PreCalc 1B Review – Functions

Standard form of a function: $f(x) = a\,f\,[b(x-c)] + d$

What does each variable control and how? Write in the correct order in which you'd perform the translations:

b a → vertical stretch/compress $(\;, ay)$

c b → horizontal " $(\frac{x}{b}, \;)$

a c → horizontal shift $(x+c, \;)$

d d → vertical shift $(\;, y+d)$

Directions: Name the parent function, and then describe the translation that will occur in words or as algebraic expressions. Graph!

1) $f(x) = -2\boxed{(x-1)^2} + 4$

Quadratic

~~b~~ $(x+1 \quad)$ 1 Rt.

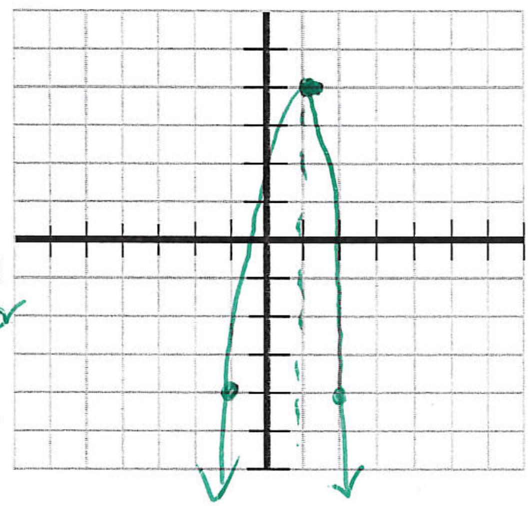
c

a $(x+1, -2y)$ vert. stretch by factor of 2, reflect over x axis

d $(x+1, -2y+4)$

up 4

x	y
-2	4
0	0
2	4



2) $f(x) = \sqrt{-2(x-1)} - 3$

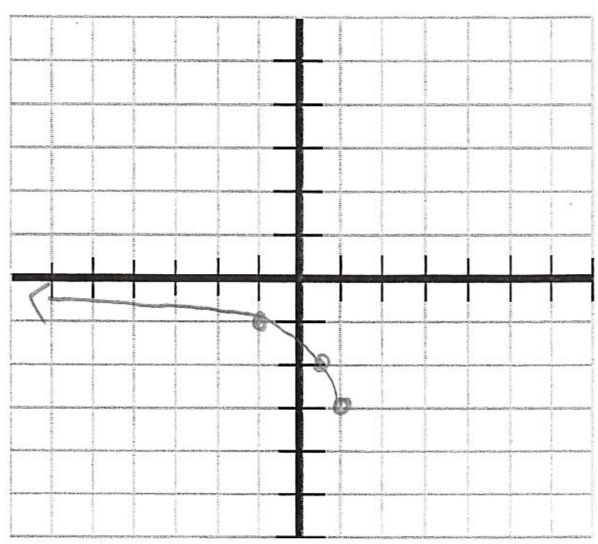
b $(\frac{x}{-2} \quad)$

c $(\frac{x}{-2} + 1, y)$

a ——

d $(\frac{x}{-2} + 1, y-3)$

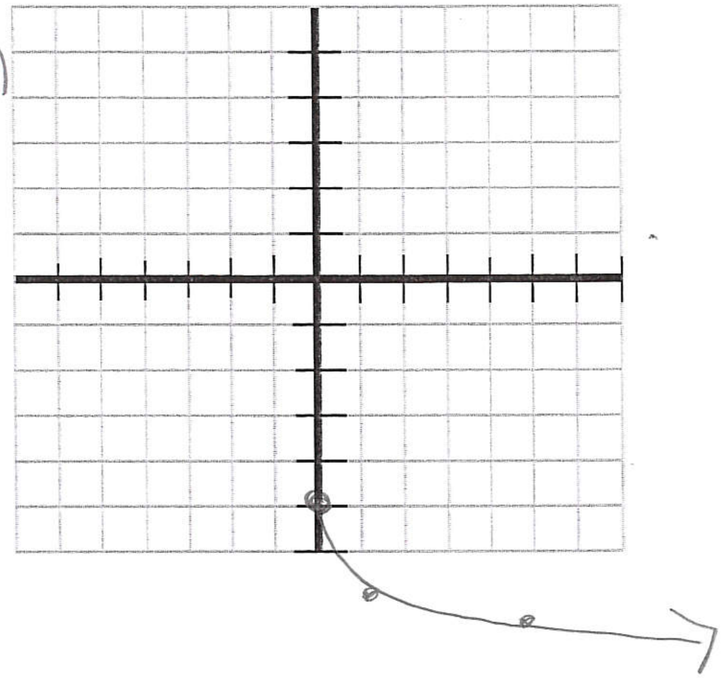
x	y		
0	0	→ 1,	-3
1	1	1/2	-2
4	2	-1,	-1



3) $f(x) = -2\sqrt{x} - 5$

$y = \sqrt{x} \Rightarrow (x, -2y-5)$

