Pre-Calculus CP Review - Unit 5 4.1_4.4 UNIT CIRCLE/TRIG FNS/IDENTITIES

Name: _____ per. _____

Key

multiply by $\frac{180^\circ}{\pi}$

1. Change each radian measure to a degree measure (to the nearest minute). Then, determine what quadrant the terminal side lies in.

a) $\frac{7\pi}{5}$ 252° Quad 3

b) $-\frac{11\pi}{12}$ -165° Quad 3

c) 8.25 472.7° Quad 2

$8.25 \text{ rad} \cdot \frac{180^\circ}{\pi} =$

d) -3.6 -206.26° Quad 2

$-3.6 \text{ rad} \cdot \frac{180^\circ}{\pi} = -206.26$

Multiply by $\frac{\pi}{180}$

2. Change each degree measure to a radian measure (in terms of π). Then, determine what quadrant the terminal side lies in.

a) -24° $\frac{-2\pi}{15}$ Quad 4

b) 325° $\frac{65\pi}{36}$ Quad 4

c) 172.5° $\frac{43\pi}{45}$ Quad 2

$\pm 360^\circ$ $\pm 2\pi$

3. Find one positive and one negative coterminal angle for each of the given angle.

a) -138° 222° & -498°

b) 876° -204° & 156°

c) $\frac{7\pi}{8}$ $\frac{23\pi}{8}$ & $\frac{-9\pi}{8}$

d) $-\frac{16\pi}{5}$ $\frac{-6\pi}{5}$ & $\frac{4\pi}{5}$

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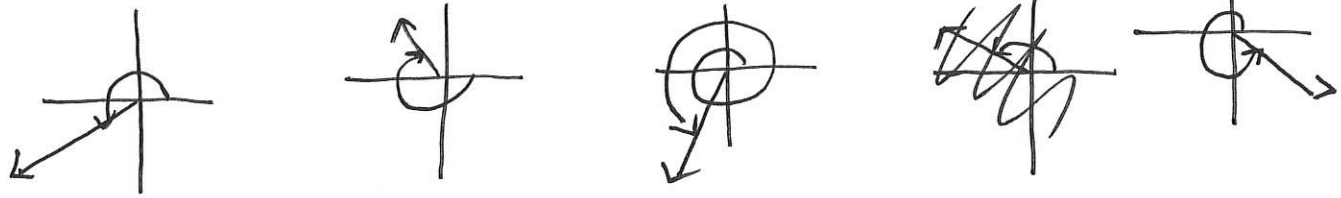
4. Write and expression for all angles that are coterminal with each of the following:

a) 289° $289 \pm 360k$ b) $\frac{15\pi}{13}$ $\frac{15\pi}{13} \pm 2\pi k$

5. Sketch and Find the reference angle for each.

a) 200° $20°$ b) -225° $45°$ c) $\frac{10\pi}{3}$ $\frac{\pi}{3}$ d) $\frac{12\pi}{7}$ $\frac{2\pi}{7}$



6. Given the measure of a central angle, find the measure of its intercepted arc in terms of π in a circle of radius 10. Recall, arc length = radius x measure of the central angle in RADIANS

a. 110° $\cdot \frac{\pi}{180°} = \frac{11\pi}{18}$

$s = \theta \cdot r$

$s = \frac{11\pi}{18} \cdot 10 = \frac{110\pi}{18}$

on test b. The ~~second~~ MINUTE hand on Big Ben measures 8 feet. How far does the tip of the hand travel in 45 minutes? $= \frac{55\pi}{9}$

$r = 8$ ft $\quad 45 \text{ mins} = 9 \cdot (30°) = 270° = \frac{3\pi}{2}$

on test $s = \frac{3\pi}{2} \cdot 8 = 12\pi$ feet OR $45 \text{ min} \cdot \frac{1 \text{ rev}}{60 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{3\pi}{2} = \theta$

C. Ms. Miller loves pizza crust. She would like to stretch the outmost side of the pizza crust so that she would be eating an 8 inch length. What angle measure should she use to cut her slice of pizza in radians? In degrees?

