Key '17

Unit 1B Review - Functions Name: _____

CP Pre-Calculus

Section A: ODD, EVEN, NEITHER

Determine if the function is odd, even or neither:

$f(-x) = f(x) \Rightarrow$ even

$f(-x) = -f(x) \Rightarrow$ odd

A. $f(x) = \dfrac{x^2-4}{x^4+2}$

$f(-x) = \dfrac{(-x)^2-4}{(-x)^4+2}$

$f(-x) = \dfrac{x^2-4}{x^4+2}$

$\therefore$ fn is even

B. $h(x) = \dfrac{1+x^2}{x}$

$h(-x) = \dfrac{1+(-x)^2}{-x} = \dfrac{1+x^2}{-x}$

$h(1) = 2$

$h(-1) = -2$

odd.

c. $f(x) = 1-2x^5$

$f(-x) = 1-2(-x)^5$

$= 1--2x^5$

$= 1+2x^5$

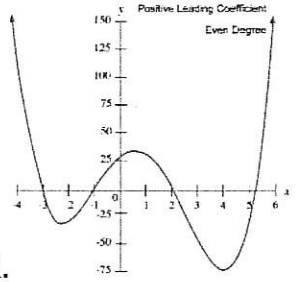
$f(1) = -1$

$f(-1) = 3$

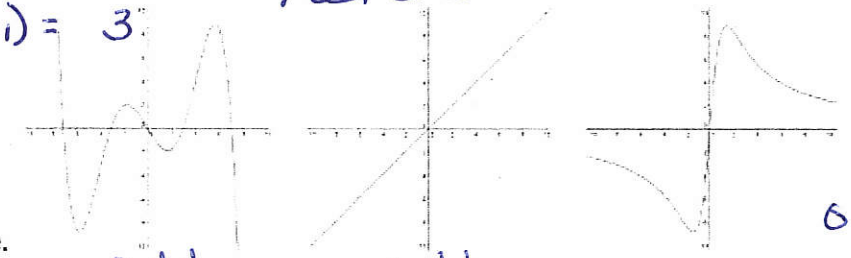
neither

neither $\Rightarrow$ not symmetrical over y axis

d.

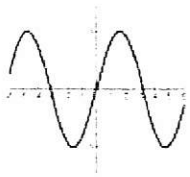


e.



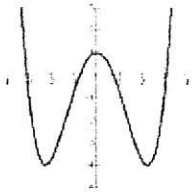
odd odd odd

f.



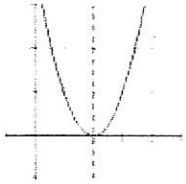
odd

g.



even

h.



Neither!

SECTION B: Parent Functions and Transformations

Standard form of a function: $af[b(x-c)]+d$

What does each variable control and how? Write in the correct order in which you'd perform the translations. Don't forget how the "b" impacts the x!

$\left(\frac{x}{b}+c, ay+d\right)$

b – horizontal stretch/compress; neg → reflect over y.

c – shifts c units right

a – vertical stretch/compress by a.f.o. $|a|$; neg a → reflects over x axis

d – shifts up or down d units

Directions: Name the parent function, and then describe the translation that will occur in words or as algebraic expressions. Graph!

