CP Pre-Calculus Unit 3 Summative Review Exponentials and Logs A

Key

Solve radical equations:

Solve: 1. $\sqrt{x^2-6x}+6=x$

$\sqrt{x^2-6x} = x-6$

$x^2-6x = x^2-12x+36$

$6x-36=0$

$x-6=0$

$\boxed{x=6}$ ✓

2. $-2\sqrt{x-4}+6=10$

$\sqrt{x-4} = -2$

$\emptyset$

3. If your parents wanted to have \$50,000 in your bank account on your 21st Birthday, how much would they have had to deposit on the day you were born if the account paid 5% compounded monthly?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$50{,}000 = P\left(1+\frac{.05}{12}\right)^{21(12)}$

4. Given the exponential model, state the initial amount, whether it represents growth or decay, and the rate as a percent.

	Initial amount	growth/decay	rate ask a percent
$y=100(1.043)^t$	100	↑	4.3%
$A=2000e^{-.036t}$	2000	↓	3.6
$A=5.3(.97)^t$	5.3	↓	↓3%

5. DEFINITION OF LOGS states: $\log_b a = p$ iff $b^p = a$

Write in exponential form. Solve if appropriate

6) $\log_2 1024 = 10$

$2^{10} = 1024$

6) $\log_2 25 = x$

$2^x = 25$

$\frac{1}{5}x^2 = 25$ $\quad 5^{-1x\cdot 2} = 5^2$

$x=-2$

8) $\ln 1 = x$

$\log_e 1 = x$

$e^x = 1$

$e=0$

When working with logs, always rewrite radicals as RATIONAL EXPONENTS!

Write in logarithmic form.

9) $3^x = 27$

$\log_3 27 = x$

$x=3$

10) $x^5 = 98$

$\log_x 98 = 5$

11) $4^{2.5} = x$

$4^{\frac{5}{2}} = x$

$\log_4 x = \frac{5}{2}$

$4^{\frac{5}{2}} = \pm 2^5$

$= \pm 32$

32

12. Find 3 points on the graph of $f(x) = 3^x$ 13. Use the points in #12 to find 3 points on $g(x) = \log_3 x$

x	f(x)
0	1
1	3
2	9

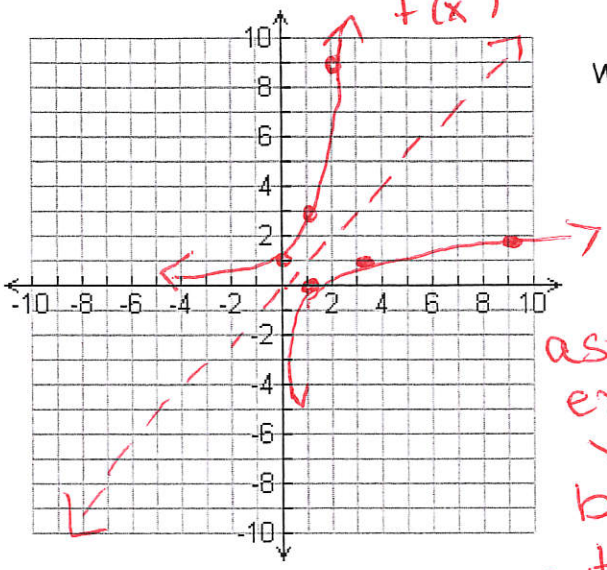
inverse fns →

x	f(x)
1	0
3	1
9	2

14. Sketch the graphs of each function above.

A. What property of inverse functions is evident from the graph? reflection across line $y = x$

B. What is the y intercept of each the exponential function? Why? $(0, 1)$ $base^0 = 1$



What is the x intercept of the log function? Why? $(1, 0)$ $base^0 = 1$

asymptote exp. fn $y = 0$ because # power $\neq 0$ non-zero →

$y = \log_3 x$

$0 = \log_3 1$

iff $3^0 = 1$ "

asympt. log fn $x = 0$

$y = \log_3 0$

$3^y \neq 0$

impossible

15. Using what you know about logs, what is the domain of y = log(x-5) ?

Argument must be > 0.