16 inches diameter.

$r = 8$ $\quad s = 8$

$s = \theta \cdot r$ $\quad 8 = 8\theta$ $\quad 1 = \theta$

$1 \text{ radian} \cdot \frac{180°}{\pi}$

$= \frac{180°}{\pi} \approx 57.3°$

D. If she wanted to slice the pizza into equal slices using the above angle measure, how much pizza would be left over (wasted) if area of a sector(slice) = $\pi r^2 \left(\frac{\text{angle in degrees}}{360}\right)$?

Each slice = 57.3°

$360° \div 57.3° = 6 +$

$6 \cdot 57.3 = 343.8° = 6$ slices +

so 16.2° left over

$A = \pi \cdot 64 \cdot \frac{16.2}{360} = 9 \text{ in}^2$

7. Given the measure of an arc, find the degree measure to the nearest tenth of a radian of the central angle it subtends in a circle of radius 16 cm.

a. 87 cm $= 311.5°$

$s = \theta r$ $\quad 87 = 16\theta$ $\quad \theta = \frac{87}{16}$

$\theta = 5.4375$ radians

$5.4375 \text{ rad} \cdot \frac{180}{\pi}$

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8. A pulley of radius 12 cm turns at 7 revolution per second. What is the linear velocity of the belt driving the pulley in meters per second?

$\frac{7 \text{ rev}}{1 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{12 \text{ cm}}{1 \text{ rad}} \cdot \frac{1 \text{ m}}{100 \text{ cm}} =$

$\approx 5.28 \text{ m/sec}$

9. A trucker drives 55 miles per hour. His truck's tires have a diameter of 26 inches. What is the angular velocity of the wheels in revolutions per second? $r = 13$ in

$\frac{55 \text{ mi}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ rad}}{13 \text{ in}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}}$

$\approx 11.85 \text{ rev/sec}$

10. Jason's bicycle wheel is 26 inches in diameter. $r = 13$

a. To the nearest revolution, how many times will the wheel turn if it is ridden for 1 mile.

Want $\frac{\text{rev}}{\text{mile}}$

$\frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{1 \text{ rad}}{13 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}}$ ≈ 776 rev.

b. Suppose the wheel turns at a constant rate of 2.5 revolutions per second. To the nearest tenth, what is the linear speed of a point on the tire in feet per second? In miles per hour?

$\frac{2.5 \text{ rev}}{1 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{13 \text{ in}}{1 \text{ rad}} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$ $\approx 17 \text{ ft/sec}$

$\frac{17 \text{ ft}}{1 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}}$

$\approx 11.6 \text{ mph}$.

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c. Mr. Mooney can ride his bike at a speed of 18 mph. If the wheels on his bike are 26 inches in diameter, how many revolutions does a wheel make in one minute?

$\frac{18 \text{ miles}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 \text{ rad}}{13 \text{ in}} \cdot \frac{1 \text{ rev}}{2\pi}$

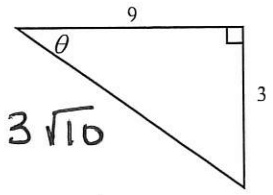
232 rev/min

SOH - CAH - TOA $\qquad a^2 + b^2 = c^2$

11. Find the indicated information. Leave answers in EXACT form.

a) $\sin\theta = \frac{\sqrt{10}}{10}$ $\qquad$ b) $\sec\theta = \frac{\sqrt{10}}{3}$

c) $\tan\theta = \frac{1}{3}$ $\qquad$ d) $\csc\theta = \sqrt{10}$



e) Find θ to the nearest tenth of a degree using your calculator set on degree mode (Think – what key might you use?)

