Unit 8 Trig Application Summative Review #1 ______ Key
CP Pre-Calculus

Law of SINES states that: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

LOS is used with ASA, SAA and SSA triangles.

You must check for the ambiguous case with which type of triangle? SSA

State the Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

In solving an SAS triangle using LOC, after you find the side that corresponds to the given angle, you use the LOS to determine the measure of the smallest remaining angle first.

In solving an SSS triangle, you must use the LOC and determine the largest angle first in order to avoid the ambiguous case with the LOS.

State the 2 area formulas for triangles used in this unit:

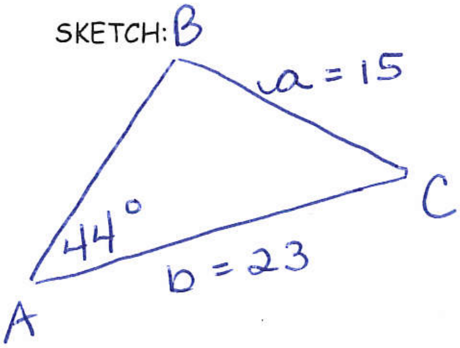
$K = \frac{1}{2}ab\sin C$ $\quad A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

You may only use SOL-CAH-TOA (SOH-CAH-TOA) if you have a RIGHT triangle.

1) In triangle ABC, identify the side-angle pattern given. Then identify the number of possible solutions. Then solve each triangle, if possible. Round all angles to the nearest degree and all side lengths to the nearest hundredth. Show all work.

a) m∠A = 44°, b = 23, a = 15 Number of Solutions______ B = ______ C = ______

none

SKETCH: 

Side-Angle Pattern: ______ c = ______

Use the ______.

$\frac{\sin 44}{15} = \frac{\sin B}{23}$ $\quad \sin B = \frac{23\sin 44}{15}$

$\sin B = 1.065$

→ greater than 1

∴ no Δ exists

OR $\sin^{-1}(1.065) = \text{ERROR}$

$\emptyset$

ASS - SSA
check for
amb. case.

b) m$\angle A = 52°$, $a = 51$, $b = 63$

Number of Solutions 2

$B = 77°/103$ $C = 51°/25$

Side-Angle Pattern: SSA

$c_1 = 50.30$ / $c_2 = 27.35$

Use the LOS

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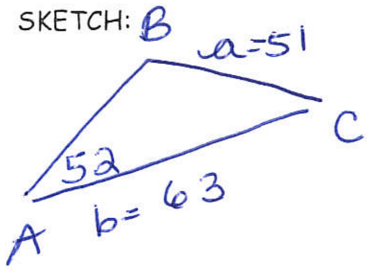


$$\frac{\sin 52}{51} = \frac{\sin B}{63} \qquad \sin B = \frac{63 \sin 52}{51}$$

$\sin B = .9734$

$\sin^{-1}(.9734) =$

$B_1 = 76.755$

SSA → check for 2 ∆s. Is there a ∆2?

$\therefore B_1 \approx 77°$

if $B_1 = 77°$

the $B_2 = 180 - 77$, if Δ_2 exists

$= 103°$

$C_1 = 180 - (52 + 77)$
$= 180 - 129$
$= 51°$

$$\frac{\sin 52}{51} = \frac{\sin 51}{c}$$

$$c = \frac{51 \sin 51}{\sin 52} = \boxed{c = 50.30}$$

will this work? (add given ∡)

$$\begin{array}{r} 52 \\ \hline 155° \end{array}$$

✓ yes → 2∆s.

so $C_2 = 180 - 155 = 25°$

$c_2 \rightarrow \dfrac{\sin 52}{51} = \dfrac{\sin 25}{c_2}$

$$c_2 = \frac{51 \sin 25}{\sin 52}$$

$c_2 = 27.35$

c) $a = 64$, $c = 90$, m$\angle C = 98°$

Number of Solutions 1

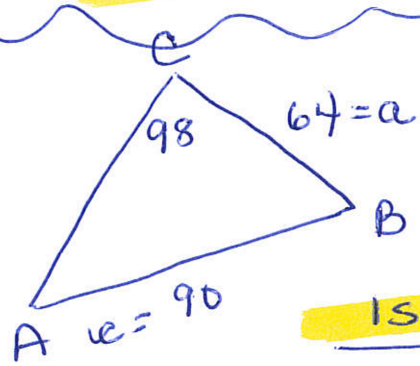
$A = 45°$ $B = 37°$

Side-Angle Pattern: SSA

$b = 54.7$

Use the LOS

SKETCH:



$$\frac{\sin 98}{90} = \frac{\sin A}{64}$$

$$\sin A = \frac{64 \sin 98}{90} \approx .7042$$

$\sin^{-1} .7042 \approx \boxed{45° = A_1}$

Is there a 2nd∆?