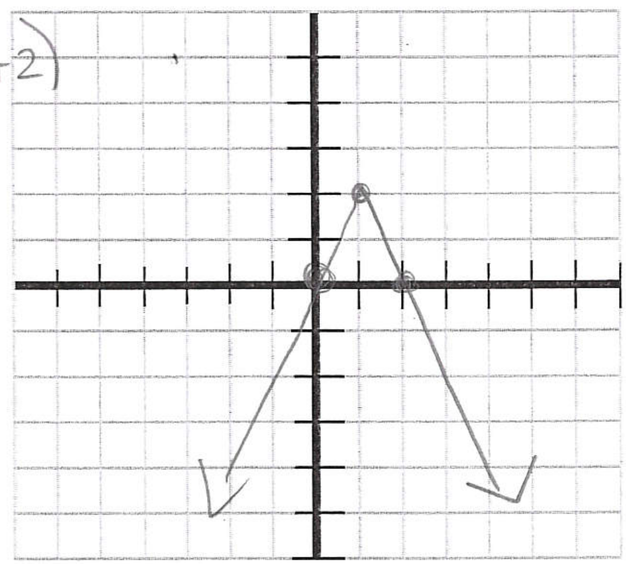
x	y	
0	0	→ 0, -5
1	1	1, -7
4	2	4, -9



4. $g(x) = -|2(x-1)| + 2$

$f(x) = |x| \quad \left(\frac{x}{2} + 1, -1y + 2\right)$

x	y		
-2	2	0, 0	(~~1, 2~~)
0	0	(1, 2)	(~~2, 0~~)
2	2	(2, 0)	(~~0, 0~~)



b = 2 c = 1

a = -1 d = 2

Directions: Now, word backwards. I'll give you the translation, you write the function.

5) $f(x) = \sqrt{x}$

Reflect over the y axis.

Translate 5 units left and 1 unit down. $g(x)$ $\sqrt{-1(x+5)} - 1$

6) $f(x) = x^2$

Reflect over the x axis.

Vertical shrink by a factor of ½

Translate 1 unit left and 3 units down $f(x) = -\frac{1}{2}(x+1)^2 - 3$

7) $f(x) = x^3$

Multiply the x values by 2. $b = \frac{1}{2}$

Translate 2 units left $C \quad +2$

Reflect around the x axis. $a = -1$

Translate 1 unit up $d = 1$

$f(x) = -1\left[\frac{1}{2}(x+2)\right]^3 + 1$

8) $f(x) = |x|$

Compress the x values by dividing the x values by 2. $b = 2$

Horizontal shift 3 left. $C \quad +3$

Stretch the y values by factor of 5. $a = -5$

Reflect over the x.