$\tan^{-1}(1 \div 3) =$

or $\sin^{-1}(\sqrt{10} \div 10) =$

18.43°

12. Find a co-function with the same value.

a. $\sin 76°$ _____ $\qquad$ b. $\cos 12°$ _____

13. Let β be in standard position. In which quadrant(s) can β lie under the given conditions? Don't guess! Use your ASTC to help you!

a. $\cos\beta > 0$ and $\tan\beta < 0$ IV

S A
T C

b. $\cot\beta$ and $\csc\beta$ have the same sign I, IV

TAN SIN
A
C

14. Find the exact value. **DO NOT USE A CALCULATOR.**

$1 \div \frac{\sqrt{3}}{2}$

a. $\cot 60° = \frac{\sqrt{3}}{3}$ $\qquad$ $\frac{\pi}{3}$

b. $\cos 30° = \frac{\sqrt{3}}{2}$ $\qquad$ $\frac{\pi}{6}$

c. $\sec 45° = \sqrt{2}$ $\qquad$ $\frac{1}{\cos}$

d. $\csc 60° = \frac{2\sqrt{3}}{3}$ $\qquad$ $\frac{1}{\sin \pi/3}$

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e. $\tan 30^\circ = \frac{\sqrt{3}}{3}$ $\frac{\pi}{6}$

f. $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

g. $\cot 330^\circ = -\sqrt{3}$ $\frac{\pi}{6} \to \frac{11\pi}{6}$ $\frac{1}{\sin}$ cot neg

h. $\csc\frac{5\pi}{4} = -\sqrt{2}$

i. $\cos 210^\circ = -\sqrt{3}/2$ $\frac{7\pi}{6}$ Q3 +

j. $\sec 300^\circ = 2$ $\frac{1}{\cos} \to +$ pos $\frac{5\pi}{3}$ $\cos = \frac{1}{2}$ $\sec = 2$

k. $\tan\frac{5\pi}{2} =$ und. $\frac{1}{+}$ (0,1)

l. $\sin\frac{3\pi}{4}$ $\frac{\sqrt{2}}{2}$

m. $\csc 270^\circ$ -1

n. $\tan\frac{11\pi}{4}$ -1

o. $\cos\frac{10\pi}{4}$ 0

p. $\sin\frac{13\pi}{4}$ $\frac{-\sqrt{2}}{2}$

q. $\cos\frac{3\pi}{4}\tan\frac{7\pi}{3} =$ $\frac{-\sqrt{6}}{2}$ $-\frac{\sqrt{2}}{2}$, $\sqrt{3}$

r. $\csc 120^\circ + \cot 315^\circ =$ $\frac{-2\sqrt{3}}{3} + -1$ $= \frac{-2\sqrt{3}}{3} + \frac{-3}{3}$

s. $4\cos 60^\circ + 3\tan\frac{\pi}{3} =$ $4\left(\frac{1}{2}\right) + 3\sqrt{3}$ $2 + 3\sqrt{3}$

t. $6\cos\frac{3\pi}{4} + 2\tan\left(-\frac{\pi}{3}\right) =$ $6\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\sqrt{3}\right)$ $-3\sqrt{2} - 2\sqrt{3}$

u. $\cos 540^\circ - \tan(-405^\circ) =$ cos 180 $\downarrow$ 45° Q4 $-1, 0$ $-1 - -1 = 0$

******Use your knowledge of reciprocal trig functions to simplify. Rewrite each reciprocal function in terms of its base function. Think reference angles as well!

Example:

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w. tan 68° • cot 68°=_____

$= \tan 68^0 \bullet \frac{1}{\tan 68^0} = 1$

x. sin 72° • csc 432°= 1

-360

72^0

$\sin 72 \cdot \csc 72$

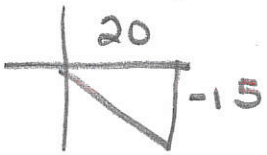
$\sin\theta \cdot \frac{1}{\sin\theta} = 1$

y. tan 20° • cos (-20°) • csc 20°=_____

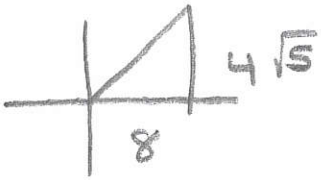
z. sin 405° • sec (-45°)=_____

15. The terminal side of an angle α in standard position passes through point G. Sketch α and find the exact values of the six trigonometric functions of α.