So $x - 5 > 0$

$x > 5$

$(5, \infty)$

PROPERTIES OF LOGS:

Product Property $\log MN = \log M + \log N$

Quotient Property $\log \frac{M}{N} = \log M - \log N$

Power Property $\log m^p = p \log m$

LNe = 1 ($\log_e e$) LOG1 = 0 ($\log_{10} 1$) $b^{\log_b x} = x$ $\log_b b^x = x$

Write as a single logarithm. (CONDENSE)

16) $2\log_a 3 + 3\log_a 2$

$= \log_a 3^2 + \log_a 2^3$

$= \log_a 9 + \log_a 8$

$= \log_8 72$

17) $\frac{1}{2}\log_a 16 + \frac{1}{3}\log_a 27$

$\log_a 4 + \log_a 3$

$= \log_a 12$

18) $\log x^3 - 2\log x$

$\log x^3 - \log x^2$

$= \log \frac{x^3}{x^2} = \log x$

20. $3\text{Log}_4 2 + \frac{1}{3}\text{Log}_4 x$

$\log_4 2^3 + \log_4 \sqrt[3]{x}$

$= \log_4 8 + \log_4 \sqrt[3]{x}$

$= \log_4 8\sqrt[3]{x}$

21. $\frac{1}{2}\text{Log}_2 25 - 3\text{Log}_2 x$

$\log_a 5 - \log_a x^3$

$= \log_a \frac{5}{x^3}$

22. $\text{Log } 4 - (\frac{1}{3}\text{Log } 64 + 3\text{Log } 2)$

$\log 4 - (\log 4 + \log 8) = \log 4 - (\log 32)$

$= \log \frac{4}{32} = \log \frac{1}{8}$

EXPAND THE FOLLOWING

23. $\text{Log}_6 \frac{5x^3}{y}$ ← binding op

$\log_6 5x^3 - \log_6 y$

$\log_6 5 + \log_6 x^3 - \log_6 y$

$\log_6 5 + 3\log_6 x - \log_6 y$

24. $\log(4x)^3$

$3[\log(4x)]$

$= 3[\log 4 + \log x]$

$= 3\log 4 + 3\log x$

25. Expand. $\text{Log}_{10} \frac{x\sqrt{y}}{z^5}$ **this might be tricky...remember your rational exponents!!

↑ binding op

$\log \frac{x y^{\frac{1}{2}}}{z^5}$

$\log x + \log y^{\frac{1}{2}} - \log z^5 = \log x + \frac{1}{2}\log y - 5\log z$

NO CALC

Simplify each expression or solve for x. Round ALL Decimals to the ten-thousandths place

26) $\log_{.25} 64$

$\log_{\frac{1}{4}} 64 = x$

$\frac{1}{4}^x = 64$

$4^{-1x} = 4^3$

$x = -3$

27) $\log_2 (1/8) = x$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$x = -3$

28) $\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$

$5^x = 5^{-3}$

$x = -3$

29) $\log_{\frac{1}{2}} 16 = x$

$\frac{1}{2}^x = 16$

$2^{-1x} = 2^4$

$x = -4$

30) $\log_2 1 = x$

$2^x = 1$

$x = 0$

31) $5\ln e$

$5(1)$

5

SOLVE

32) $4\log_2 x + \log_2 5 = \log_2 405$

condense; m=N

$\log_2 x^4 + \log_2 5 = \log_2 405$

$\log_2 5x^4 = \log_2 405$

$5x^4 = 405$ $\quad x^4 = 81$

$x = \pm 3$

$\boxed{x = 3}$

33) $\log_6(2x-5) + 1 = \log_6(7x+10)$

isolate, condense, def.

$\log_6 \frac{(2x-5)}{(7x+10)} = -1$

$\frac{1}{6} = \frac{2x-5}{7x+10}$ $\quad \rightarrow 12x - 30 = 7x + 10$

$5x = 40$

$\boxed{x = 8}$

34) $140 = 7e^{3k} + 28$

isolate; ln both sides

$112 = 7e^{3k}$

$\frac{112}{7} = e^{3k}$

$16 = e^{3k}$

$\ln 16 = \ln e^{3k}$

$\ln 16 = 3k$

$\ln 16 \div 3 = k$

$k = \_\_\_\_$

35) $3(\ln 5x) = 24$

isolate defn.