Translate 6 units down. $d = -6$

$f(x) = -5\,|2(x+3)| - 6$

9. Use the graph of f(x) to graph g(x).

A. $g(x) = -f(x+1)$ $\Rightarrow (x-1, -1y)$

ORIG

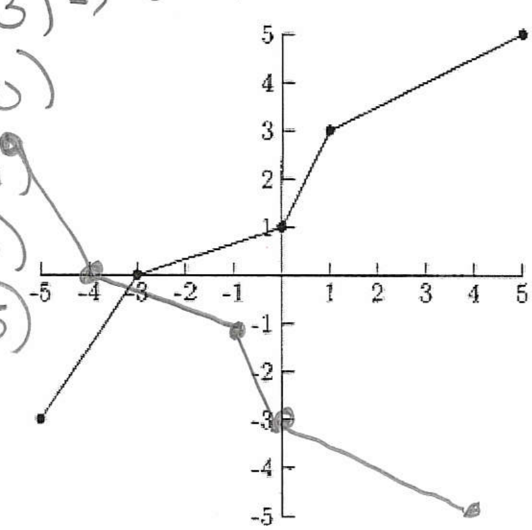
$(-5,-3) \Rightarrow (-6,3)$

$(-3,0)$

$(0,1)$

$(1,3)$

$(5,5)$



B. h(x) = f(- x) + 2 $(-1x, y+2)$

$(5,-1)$

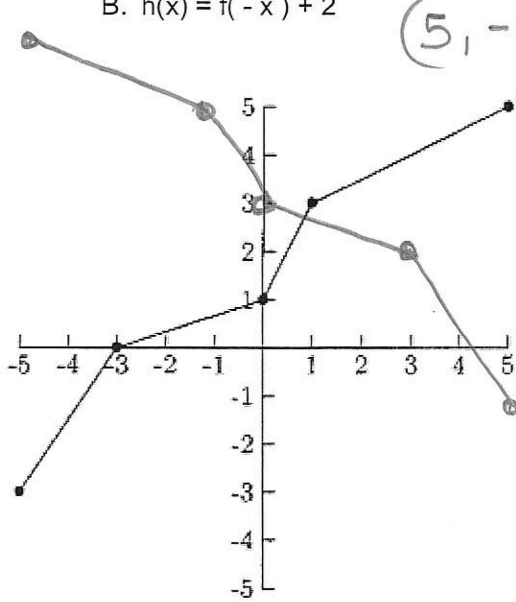
$(3,2)$

$(0,3)$

$(-1,5)$

~~(−1,5)~~

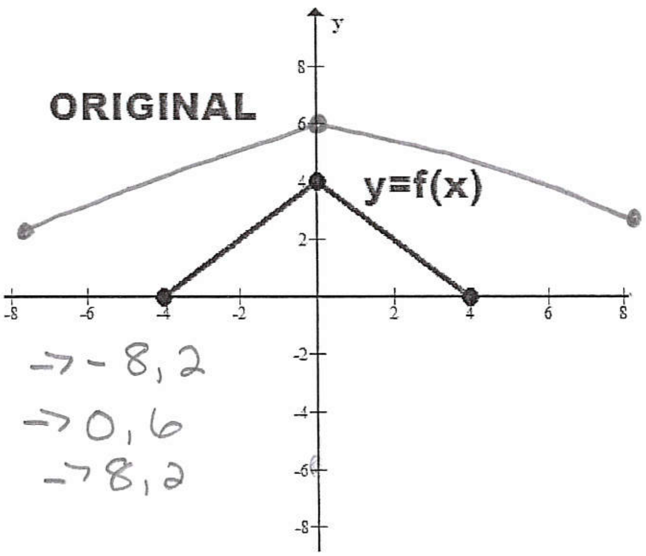
$(-5,7)$



Think of f(x) as the parent function.

10A. Use the graph of f(x) to graph g(x).

