Pre-Calculus Midterm Exam Review

This is a suggested list of review concepts and is not limited to the following:

KEY

Unit 1 Functions

- Finding domain and range
- Composition of Functions
- Inverse functions
- Transformations

Unit 2 Exponential and Logarithmic Functions

- Properties of exponential and logarithmic graphs
- Re-writing exponential and logarithmic expressions
- Logarithmic Properties
- Solving exponential and logarithmic equations
- Growth and Decay/Interest/Newton's Law of Cooling

Unit 3 Rational Functions

- Simplifying Rational Expressions
- Adding/Subtracting/Multiplying/Dividing Rational Expressions
- Solving Rational Equations
- Solving Rational Inequalities
- Graphing Rational Functions

Unit 4 Sequences and Series

- Arithmetic Sequences and Series
- Geometric Sequences and Series
- Applications

Unit 1 Functions

1) Using the standard form of a function $y = a\,f[b(x-c)]+d$

What does each variable control and how? Write in the correct order in which you'd perform the translations:

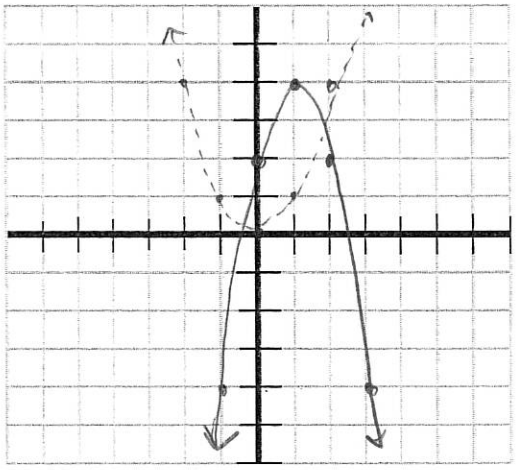
b → horz. stretch / shrink, reflect y-axis (compress)
c → shift left / right
a → vertical stretch / shrink, reflect x-axis
d → shift up / down

Directions: Name the parent function, and then describe the translation that will occur in words or as algebraic expressions. Graph!

→ QUADRATIC

2) $f(x) = -2(x-1)^2 + 4$

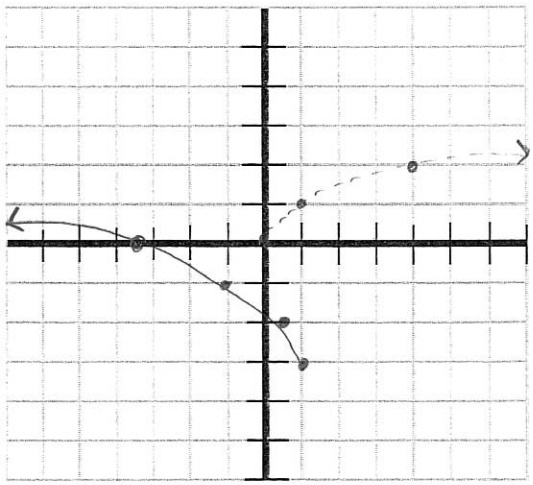
1. None (x, y)
2. right 1 $(x+1, y)$
3. reflect x-axis, vert stretch $(x+1, -2y)$
4. up 4 $(x+1, -2y+4)$



$f(x) = [-2(x-1)]^{1/2} - 3$

3) $f(x) = \sqrt{-2(x-1)} - 3$ SQUARE ROOT

1. y-axis reflection, horz. shrink $(-\frac{x}{2}, y)$
2. right 1 $(-\frac{x}{2}+1, y)$
3. None $(-\frac{x}{2}+1, y)$
4. down 3 $(-\frac{x}{2}+1, y-3)$



Directions: Now, word backwards. I'll give you the translation, you write the function.

4) $f(x)=|x|$

b Compress the x values by dividing the x values by 2.

c Horizontal shift 3 left.

a Stretch the y values by factor of 5.

a Reflect over the x.

d Translate 6 units down.