SSA

$A_1 = 45$

$A_2 = 180 - 45$ if it exists

$A_2 = 135°$
$+98°$

TOO BIG → one ∆

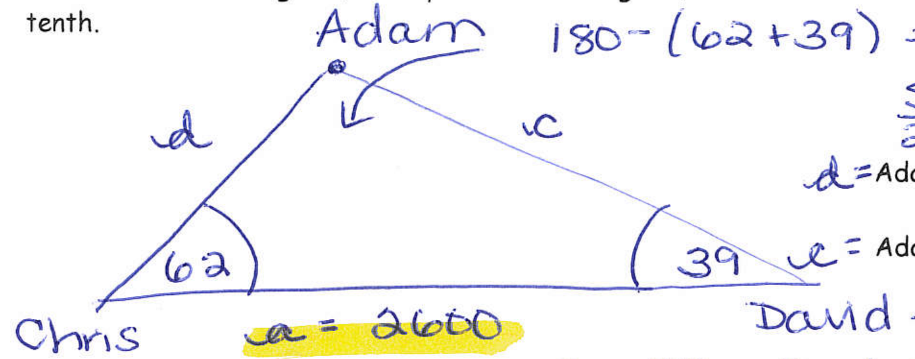
$\therefore B_1 = 180 - (45 + 98)$
$= 180 - 143$

$\boxed{B_1 = 37°}$

$$\frac{\sin 98}{90} = \frac{\sin 37}{b}$$

$b_1 = 54.7$

2) Adam is directly over a 2600 meter landing strip in his hot-air balloon and is observed at one end of the strip by David with an angle of elevation measuring 39°. Meanwhile, at the other end of the strip, Chris observes the balloon with an angle of elevation of 62°. Find the distances between Adam and each of his friends. Draw a diagram, clearly label each diagram, and show all work. Round all answers to the nearest tenth.



$180-(62+39) = 180-101 = 79^{0}$

$\frac{\sin 79}{2600} = \frac{\sin 39}{d}$ $d = \frac{2600\sin 39}{\sin 79}$

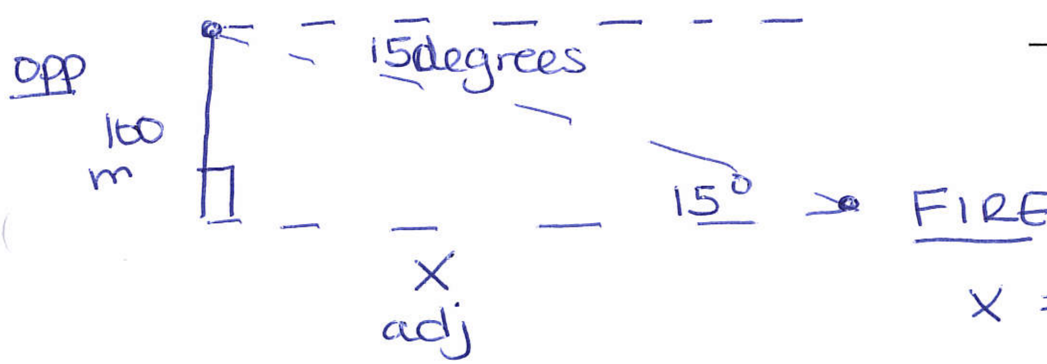
d = Adam to Chris 1666.86 meters

c = Adam to David 2459 meters

ASA → LOS:

$\frac{\sin 79}{2600} = \frac{\sin 62}{c}$ $c = \frac{2600\sin 62}{\sin 79}$

3) A forest ranger spots a forest fire from the top of an observation tower 100 meters high. The angle of depression from the tower is measured to be 15°. To the nearest meter, how far is the fire from the base of the tower?