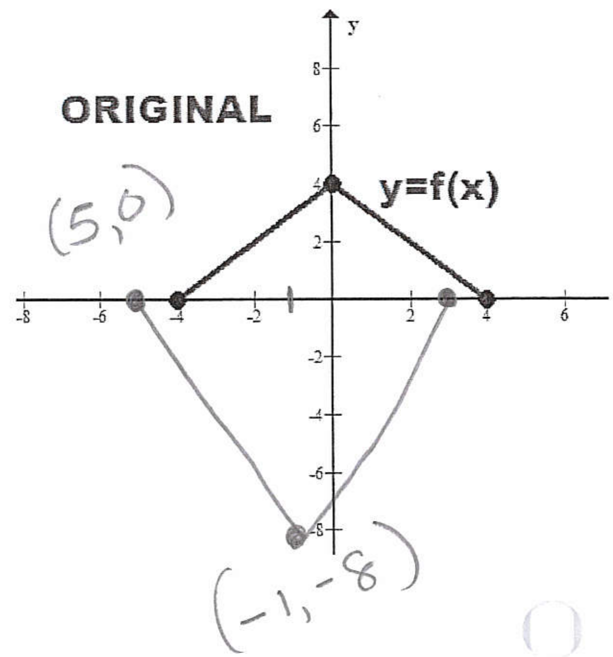
$g(x) = f\left(\frac{1}{2}x\right) + 2$ $\quad \frac{x}{b} = (2x, y+2)$



$-4, 0 \to -8, 2$
$0\ 4 \to 0, 6$
$4, 0 \to 8, 2$

10B. Use f(x) to graph(x)
h(x) = -2f (x + 1)

$(x-1, {}^{-}2y)$



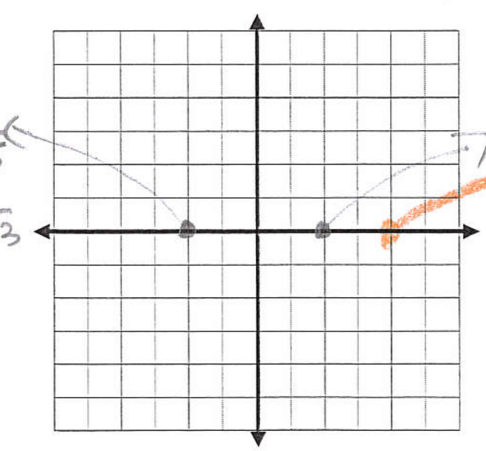
SET/INTERVAL NOTATION RECAP:

11. State the domain for each of the following functions. Think – What x values must be excluded? What does the graph look like compared to the parent function? Only graph if you have to! Then write in interval notation.

A) $h(p) = \sqrt{p^2 - 4}$ $\quad$ or $\sqrt{p-4}$

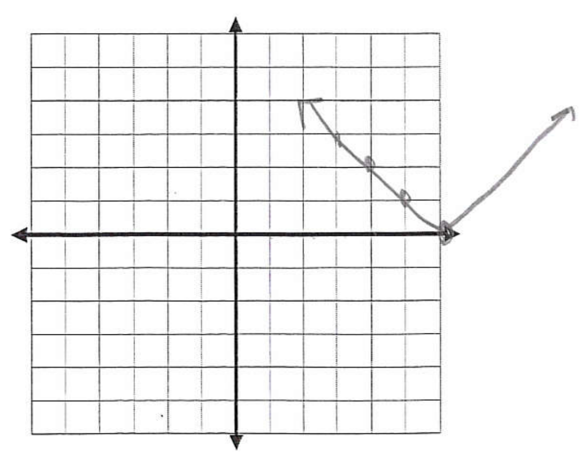
$(-\infty, -2] \cup [2, \infty)$ D: $[4, \infty)$

$-2 \to 0$
$-3 \to \sqrt{5}$
$-4 \to 2\sqrt{3}$



b) $f(x) = |x - 6|$

D: $(-\infty, \infty)$



12-13. Find $f(g(x))$ *and* $g(f(x))$ and verify wheather the pair of functions given below are inverses of each other using function composition.

12. $f(x) = 6x + 7$ and $g(x) = \frac{x-7}{6}$.