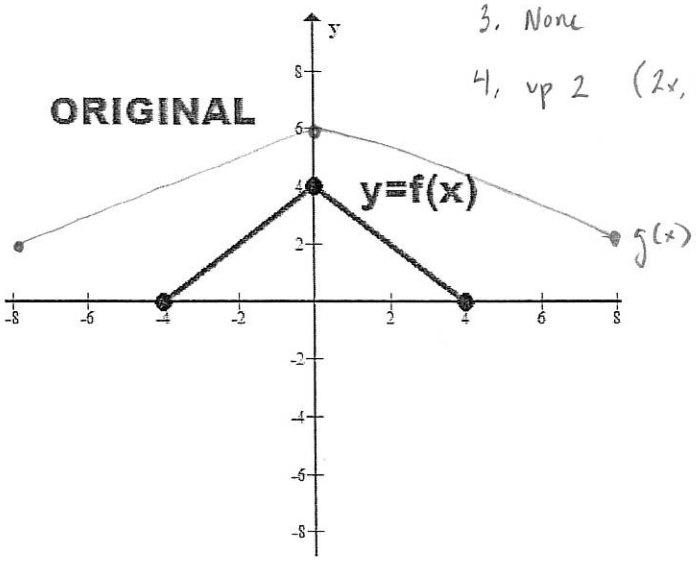
$g(x) = -5|2(x+3)|-6$

↳ or $g(x) = -5|2x+6|-6$

5) Think of f(x) as the parent function. Use the graph of f(x) to graph g(x)

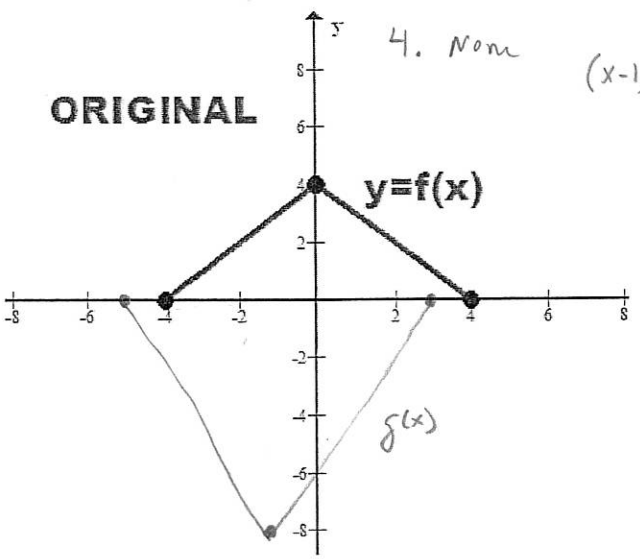
a) $g(x)=f\left(\frac{1}{2}x\right)+2$

1. horz stretch $(2x, y)$
2. None
3. None
4. up 2 $(2x, y+2)$



b) $h(x) = -2f(x+1)$

1. None
2. left 1 $(x-1, y)$
3. vert stretch, reflect x-axis
4. None $(x-1, -2y)$



6) Write the inverse of the function: $f(x)=4x^2-16$ →

$y = 4x^2-16$

$x = 4y^2-16$

$\frac{x+16}{4} = \frac{4y^2}{4}$

$y^2 = \frac{x+16}{4}$

$y^{-1} = \frac{\pm\sqrt{x+16}}{2}$

7) Find the inverse of $f(x)=\frac{5-3x}{2}$. Is the function one-to-one? Is the inverse a function?

$x = \frac{5-3y}{2}$

$2x = 5-3y$

$2x-5 = -3y$

$y^{-1} = \frac{2x-5}{-3}$

yes → passes the vertical line test AND the horizontal line test

8-9. Find $f(g(x))$ *and* $g(f(x))$ and verify wheather the pair of functions given below are inverses of each other using function composition.

8. $f(x) = 6x + 7$ and $g(x) = \dfrac{x-7}{6}$.

$f[g(x)] = 6\left(\frac{x-7}{6}\right) + 7 = x - 7 + 7 = \boxed{x}$ ✓

$g[f(x)] = \dfrac{(6x+7)-7}{6} = \dfrac{6x}{6} = \boxed{x}$ ✓

Yes, inverses

9. $f(x) = 1 - x^3$
$g(x) = \sqrt[3]{1-x}$

$f[g(x)] = 1 - \left(\sqrt[3]{1-x}\right)^3$

$= 1 - (1-x) = 1 - 1 + x = \boxed{x}$ ✓

$g[f(x)] = \sqrt[3]{1-(1-x^3)}$

Yes, inverses

$= \sqrt[3]{1-1+x^3} = \sqrt[3]{x^3} = \boxed{x}$ ✓

10. Given $f(x) = 2x^2 + 3$

a. Is the above function one-to one for all values of x?