$f(x) = x^2$ Quadratic

1) $f(x)=-2(x-1)^2+4$

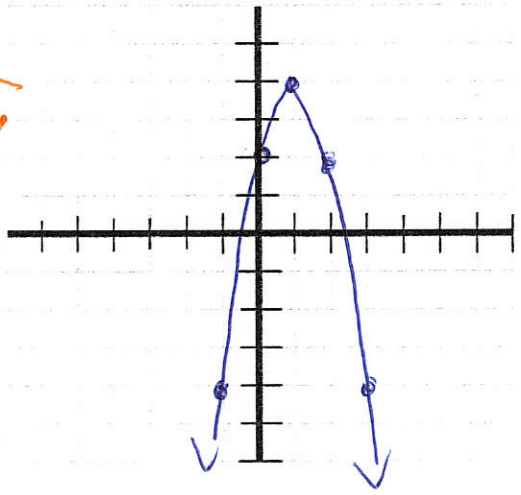
$(x+1, -2y+4)$

$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

→

x+1	-2y+4
-1	-4
0	2
1	4
2	2
3	-4



2) $f(x)=\sqrt{-(x-1)}-3$ ⇒ $(-1x+1, y-3)$

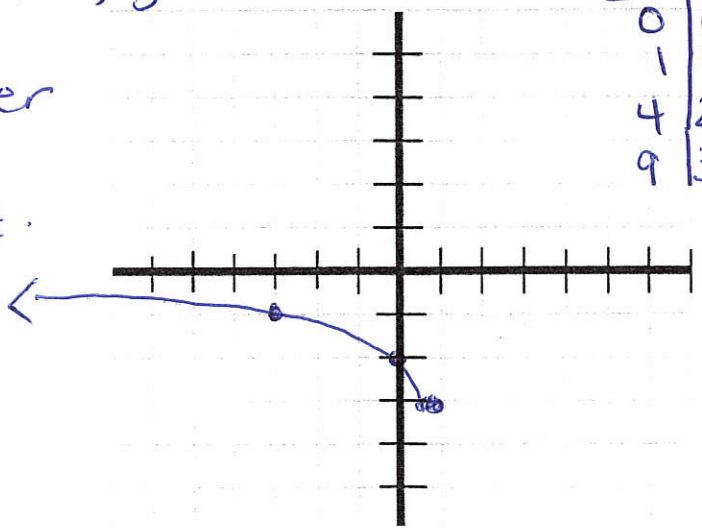
x	y	x'	y'
0	0	1	-3
1	1	0	-2
4	2	-3	-1
9	3	-8	0

$b = -1$ reflect over y

$c = 1$ 1 unit Rt.