$f(g(x))$	$g(f(x))$
$f\left(\frac{x-7}{6}\right)$	$g(6x+7)$
$= 6\left(\frac{x-7}{6}\right) + 7$	$= \frac{6x+7-7}{6}$
$= x - 7 + 7$	$= \frac{6x}{6}$
$= x$ ✓	$= x$ ✓
☺	☺

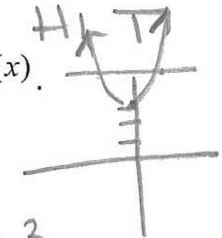
13. $f(x) = 1 - x^3$
$g(x) = \sqrt[3]{1-x}$

$f(g(x))$	$g(f(x))$
$f\left(\sqrt[3]{1-x}\right)$	$g(1-x^3)$
$= 1 - \left(\sqrt[3]{1-x}\right)^3$	$= \sqrt[3]{1-(1-x^3)}$
$= 1 - (1-x)$	$= \sqrt[3]{1-1+x^3}$
$= 1 - 1 + x$	$= \sqrt[3]{x^3}$
$= x$ ☺	$= x$

14. Given $f(x) = 2x^2 + 3$

$F(x) = 2x^2 + 3$

a. Is the above function one-to one for all values of x? NO → does not pass HLT



a. Find the equation for $f^{-1}(x)$.

$y = 2x^2 + 3$

$x = 2y^2 + 3$

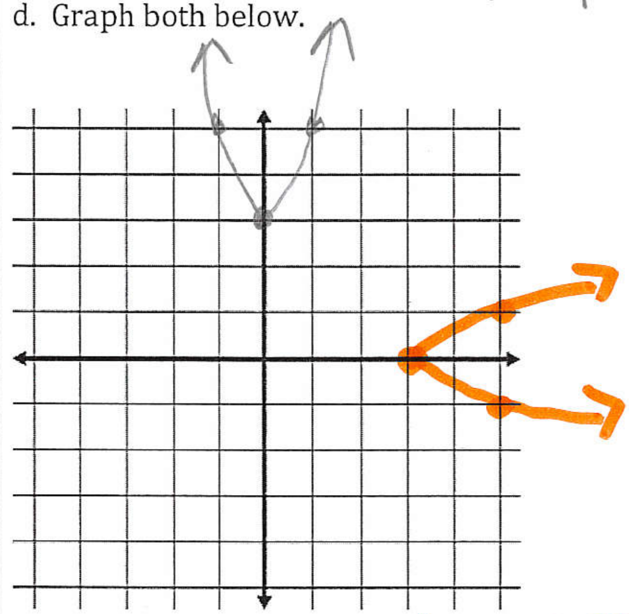
$x - 3 = 2y^2$

$\frac{x-3}{2} = y^2$ $\quad y = \pm\sqrt{\frac{x-3}{2}}$

c. Is the inverse a function? NO

$f^{-1}(x) = \pm\sqrt{\frac{x-3}{2}}$

d. Graph both below.



$y = 2x^2 + 3$

x	y
-1	5
0	3
1	5

Inverse

x	y
5	-1
3	0
5	1

15. Write the inverse of the function: $f(x) = 4x^2 - 16$

$y = 4x^2 - 16$

$x = 4y^2 - 16$

$\frac{x+16}{4} = y^2$

$y = \pm\sqrt{\frac{1}{4}x + 4}$

$f^{-1}(x) = \pm\sqrt{\frac{1}{4}x + 4}$

16. Find the inverse of $f(x)=\dfrac{5-3x}{2}$. Is the function one-to-one? Is the inverse a function?

f(x) → linear ∴ one to one ∴ inverse is a fn (also linear)