No, fails horizontal line test

a. Find the inverse of f(x).

$x = 2y^2 + 3$

$2y^2 = x - 3$

$y^2 = \dfrac{x-3}{2}$

$y^{-1} = \pm\sqrt{\dfrac{x-3}{2}}$

c. Is the inverse a function?

No

Unit 2 Logarithmic and Exponential Functions

11. Evaluate each logarithm.

a) $\log_2 8$ (3)

b) $\log_7 7$ (1)

c) $\log_{\frac{1}{5}} 125 = x$ (-3)

$\left(\frac{1}{5}\right)^x = 125$

$\left(\frac{1}{5}\right)^x = 5^3$

$5^{-x} = 5^3$

d) $\log_{11} 1$

$11^x = 1$

$x = 0$

12. Use the properties of logarithms to evaluate each expression.

-2-3

a) $\log_3 27 - \log_3 9$

$\log_3 \frac{27}{9}$

$\log_3 3 = 1$

b) $2\log_2 64 + \log_2 2$

$\log_2 64^2 + \log_2 2$

$\log_2 (2^6)^2 + \log_2 2$

$\log_2 2^{12} + \log_2 2$

$\log_2 2^{13} = 13$

c) $-\log_4 \frac{1}{16} - \log_4 64$

$-\log_4 4^{-2} - \log_4 4^3$

$-(-2) - 3$

$2 - 3 = -1$

13. Solve each equation.

a) $8^{2x-1} = 16^x$

$(2^3)^{2x-1} = (2^4)^x$

$2^{6x-3} = 2^{4x}$

$6x - 3 = 4x$

$2x = 3$

$x = \frac{3}{2}$

b) $2 \cdot 3^x - 100 = 62$

$\frac{2 \cdot 3^x}{2} = \frac{162}{2}$

$3^x = 81$

$3^x = 3^4$

$x = 4$

c) $5^{\sqrt{x+8}} = 125^{\sqrt{x}}$

$5^{\sqrt{x+8}} = 5^{3\sqrt{x}}$

$(\sqrt{x+8})^2 = (3\sqrt{x})^2$

$x + 8 = 9x$

$8 = 8x$

$x = 1$

d) $\log_3 2x + \log_3 x - \log_3 9 = \log_3 8$

$\log_3 2x^2 - \log_3 9 = \log_3 8$

$\log_3 \frac{2x^2}{9} = \log_3 8$

$\frac{2x^2}{9} = 8$

$2x^2 = 72$

$x^2 = 36$

$x = 6, -6$

e) $\log_3(2x-5) = \log_3(7x+10) - 1$

$\log_3(2x-5) - \log_3(7x+10) = -1$

$\log_3 \frac{2x-5}{7x+10} = -1$

$\frac{2x-5}{7x+10} = \frac{1}{3}$

$6x - 15 = 7x + 10$

$-25 = x$

$\emptyset$

h) $\log 4x = 3$

$10^3 = 4x$

$x = 250$

i) $2\log x = -4$

$\log x = -2$

$10^{-2} = x$

$x = 1/100$

j) $3x^{\frac{5}{4}} = 96$

$(x^{5/4})^{4/5} = (32)^{4/5}$

$x = 16$

k) $12 = 7e^{2k}$

$e^{2k} = \frac{12}{7}$

$\ln e^{2k} = \ln(12/7)$

$2k = \ln(12/7)$

$k = .2695$

l) $3^{x-1} = 24$

$(x-1)\ln 3 = \ln 24$

$x - 1 = \frac{\ln 24}{\ln 3}$

$x = 3.8928$

m) $4^{2x} = 3^{x+1}$

$2x \ln 4 = \ln 3 (x+1)$

$2x\ln 4 - x\ln 3 = \ln 3$

$x(2\ln 4 - \ln 3) = \ln 3$

$x = \frac{\ln 3}{(2\ln 4 - \ln 3)} = .6563$

n) $e^{2x} - e^x - 6 = 0$

$(e^x - 3)(e^x + 2) = 0$

$e^x - 3 = 0$ $\quad e^x + 2 = 0$

$\ln e^x = \ln 3$ $\quad \ln e^x = \ln(-2)$

$x = 1.0986$ $\quad$ ~~0~~

p) $\log_2(-x) = 3 - \log_2(2-x)$

$\log_2(-x) + \log_2(2-x) = 3$

$\log_2(-2x + x^2) = 3$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = \cancel{4}, -2$

14. Solve each word problem.

a) How long will it take an investment of $300 to triple if the interest rate is 9.5% per year, compounded continuously?

