

PC REVIEW – SOLVING RADICAL EQUATIONS

Solving Radical/Root Equations

Name: Key Period:

OBJECTIVE: I can solve radical and root equations, checking for valid vs. extraneous solutions.

Example A: $\sqrt{x} = 3$

$$\sqrt{x}^2 = 3^2$$

$$x = 9$$

check $\rightarrow \sqrt{9} = 3$

Example B: $\sqrt{x} = -3$

$$(\sqrt{x})^2 = (-3)^2$$

$$x = 9$$

check $\sqrt{9} = -3$
 $3 = -3$ false

This is going to happen with radical equations; we may get an extraneous solution. In this context, it means that the solution is "mathematically correct, but not relevant or useful, as far as the original question is concerned".

Solving an Equation with Radicals

Step 1 Isolate the radical. Make sure that one radical term is alone on one side of the equation.

Step 2 Apply the power rule. Raise both sides of the equation to a power that is the same as the index of the radical.

Step 3 Solve the resulting equation; if it still contains a radical, repeat Steps 1 and 2.

Step 4 Check all proposed solutions in the original equation.

Example C:

$$\sqrt[3]{4x-8} - 4 = 0$$

$$\left(\sqrt[3]{4x-8}\right)^3 = (4)^3$$

$$4x - 8 = 64$$

$$4x = 72$$

$$x = 18$$

check

$$\sqrt[3]{72-8} - 4 \stackrel{?}{=} 0$$

$$\sqrt[3]{64} - 4 \stackrel{?}{=} 0 \rightarrow 4 - 4 = 0 \checkmark$$

Example D:

$$3\sqrt{2x-1} - 9 = -3$$

$$3\sqrt{2x-1} = 6$$

$$\sqrt{2x-1} = 2$$

$$2x - 1 = 4$$

$$2x = 5$$

$$x = 5/2$$

check

$$3\sqrt{5-1} - 9 \stackrel{?}{=} -3$$

$$3\sqrt{4} - 9 \stackrel{?}{=} -3$$

$$6 - 9 \stackrel{?}{=} -3$$

$$-3 = -3 \checkmark$$

Example E:

$$\sqrt{x+1} + 2 = -4$$

$$\sqrt{x+1} = -6$$

stop
right
there!

APPLICATION: Helen drops a ball from 25 feet above a lake. The formula $t = \frac{1}{4}\sqrt{25-h}$ describes the time in seconds that the ball is h feet above the water.

A) How long is the ball in the air when the ball is at 16 feet? $t = \frac{1}{4}\sqrt{25-16}$
 $t = \frac{1}{4}(\sqrt{9})$ $t = \frac{3}{4}$ sec.

B) How many feet above the water will the ball be after 1 second?
 $1 = \frac{1}{4}\sqrt{25-h}$ $4 = \sqrt{25-h}$
 $16 = 25-h$

Practice

OBJECTIVE: I can solve radical and root equations with multiple roots. $h=9$ ft.

Example A: $\sqrt{x-4} = \sqrt{2x-13}$

$x-4 = 2x-13$
 $9 = x$

check
 $\sqrt{5} = \sqrt{5} \checkmark$

Example B: $\sqrt[3]{2x+7} = \sqrt[3]{3x-2}$

$2x+7 = 3x-2$
 $9 = x$
 $\sqrt[3]{25} = \sqrt[3]{25} \checkmark$

Example C: $x = \sqrt{x^2 - 5x + 15}$

$x^2 = x^2 - 5x + 15$
 $0 = -5x + 15$
 $5x = 15$

$x = 3$

check
 $\sqrt{9} = -3$

Example D: $\sqrt{5-x} = x+1$

$5-x = (x+1)^2$
 $5-x = x^2 + 2x + 1$
 $0 = x^2 + 3x - 4$
 $0 = (x+4)(x-1)$
 $x = -4$ $x = 1$

~~$x = -3$~~
 ~~$x = -1$~~
 $(x-3)(x+1) \neq 0$

Example E: $\sqrt{1-2p-p^2} = p+1$

$1-2p-p^2 = p^2 + 2p + 1$
 $0 = 2p^2 + 4p$
 $0 = 2p(p+2)$
 $p = 0$ ~~$p = -2$~~

Example F: $\sqrt{2x+3} + \sqrt{x+1} = 1$

$\sqrt{2x+3} = 1 - \sqrt{x+1}$
 $2x+3 = 1 - 2\sqrt{x+1} + x+1$
 $2x+3 = 2 + x - 2\sqrt{x+1}$
 $x+1 = -2\sqrt{x+1}$
 $x^2 + 2x + 1 = 4(x+1)$
 $x^2 + 2x + 1 = 4x + 4$
 $x^2 - 2x - 3 = 0$