a) G(20, -15) $r = 25$



b) $G\left(8, 4\sqrt{5}\right)$ $r = 12$



$\frac{y}{r}$ sin α -3/5 csc α -5/3 sin α $\sqrt{5}/3$ csc α $3\sqrt{5}/5$

$\frac{x}{r}$ cos α 4/5 sec α 5/4 cos α 2/3 sec α 3/2

$\frac{y}{x}$ tan α -3/4 cot α -4/3 tan α $\sqrt{5}/2$ cot α $2\sqrt{5}/5$

16. Find the exact values of the other 5 trigonometric functions for an angle α in standard position lying in the given quadrant

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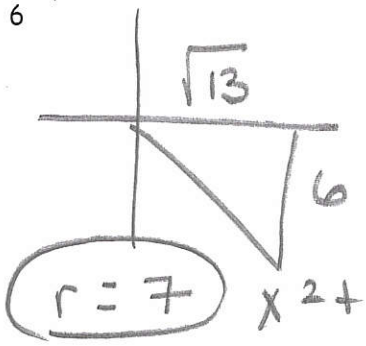
a) Cot $\alpha = \frac{-\sqrt{13}}{6}$; IV

$\cot = \frac{x}{y}$

$x = \sqrt{13}$

$y = -6$



$x^2+y^2=r^2$

Sin α $\frac{-6}{7}$ Csc α $\frac{-7}{6}$

Cos α $\frac{\sqrt{13}}{7}$ Sec α $\frac{7\sqrt{13}}{13}$

Tan α $\frac{-6\sqrt{13}}{13}$

b) Sin $\alpha = \frac{8}{17}$; II

$x^2+y^2=r^2$

$r = 17$

$y = 8$

$x = -15$

Cos α $\frac{-15}{17}$ Csc α $\frac{17}{8}$

Tan α $\frac{-8}{15}$ Sec α $\frac{-17}{15}$

Cot α $\frac{-15}{8}$

17. Given the following, find tan, cot, sec, csc. Think: x = 8 y = 5 r = 13

$\sin\theta = \frac{5}{13}, \cos\theta = \frac{8}{13}$

tan $\frac{y}{x} = \frac{5}{8}$ cot $\frac{x}{y} = \frac{8}{5}$ sec $\frac{r}{x} = \frac{13}{8}$ csc $\frac{r}{y} = \frac{13}{5}$

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18. Use your knowledge of the unit circle and the Pythagorean Theorem to find the missing value. Be sure to sketch a right triangle in the correct quadrant first. Then, identify what you know (x, y, r) and label your triangle. Use the PT to find the length of the missing side. Use the quadrant to determine the sign. Recall, the radius is always positive!

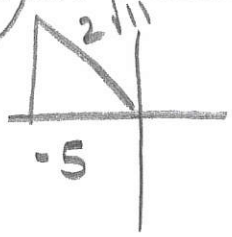
a. $\sec\theta = -\frac{2\sqrt{11}}{5}, \frac{\pi}{2} < \theta < \pi$ tan Θ = $-\frac{\sqrt{19}}{5}$

Q2

$\sec\theta = \frac{r}{x}$

$r = 2\sqrt{11}$

$x = -5$



$x^2+y^2=r^2$

$25+y^2=44$

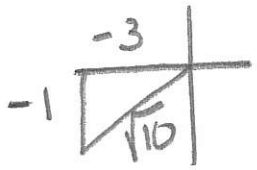
$y^2=19$

$y=\sqrt{19}$

b. $\cos t = -\frac{3}{\sqrt{10}}, \pi < \theta < \frac{3\pi}{2}$ csc Θ = $-\sqrt{10}$

Q3

$x = -3$ $r = \sqrt{10}$



$9+y^2=10$

$y^2=1$

$y=-1$

$\csc = \frac{r}{y} = \frac{\sqrt{10}}{-1} = -\sqrt{10}$