$3 = e^{.095t}$

$t \approx 11.6$ yrs.

b) An investment was made at 9.5% interest compounded quarterly. How long will it take for the investment to double in value?

$$2 = 1\left(1 + \frac{0.095}{4}\right)^{4t}$$

$t = 7.4$ yrs.

c) A new computer that cost \$1,600 has a depreciated value of \$900 after 2 years. Find the value of the computer after 3 years if it depreciates exponentially.

$V_n = P(1 \pm r)^n$

$900 = 1600(1-r)^2$

$r = 1/4$

$A = 1600(1 - 1/4)^3$

$A = \$675$

d) Bacteria usually reproduce by a process known as binary fission. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal condition, some bacteria can reproduce every 20 minutes. Find the constant k for the growth of these types of bacteria under ideal conditions and write the growth equation.

$$2 = e^{20k}$$

$K = 0.0347$

e) A cake removed from an oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F. Find a model for the temperature of the cake, T, after t minutes.

a) What is the temperature of the cake after 40 minutes?

$T = 70 + (210 - 70)e^{30k}$

$K = -.0231$

$T = 70 + 140e^{-.0231(40)}$

$T = 125.5699^\circ$

b) When will the temperature of the cake be 90°F?

$$90 = 70 + 140e^{-.0231t}$$

$t = 84.2255$ min

Unit 3 Rational Functions

Simplify each expression.

1. $\dfrac{\dfrac{p^2+7p}{3p}}{\dfrac{49-p^2}{3p-21}}$

$\dfrac{p(p+7)}{3p} \cdot \dfrac{3(p-7)}{-1(p-7)(p+7)} = -1$

2. $\dfrac{8x^3-27}{4x^2-9}$

$\dfrac{(2x-3)(4x^2+6x+9)}{(2x-3)(2x+3)}$

$\dfrac{4x^2+6x+9}{2x+3}$

3. $\dfrac{3r}{2r-s} - \dfrac{2r}{2r+s} + \dfrac{2s^2}{4r^2-s^2}$ $\quad (2r+s)\ (2r-s)$

LCM: $(2r-s)(2r+s)$

$\dfrac{3r(2r+s) - 2r(2r-s) + 2s^2}{\text{LCM}}$

$\dfrac{r+2s}{2r-s}$

Solve each equation. Be sure to check your solutions.

4. $\dfrac{1}{x+1} + \dfrac{1}{x-1} = \dfrac{2}{x^2-1}$ $\quad (x-1)\ (x+1)$

LCM: $(x+1)(x-1)$

$(x-1) + (x+1) = 2$

$x = 1$

$\{1\}$

5. $\dfrac{4}{x^2-2x-3} = \dfrac{-1}{x+1} + \dfrac{x}{3-x}$ $\quad (x-3)(x+1) \quad -1(x-3)$

LCM: $(x-3)(x+1)$

$4 = -1(x-3) + x(x+1)$

$0 = x^2 - 1$

$x = 1, -1$

6. $\dfrac{5x+2}{x^2-4} = \dfrac{5x}{2-x} + \dfrac{2}{x+2}$ $\quad (x+2)(x-2) \quad -1(x-2)$

LCM: $(x-2)(x+2)$

$5x+2 = 5x(x+2) + 2(x-2)$

$5x^2 + 7x - 6 = 0$

$(5x-3)(x+2) = 0$

$x = 3/5, -2$

7. $\dfrac{5}{x-5} = \dfrac{x}{x-5} - 1$

LCM: $x-5$

$5 = x - x + 5$

$5 = 5$

all $\mathbb{R}$; except $x \neq 5$

8. $\frac{4}{x-2}-\frac{x+6}{x+1}=1$ LCM: $(x-2)(x+1)$

$4(x+1)-(x+6)(x-2)=(x-2)(x+1)$

$2x^2-x-18=0$

$x=\frac{1\pm\sqrt{145}}{4}$

9. $\frac{1}{x+2}+\frac{1}{x-2}=\frac{3}{x+1}$ LCM: $(x+2)(x-2)(x+1)$

$(x-2)(x+1)+(x+2)(x+1)=3(x+2)(x-2)$

$x^2-2x-12=0$

$x=\frac{2\pm\sqrt{52}}{2}=\frac{2\pm2\sqrt{13}}{2}$

$=1\pm\sqrt{13}$

Solve each inequality.