$a = 1$

$d = -3$ ↓ 3



$(x, -2y-5)$

$-2y-5$

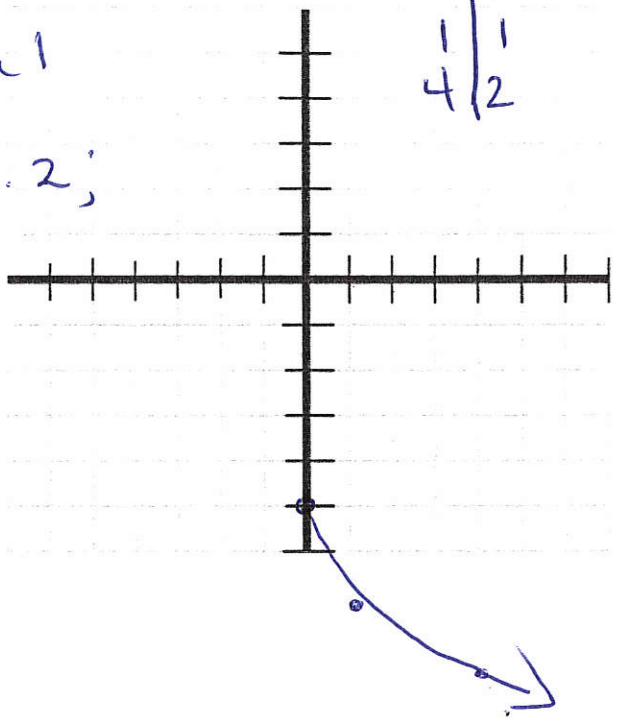
x	y
0	0
1	1
4	2

x'	y'
0	-5
1	-7
4	-9

3) $f(x) = -2\sqrt{x} - 5$

$a = -2$ - vertical stretch by a f. 0.2; reflect over x axis,

$d = -5$ down 5



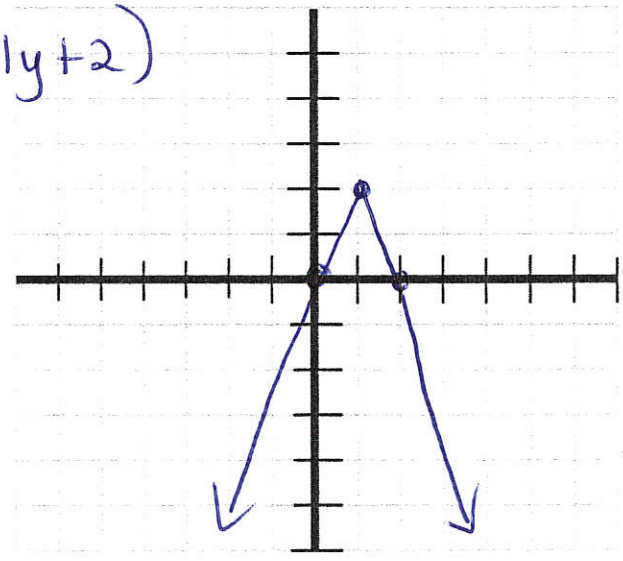
4. $g(x) = -|2(x-1)| + 2$

$\left(\frac{x}{2} + 1, -1y + 2\right)$

$y = |x|$

x	y
-2	2
0	0
2	2

x'	y'
0	0
1	2
2	0



*****Directions: Now, word backwards. I'll give you the translation, you write the function.

5) $f(x) = \sqrt{x}$

Reflect over the y axis. → $b = -1$

Translate 1 unit down. $d = -1$ $f(x) = \sqrt{-x} - 1$

6) $f(x)=x^2$

Reflect over the x axis. a = neg

Vertical shrink by a factor of ½ $a=-\frac{1}{2}$

Translate 1 unit left and 3 units down

$c=-1$ $d=-3$

$f(x)=-\frac{1}{2}(x+1)^2-3$

7) $f(x)=x^3$

Translate 2 units left $b=-2$

Reflect around the x axis. a is neg

Translate 1 unit up $d=1$

$f(x)=-(x+2)^3+1$

8) $f(x)=|x|$

Horizontal shift 3 left. $c=-3$

Stretch the y values by factor of 5. $a=5$

Reflect over the x. $a=-5$

Translate 6 units down. $d=-6$

$f(x)=-5|x+3|-6$

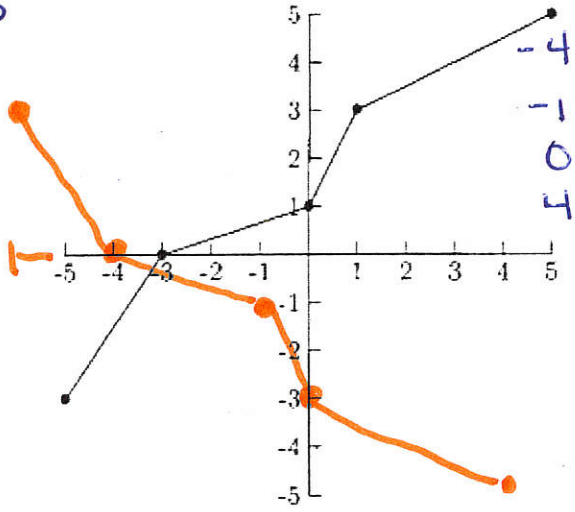
9. Use the graph of f(x) to graph g(x).

A. $g(x)=-f(x+1)$

$x-1, -y$

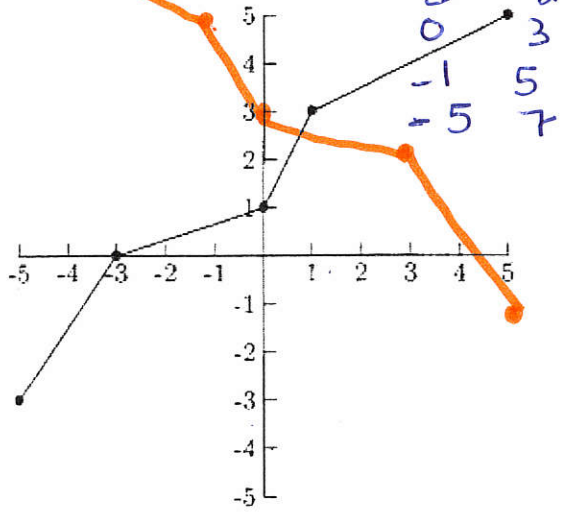
-6	3
-4	0
-1	-1
0	-3
4	-5

-5 -3
-3 0
0 1
1 3
5 5



B. h(x) = f(- x) + 2 $-1x, y+2$

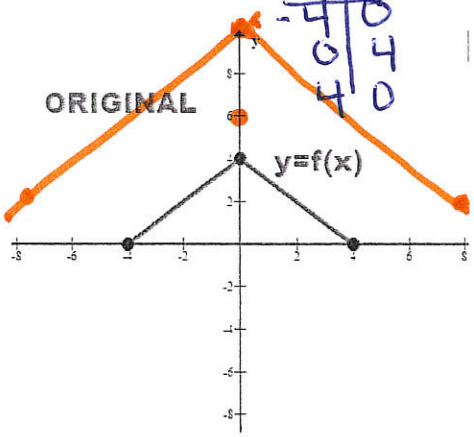
5	-1
3	2
0	3
-1	5
-5	7



Think of f(x) as the parent function.

10A. Use the graph of f(x) to graph g(x).