$\ln 5x = 8$

$\log_e 5x = 8$

$e^8 = 5x$

$\boxed{x = e^8/5}$

36) $4^{x+1} = 8^{2x+3}$

like bases

$2^{2(x+1)} = 2^{3(2x+3)}$

$2x + 2 = 6x + 9$

$-7 = 4x$

$\boxed{x = -\frac{7}{4}}$

37) $9^{x-2} = (\frac{1}{27})^{x+5}$

like bases

$3^{2(x-2)} = 3^{-3(x+5)}$

$2x - 4 = -3x - 15$

$5x = -11$

$\boxed{x = -\frac{11}{5}}$

ln ln

38) $8^{2x-5} = 5^{x+1}$

LOG OR LN both sides

$(2x-5)\ln 8 = (x+1)\ln 5$

$2x\ln 8 - 5\ln 8 = x\ln 5 + \ln 5$

$2x\ln 8 - x\ln 5 = \ln 5 - 5\ln 8$

$x(2\ln 8 - \ln 5) = \ln 5 - 5\ln 8$

$x = (\ln 5 - 5\ln 8) \div (2\ln 8 - \ln 5)$

39) $e^{2x} - 7e^x + 12 = 0$

factor

$(e^x - 4)(e^x - 3) = 0$

$e^x = 4$ $\quad e^x = 3$

$\ln e^x = \ln 4$

$x = \ln 4$

$\boxed{x = \ln 3}$

40. $\log_x 16 = \frac{2}{3}$

defn logs

$x^{\frac{2}{3}} = 16$

$(x^{\frac{2}{3}})^{\frac{3}{2}} = 16^{3/2}$

$x = (\pm\sqrt{16})^3$

$x = \pm 64$

$\boxed{x = 64}$

41. $\log_x 125 = \frac{3}{2}$

defn logs

$[x^{\frac{3}{2}}]^{\frac{2}{3}} = [125]^{\frac{2}{3}}$

$x = (\sqrt[3]{125})^2$

$x = 5^2$

$\boxed{x = 25}$

$y = 24\left(\frac{1}{2}\right)^x$

42. Write an exponential equation, $y = ab^x$, generated by hand, that passes through the points (1, 12) and (3, 3). Show all work.

point #1

$12 = ab$

$a = \frac{12}{b}$

$y = ab^x$

2nd point

$3 = \left(\frac{12}{b}\right)b^3$

$3 = 12b^2$

$\frac{1}{4} = b^2$

$\pm\sqrt{\frac{1}{4}} = b$

$\pm\frac{1}{2} = b$

$b = \frac{1}{2}$

$a = 24$

Blast from the Past – Exponents and Radicals. . . Evaluate each expression. NO CALC

1) $(e^x)(e^x)$ $= e^{2x}$

2) $(-8)^{\frac{2}{3}}$

$= \left(\sqrt[3]{-8}\right)^2$

$= (-2)^2$

$= 4$

3) $(25)^{3/2}$

$= \left(\pm\sqrt{25}\right)^3$

$= (\pm 5)^3$

$= \pm 125$

4) $49^{\sqrt{2}} \div 7^{\sqrt{12}}$

$\frac{7^{2\sqrt{2}}}{7^{\sqrt{4}\sqrt{3}}}$

$= \frac{7^{2\sqrt{2}}}{7^{2\sqrt{3}}} = 7^{2\sqrt{2}-2\sqrt{3}}$

5) $\left(x^{1/4} - y^{1/4}\right)\left(x^{1/4} + y^{1/4}\right)$

$x^{\frac{1}{2}} - \sqrt[4]{xy} + \sqrt[4]{xy} - y^{\frac{1}{2}}$

$= \sqrt{x} - \sqrt{y}$

6. $3\sqrt{24} - 5\sqrt{150}$

$3\sqrt{4}\sqrt{6} - 5\sqrt{25}\sqrt{6}$

$= 6\sqrt{6} - 25\sqrt{6}$

$= -19\sqrt{6}$

7. $16^{-3/4}$

$= \left(\pm\sqrt[4]{16}\right)^{-3}$

$(\pm 2)^{-3}$

$= (\pm 8)^{-1}$

$= \pm\frac{1}{8}$

8. $\left(\frac{1}{4}\right)^{-\frac{5}{2}}$

$\left(\pm\sqrt{\frac{1}{4}}\right)^5$

$\left(\pm\frac{1}{2}\right)^5$

$= \pm\frac{1}{32}$

9. $(x-6)^2$