373 meters

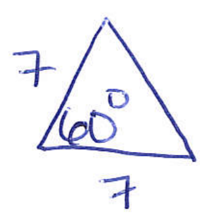
$\tan 15 = \frac{100}{x}$

$x = \frac{100}{\tan 15} \approx 373$ meters

4) A regular hexagon is inscribed in a circle with a radius of 7 cm. Find the area of the hexagon to the nearest hundredth.

hexagon

$360 \div 6 = 60^{0}$



127.31 cm^2

$A_{\Delta_1} = \frac{1}{2} ab \sin C$

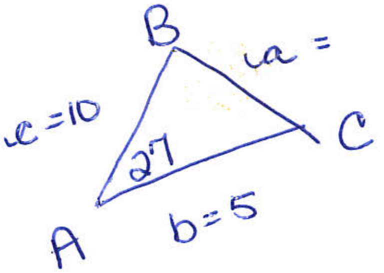
$A_{\Delta_1} = \frac{1}{2}(7)(7)\sin 60$

$A_{hex} = 6 \cdot \frac{1}{2} \cdot 7 \cdot 7 \cdot \sin 60$

$A_{hex} = 127.31$

5) Solve triangle ABC given: Find all measurements to two decimal places.

a) b = 5, c = 10, A = 27°   SAS →   a = 6   B = 22.23°   C = 130.77



LOC, then find smallest Δ first.

$a^2 = 10^2 + 5^2 - 2(10)(5)\cos 27$

$a^2 = 125 - 100\cos 27$

$a^2 = 35.8993 \approx 6$

$\boxed{a \approx 6}$

$\frac{\sin 27}{6} = \frac{\sin B}{5}$

$\sin B = (5\sin 27) \div 6 = .3783$

$\sin^{-1} .3783 \approx 22.23°$

$C = 180 - (27 + 22.23)$

$C = 180 - (49.23)$

$C = 130.77$

b) a = 8, b = 4, c = 11   SSS →   A = 24.46°   B = 12.70°   C = 142.84°

LOC lgst 1st

$11^2 = 8^2 + 4^2 - 2(8)(4)\cos C$

$121 = 64 + 16 - 64\cos C$

$121 = 80 - 64\cos C$

$51 = -64\cos C$

$-.7969 = \cos C$

$\cos^{-1}(-.7969) = 142.84°$

$\frac{\sin 142.84}{11} = \frac{\sin B}{4}$

$\sin B = .2197$

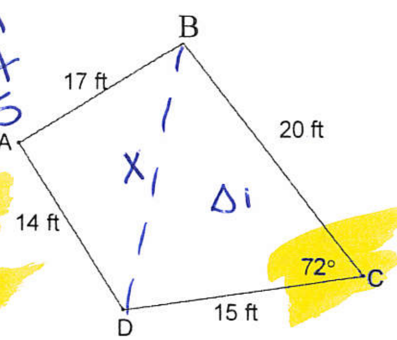
$B = \sin^{-1} .2197$

$B = 12.70$

$C = 24.46$

6) Find the area of the irregular polygon pictured below:

Total 142.7 + 118.5 = 261.2 ft²



Have SAS

Find X: LOC

$c^2 = 15^2 + 20^2 - 2(20)(15)\cos 72$

$c^2 = 225 + 400 - 600\cos 72$

$c^2 = 625 - 600\cos 72$

$c^2 = 439.6$

$c \approx 20.966 \approx 21$

$A_{\Delta 1} = \frac{1}{2} ab \sin C$

$A_{\Delta 1} = \frac{1}{2}(20)(15)\sin 72$

$= 150 \sin 72$

$\boxed{A_{\Delta 1} = 142.7}$

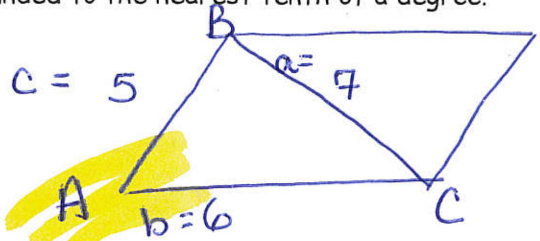
$A_{\Delta 2} = \sqrt{26(26-21)(26-17)(26-14)}$

$s = \frac{17+14+21}{2} = 26$

$= \sqrt{26(5)(9)(12)}$

$\boxed{A_{\Delta 2} \approx 118.51}$

7) Two adjacent sides of a parallelogram form an acute angle and have side lengths of 5 cm and 6 cm. The shorter diagonal measures 7 cm. Find the measure of the acute angle formed by the 5 cm and 6 cm sides rounded to the nearest tenth of a degree.



$a^2 = b^2 + c^2 - 2bc\cos A$

$7^2 = 6^2 + 5^2 - 2(6)(5)\cos A$

$49 = 36 + 25 - 60\cos A$

$49 = 61 - 60\cos A$ $\quad \cos A = .2$

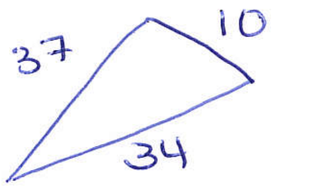
$-12 = -60\cos A$

$A = 78.46°$

8). Ms. Miller is planting a triangular garden with sides measuring 37 feet, 34 feet and 10 feet.