$g(x) = f\left(\frac{1}{2}x\right) + 2$



x	y
-4	0
0	4
4	0

$2x, y+2$ →

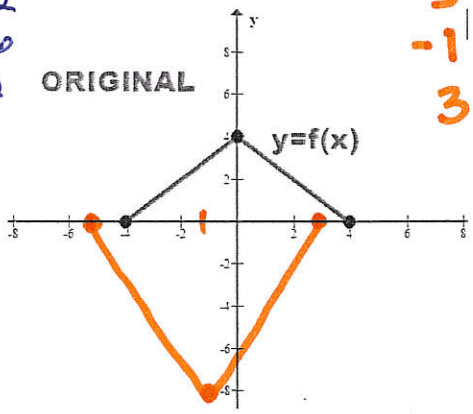
x	y
-8	2
0	6
8	2

10B. Use f(x) to graph(x)

$h(x) = -2f(x+1)$

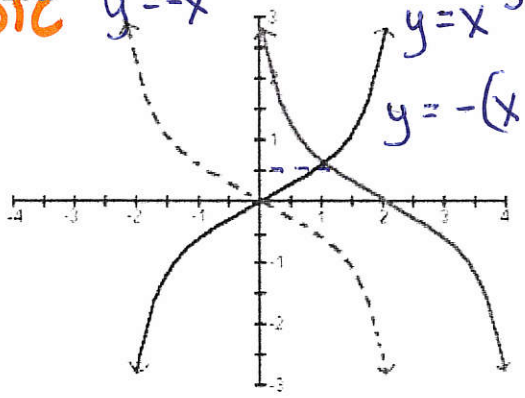
$x-1, -2y$

x	y
-5	0
-1	-8
3	0

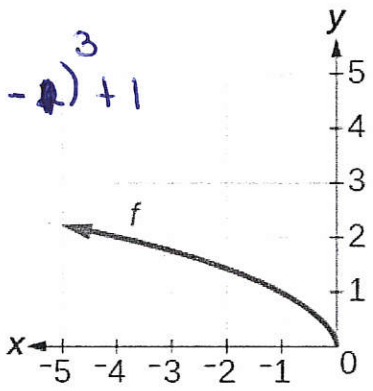


Write the equations of the following functions using your knowledge of parent functions and transformations.

10c. cubic $y = -x^3$



10d.



sq. root $\frac{x}{b}$

Parent $\frac{x}{1}$

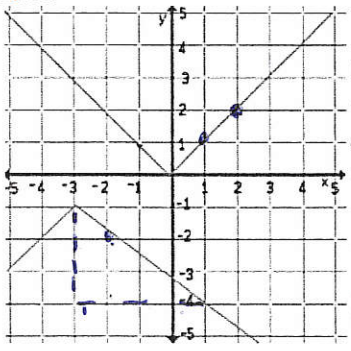
x	y
0	0
1	1
4	2

x	y
0	0
-1	1
-4	2

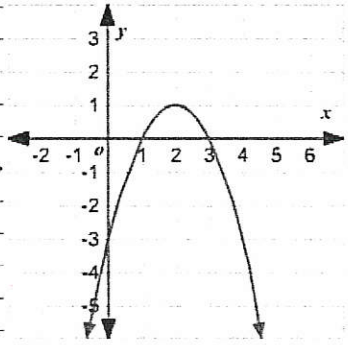
reflect over y.

$y = \sqrt{-x}$

10e. ABS. VAL



10f. Quad.



x	y
2	1
0	0
-1	1

2 right
up 1
reflect over x

$y = -(x-2)^2 + 1$

x	y
-2	2
-1	1
0	0
1	1
2	2

x'	y'
-3	-1

vertex

↑ left 3, down 1

$y = -|x+3| - 1$

11. State the domain and range for each of the following functions. Think – What x values must be excluded? What does the graph look like compared to the parent function? Only graph if you have to! Then write in interval notation. Where is the function increasing? Decreasing?