$$y=\frac{5-3x}{2}$$

$$x=\frac{5-3y}{2}$$

$$2x=5-3y$$

$$2x-5=-3y$$

$$y=\frac{2}{-3}x-\frac{5}{-3}$$

$$f^{(-1)}(x)=-\frac{2}{3}x+\frac{5}{3}$$

17. Find the inverse of f(x) = $f(x)=\sqrt{2x-3}$. State the domain restriction based on f(x).

D: $\{x \mid x \geq 1.5\}$ or D: $[1.5, \infty)$

$$y=\sqrt{2x-3}$$

$$x=\sqrt{2y-3}$$

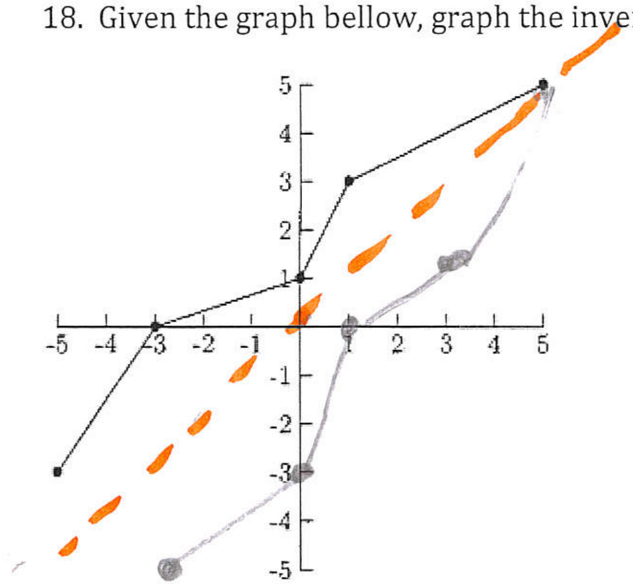
$$x^2=2y-3$$

$$x^2+3=2y$$

$$\frac{1}{2}x^2+\frac{3}{2}=y$$

$$f^{-1}(x)=\frac{1}{2}x^2+\frac{3}{2}$$

18. Given the graph bellow, graph the inverse.



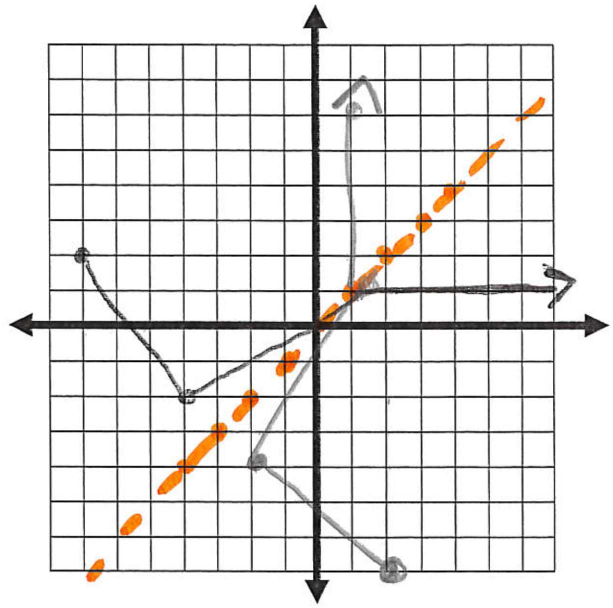
Inverse → Interchange x and y

L to R	
(-5, -3) →	(-3, -5)
(-3, 0) →	(0, -3)
(0, 1) →	(1, 0)
(1, 3) →	(3, 1)
(5, 5) →	(5, 5)

19. Given the graph, plot the inverse.

(-7,2)
(-4,-2)
(1,1)
(6,1)

(2,-7)
(-2,-4)
(1,1)
(1,6)

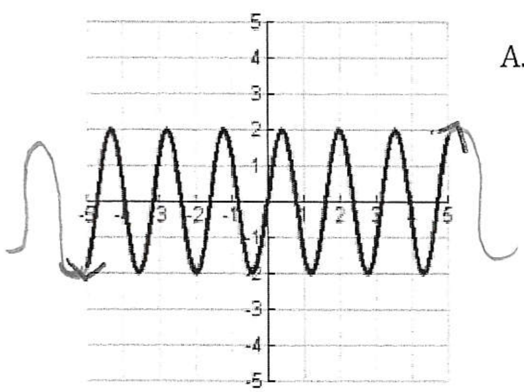


19b. Write the domain and range of the original graphed above.

