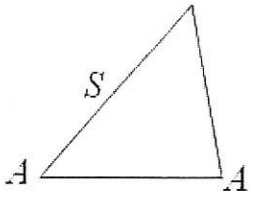
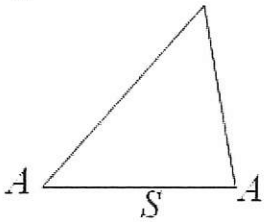
Key

Using the Law of Sines

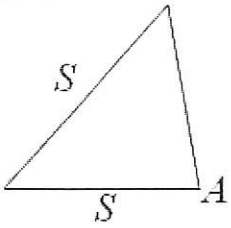
If S represents a given side of a triangle and if A represents a given angle of a triangle, then the Law of Sines can be used to solve the three oblique triangle cases shown below.



The SAA Case



The ASA Case

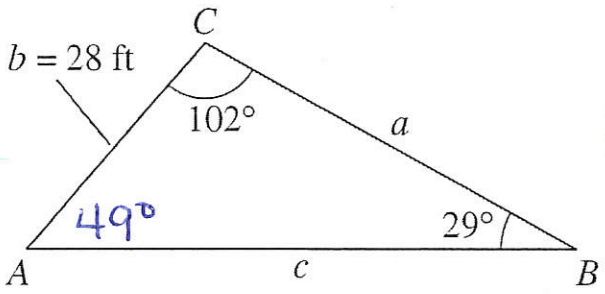


The SSA Case
(The Ambiguous Case)

To use the LOS, you must have an ANGLE-OPPOSITE SIDE 'MATCH' from which you can create a ratio.

SAA:

For the triangle below, $C = 102°$, $B = 29°$, and $b = 28$ feet. Find the remaining angle and sides.(SOLVE THE TRIANGLE)



1. Determine the third angle of the triangle -

$A = 180° - B - C$

$= 180° - 29° - 102°$

$= 49°.$

Then, apply the LOS to solve the triangle: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 29}{28} = \frac{\sin 102}{c}$

$c = \frac{28 \sin 102}{\sin 29} = 56.5 \text{ ft}$

$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 29}{28} = \frac{\sin 49}{a}$

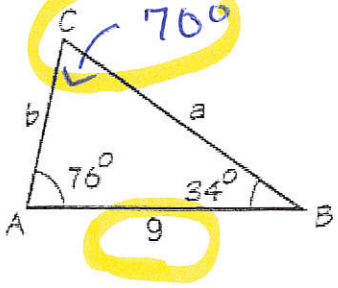
$a = \frac{28 \sin 49}{\sin 29} = 43.6 \text{ ft}$

ASA Again, use 180 degrees to determine the missing angle. Then, create a ratio! Always return to your 'BASE RATIO" to solve for other parts of the triangle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

180 − (76 + 34)

180 − (110)



$\frac{\sin C}{c} = \frac{\sin A}{a}$ $\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin 70}{9} = \frac{\sin 76}{a}$ $\frac{\sin 70}{9} = \frac{\sin 34}{b}$

$\frac{9 \sin 76}{\sin 70} = a = 9.3$

$b = \frac{9 \sin 34}{\sin 70}$

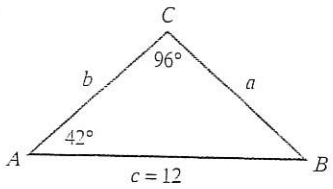
$b = 5.36$

EXERCISE SET 6.1

Practice Exercises

In Exercises 1–8, solve each triangle. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

1.



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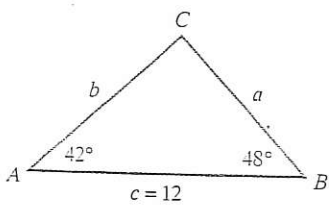

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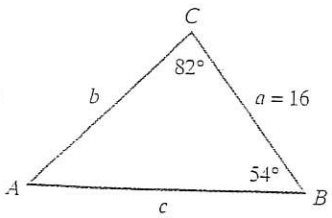

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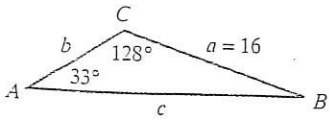

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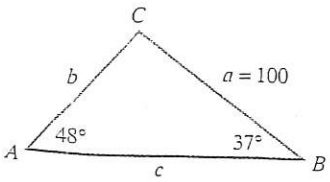

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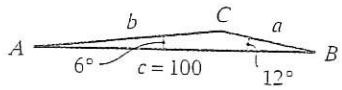

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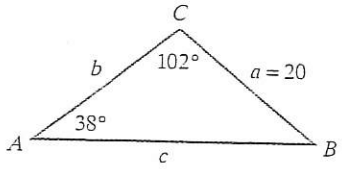

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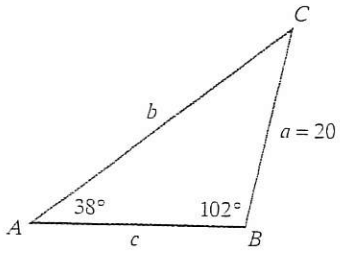

In Exercises 9–16, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

9. $A = 44°, B = 25°, a = 12$
10. $A = 56°, C = 24°, a = 22$
11. $B = 85°, C = 15°, b = 40$
12. $A = 85°, B = 35°, c = 30$
13. $A = 115°, C = 35°, c = 200$
14. $B = 5°, C = 125°, b = 200$
15. $A = 65°, B = 65°, c = 6$
16. $B = 80°, C = 10°, a = 8$

In Exercises 17–32, two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results. Round to the nearest tenth and the nearest degree for sides and angles, respectively.

17. $a = 20, b = 15, A = 40°$
18. $a = 30, b = 20, A = 50°$
19. $a = 10, c = 8.9, A = 63°$
20. $a = 57.5, c = 49.8, A = 136°$