A) $f(p)=\sqrt{p-4}$

D: $[4, \infty)$

R: $[0, \infty)$

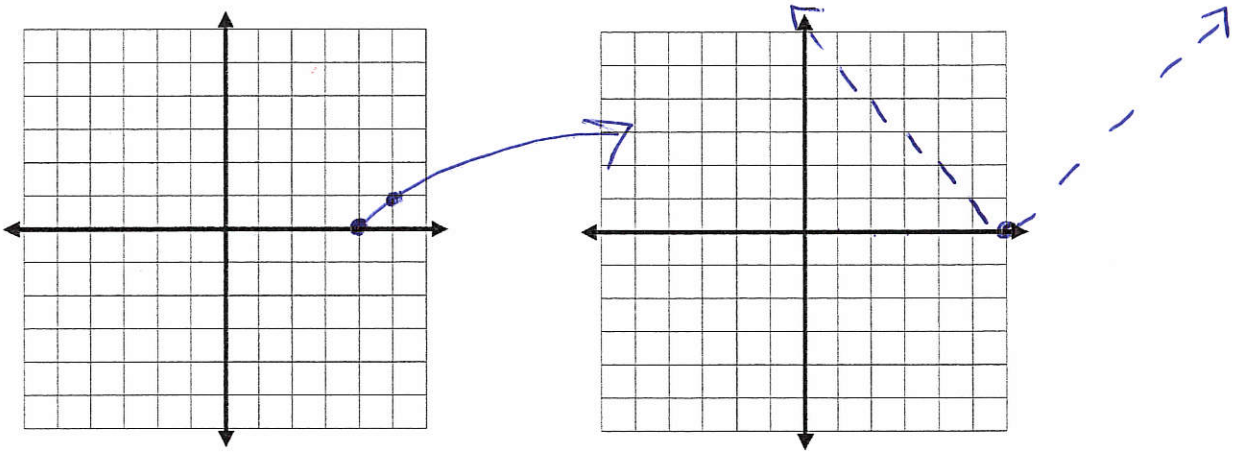
Incr: $[4, \infty)$

b) $f(x)=|x-6|$

D: $(-\infty, \infty)$

R: $[0, \infty)$

Decr. $(-\infty, 6)$



Section 3: Inverses

Functions are verified as inverses if

$f(g(x))=x$ and $g(f(x))=x$

Steps to create the equation of an inverse:

Interchange x & y

Solve for y

Rewrite y as $f^{-1}(x)$

Graphically, the inverse will be a reflection about the line $y=x$

12-13. Find $f(g(x))$ *and* $g(f(x))$ and verify wheather the pair of functions given below are inverses of each other using function composition.

12. $f(x)=6x+7$ and $g(x)=\frac{x-7}{6}$.

$f(g(x)) = f\left(\frac{x-7}{6}\right) = 6\left(\frac{x-7}{6}\right)+7 = x$ ☺

$g(f(x)) = g(6x+7) = \frac{6x+7-7}{6} = \frac{6x}{6} = x$ ☺

inverses ☺

13. $f(x)=1-x^3$
$g(x)=\sqrt[3]{1-x}$

yes ☺

$f(g(x)) = f\left(\sqrt[3]{1-x}\right) = 1-\left(\sqrt[3]{1-x}\right)^3 = 1-(1-x) = x$ ✓

$g(f(x)) = g(1-x^3) = \sqrt[3]{1-(1-x^3)} = \sqrt[3]{x^3} = x$ ☺

14. Is the function $f(x)=4x^2-16$ one to one? no → quadratic → parabola.

15. Find the inverse of $f(x)=\frac{5-3x}{2}$. Is the function one-to-one? Is the inverse a function?

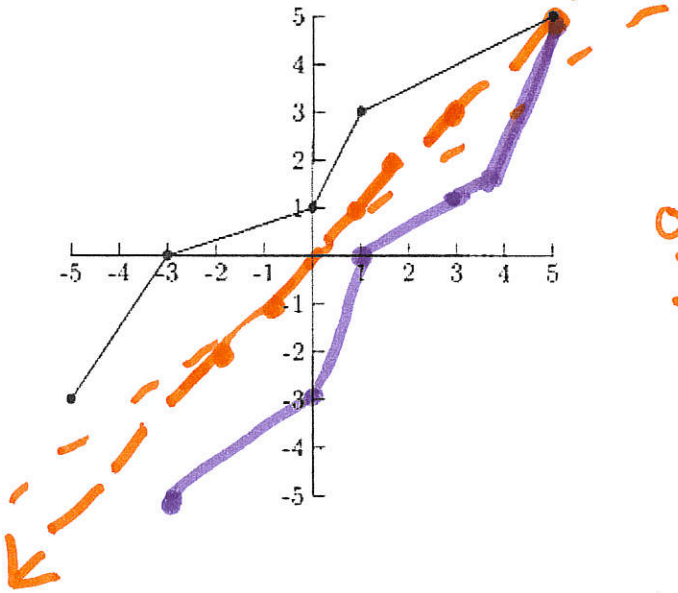
One to one → It's linear. Inverse linear

$y = \frac{5-3x}{2}$ $x = \frac{5-3y}{2}$ $2x = 5-3y$

$\frac{2x-5}{-3} = y$

$f^{-1}(x) = \frac{2x-5}{-3} = -\frac{2}{3}x+\frac{5}{3}$

16. Given the graph bellow, graph the inverse.



original		⇒ Inverse	
-5	-3	-3	-5
-3	0	0	-3
0	1	1	0
1	3	3	1
5	5	5	5