She buys a bag of fertilizer that says it covers 150 square feet. Should she have bought an additional bag?

Show and label all work, use specific calculations and explain your reasoning.



$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{40.5(40.5-10)(40.5-37)(40.5-34)}$

$A = \sqrt{40.5(30.5)(3.5)(6.5)} \approx 167.64 \text{ ft}^2$

$s = \dfrac{37+34+10}{2}$

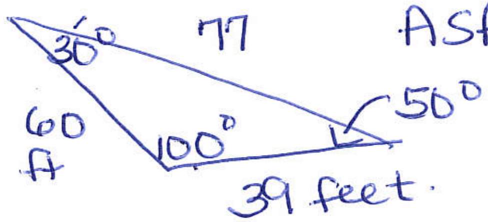
$= 40.5$

Yes!

$\div 150 = 1.117$ bags

9). Ms. Miller is considering another garden on a triangular piece of land that has angle measurements of 100 and 30 degrees, with the side IN BETWEEN these angles measuring 60 feet. She does not want the bear family to parade through her garden, so she would like to fence it in.

A. How many feet of fencing would she need?



ASA:

Perimeter ⇒ LOS

$\dfrac{\sin 50}{60} = \dfrac{\sin 100}{x}$

$x = \dfrac{60\sin 100}{\sin 50} \approx 77 \text{ ft}.$

$\dfrac{\sin 50}{60} = \dfrac{\sin 30}{y}$

$y = \dfrac{60\sin 30}{\sin 50} = \dfrac{30}{\sin 50}$

$\approx 39 \text{ ft}.$

$P = 39 + 60 + 77 = 99 + 77$

$P = 176 \text{ ft}.$

B. How big will the garden be in square feet?

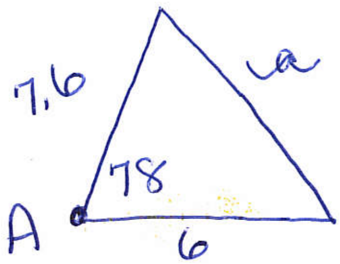
$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}\, 60 \cdot 39 \sin 100$

$\frac{1}{2}\, 77 \cdot 39 \cdot \sin 50$

$A \approx 1152;\ 1150 \text{ ft}^2$

10). Ms. Miller and her sister get into a fight at the Corner of West Avon and Country Club. They huff off in different directions, their courses diverging by 78 degrees. If Ms. Miller can walk at 3.8 miles per hour and her sister can walk at a rate of 3 miles per hour, how far apart will they be in 2 hours?



SAS → LOC

$a^2 = 36 + 7.6^2 - 2(6)(7.6)\cos 78$

$a^2 \quad 36 + 57.76 - 91.2\cos 78$

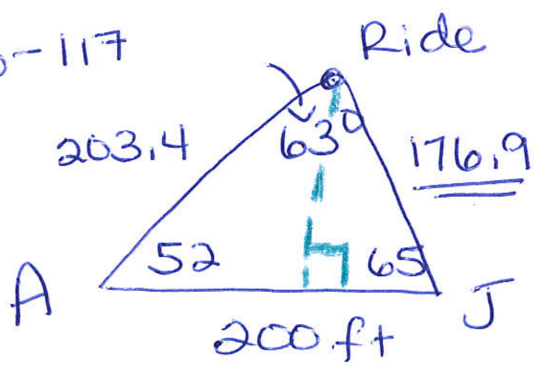
$a^2 = 74.8$

$a = 8.65$ miles apart.

11). On the Lake Compounce Field Trip, Alex and Jason determine that the height of the highest attraction was not as advertised. Alex stood on one side of the ride, while Jason stood on the other, (making a line segment as all were aligned), Each used trig to determine that Alex was standing to form an angle of elevation with the top of the ride of 52 degrees, while Jason calculated that the angle of elevation from his standing point was 65 degrees. What did Alex and Jason say was the height of the ride?

Alex and Jason are 200 feet apart.

180 − 117



$\frac{\sin 63}{200} = \frac{\sin 52}{a}$

$a = \frac{200\sin 52}{\sin 63}$

$a = 176.9$

$\sin 65 = \frac{opp}{176.9}$

$176.9\sin 65 = opp$

160.325 feet

OR

vs.

$\frac{\sin 63}{200} = \frac{\sin 65}{j}$

$j = \frac{200\sin 65}{\sin 63}$

$j = 203.4$

$\sin 52 = \frac{opp}{203.4}$

$opp = 203.4\sin 52 = 160.3$ feet.