 9 = 0$$

$$t = \frac{9}{8} \text{ second}$$

- b) How high is the kangaroo after 0.2 seconds?

$$t = 0.2$$

$$h = -16(0.2)^2 + 18(0.2) + 0$$

$$h = 2.96 \text{ feet}$$

- c) How high is the kangaroo after 0.8 seconds?

$$t = 0.8$$

$$h = -16(0.8)^2 + 18(0.8) + 0$$

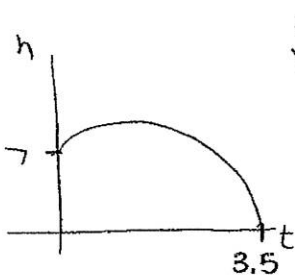
$$h = 4.16$$

- d) When does the kangaroo reach its maximum height? Approximately how high does it jump?

~ around .5 → .6 seconds the kangaroo will reach its maximum height, which is about 5 feet



6. A baseball player releases a baseball at a height of 7 feet with an initial velocity of 54 feet per second. How long will it take the ball to reach the ground?



$$S = 7$$

$$V = 54$$

$$h = -16t^2 + 54t + 7$$

$$0 = -16t^2 + 54t + 7$$

$$0 = -1(16t^2 - 54t - 7)$$

$$0 = (16t^2 + 2)(56t - 7)$$

$$0 = 2t(8t + 1) - 7(8t + 1)$$

$$0 = (2t - 7)(8t + 1) = 0$$

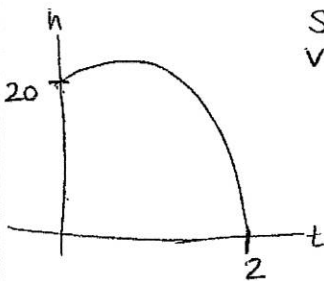
$$t = 7/2 \quad t = -1/8$$

$$\frac{ac = -112}{b = -54}$$

$$\frac{\quad}{2, 56}$$

3.5 seconds

7. A miniature rocket is launched off a roof 20 feet above the ground with an initial velocity of 22 feet per second. How much time will elapse before the rocket reaches the ground?



$$S = 20$$

$$V = 22$$

$$h = -16t^2 + 22t + 20$$

$$0 = -16t^2 + 22t + 20$$

$$0 = -2(8t^2 - 11t - 10)$$

$$0 = (8t^2 - 16t) + (5t - 10)$$

$$0 = 8t(t - 2) + 5(t - 2)$$

$$0 = (8t + 5)(t - 2) = 0$$

$$t = -5/8 \quad t = 2$$

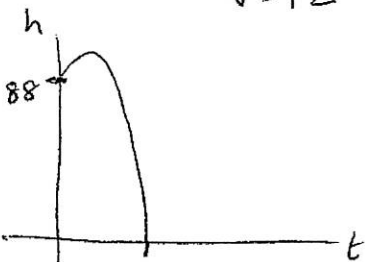
$$\frac{ac = -80}{b = -11}$$

$$\frac{\quad}{8, 10}$$

$$\frac{\quad}{5, 16}$$

2 seconds

8. A cliff diver jumps from a ledge 88 feet above the ocean with an initial upward velocity of 12 feet per second. How long will it take until the diver enters the water?



$$S = 88$$

$$V = 12$$

$$h = -16t^2 + 12t + 88$$

$$0 = -16t^2 + 12t + 88$$

$$0 = -4(4t^2 - 3t - 22)$$

$$0 = (4t^2 + 8t) + (-11t - 22)$$

$$0 = 4t(t + 2) - 11(t + 2)$$

$$0 = (4t - 11)(t + 2) = 0$$

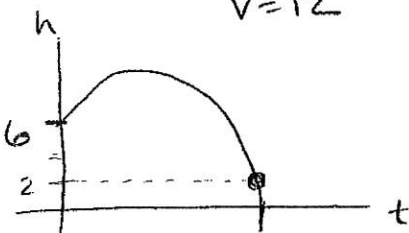
$$t = 11/4 \quad t = -2$$

$$\frac{ac = -88}{b = -3}$$

$$\frac{\quad}{8, 11}$$

2.75 seconds

9. You throw a football from a height of 6 feet into the air with an initial vertical velocity of 12 feet per second. The football is caught at a height of 2 feet. After how many seconds is the football caught?



$$S = 6$$

$$V = 12$$

$$h = -16t^2 + 12t + 6$$

$$2 = -16t^2 + 12t + 6$$

$$16t^2 - 12t - 4 = 0$$

$$4(4t^2 - 3t - 1) = 0$$

$$(4t^2 - 4t) + (t - 1) = 0$$

$$4t(t - 1) + 1(t - 1) = 0$$

$$\frac{ac = -4}{b = -3}$$

$$\frac{\quad}{1, 4}$$

$$0 = (4t + 1)(t - 1) = 0$$

$$t = -1/4 \quad t = 1$$

1 second



10. You hit a badminton birdie upward with a racket from a height of 2 feet with an initial velocity of 4 feet per second.

$$s = 2$$

$$v = 4$$

- a) How high is the birdie at 0.1 seconds?

$$h = -16(0.1)^2 + 4(0.1) + 2 = 2.24 \text{ feet}$$

$$h = -16t^2 + 4t + 2$$

- b) How high is the birdie at 0.25 seconds?

$$h = -16(0.25)^2 + 4(0.25) + 2 = 2 \text{ feet}$$

- c) How long will it take the birdie to reach the ground?

0.5 seconds

$$0 = -16t^2 + 4t + 2$$

$$0 = \cancel{16}(16t^2 - 4t - 2)$$

$$0 = (16t^2 + 4t)(8t - 2)$$

$$0 = 4t(4t + 1) - 2(4t + 1)$$

$$0 = (4t - 2)(4t + 1)$$

$$4t - 2 = 0$$

$$t = \frac{2}{4}$$

$$t = \frac{1}{2}$$

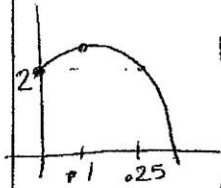
$$4t + 1 = 0$$

$$t = -\frac{1}{4}$$

$$a.c. = -32$$

$$b = -4$$

$$\frac{4}{8}$$



11. A spittlebug jumps into the air with an initial vertical velocity of 10 feet per second.

$$v = 10$$

$$s = 0$$

- a) When will the spittlebug land on the ground?

$$h = -16t^2 + 10t + 0$$

$$0 = -16t^2 + 10t$$

$$0 = -2t(8t - 5)$$

$$-2t = 0 \quad 8t - 5 = 0$$

$$t = 0$$

$$8t = 5$$

$$t = \frac{5}{8} \text{ second}$$

- b) The spittlebug reaches its maximum height after 0.3125 second. How high can it jump?

$$t = 0.3125$$

$$h = -16(0.3125)^2 + 10(0.3125)$$

$$h = 1.5625 \text{ feet}$$

- c) How does the time in part (b) relate to the time in part (a)?

$$0.3125$$

$$\frac{5}{8} = .625$$

$$\rightarrow 0.3125 = \frac{1}{2} (.625)$$

- d) What relationship do you think there is between these coordinates in parts (a) and (b)? Examine a sketch of the spittlebug's jump to help.

The maximum height occurs halfway between the 2 x-intercepts (when the bug starts & lands the jump)