D: $\{x \mid x \geq -7\}$ D: $[-7, \infty)$

R: $\{x \mid -2 \leq y \leq 2\}$ R: $[-2, 2]$

20. State the domain and range of each function graphed below. Write in set and interval notation.



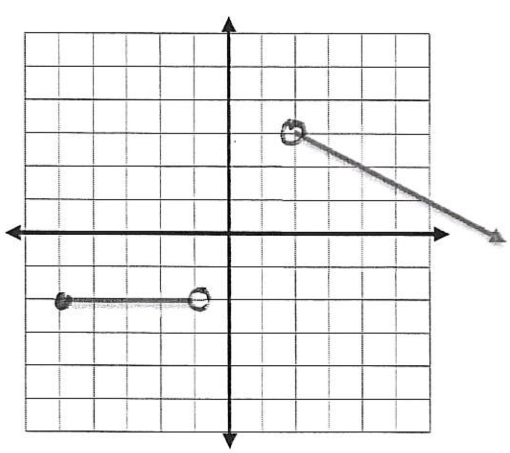
A.

Domain interval: $(-\infty, \infty)$

Domain set: $\{x \mid x \in \mathbb{R}\}$

Range interval: $[-2, 2]$

Range set: $\{y \mid -2 \leq y \leq 2\}$



B.

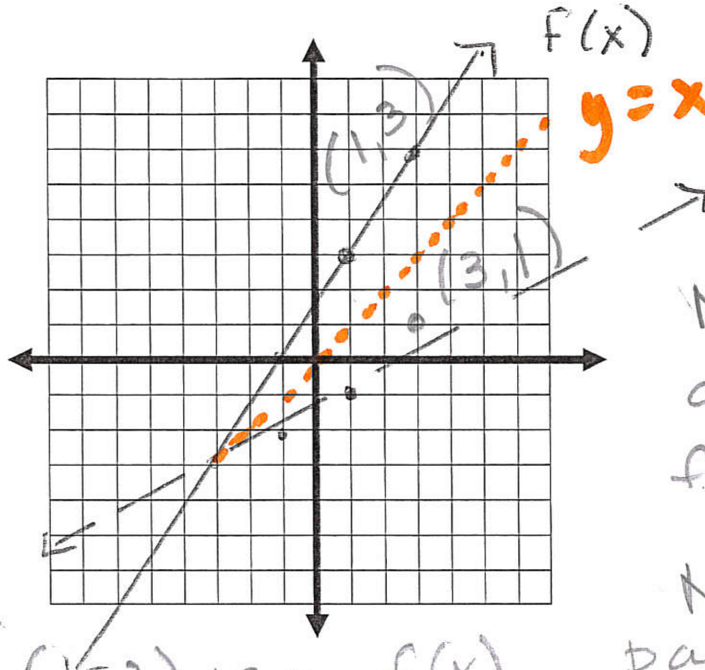
Domain interval: $[5, -1) \cup (2, \infty)$

Domain set: $\{x \mid 5 \leq x < -1 \cup x > 2\}$

Range interval/Range set:

$(-\infty, 3)$ $\{y \mid y < 3\}$

21. Given the graph of f(x) and g(x) below, do the functions appear to be inverses? Why or why not? Support your reasoning with mathematical evidence and definitions.



NO - g(x) is not a reflection of f(x) over x axis.

NO → each ordered pair of f(x) when x and y valued are interchanged, don't create points on g(x)

Ex (1,3) is on f(x). So (3,1) should be on g(x), but its not.

22. Create your own one-to-one function that is unique and unlike any function you have studied. Graph it. Then graph it's inverse in a different color and label it using inverse notation.