10. $1+\frac{5}{a-1}\le\frac{7}{6}$

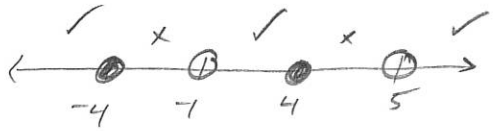
$-\frac{1}{6}+\frac{5}{a-1}\le0$

$\frac{-a+31}{6(a-1)}\le0$

$(-\infty,1)\cup[31,\infty)$

11. $\frac{x^2-16}{x^2-4x-5}\ge0$

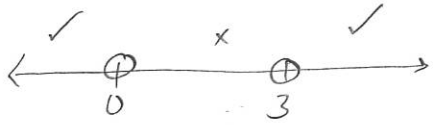
$\frac{(x-4)(x+4)}{(x-5)(x+1)}\ge0$



$(-\infty,-4]\cup(-1,4]\cup(5,\infty)$

12. $5+\frac{1}{x}>\frac{16}{x}$

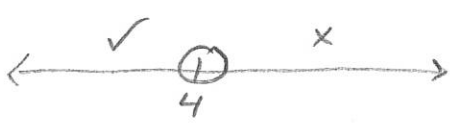
$\frac{5x-15}{x}>0$



$(-\infty,0)\cup(3,\infty)$

13. $\frac{2a-5}{6}-\frac{a-5}{4}<\frac{3}{4}$

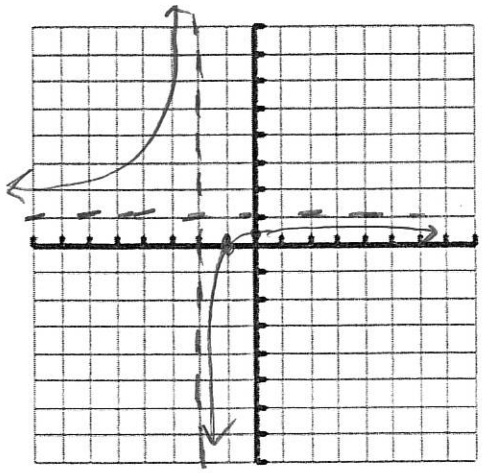
$\frac{a-4}{12}<0$



$(-\infty,4)$

14. Identify any holes, vertical asymptotes, horizontal asymptotes, and slant asymptotes of each graph. If they do not exist write **none** in the space provided. Accurately graph each rational function.

a. $R(x)=\dfrac{x^2+3x+2}{(x+2)^2}$



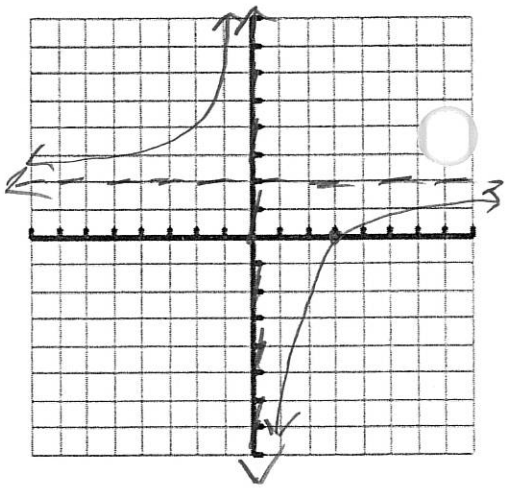
Domain $\{x \mid x \neq -2\}$

Hole(s) None Vertical Asymptotes $x=-2$

X-intercepts $(-1,0)$ Horizontal Asymptotes $y=1$

Y-intercepts $(0, \frac{1}{2})$ Slant Asymptotes None

b. $F(x)=\dfrac{2x-6}{x}$



Domain $\{x \mid x \neq 0\}$

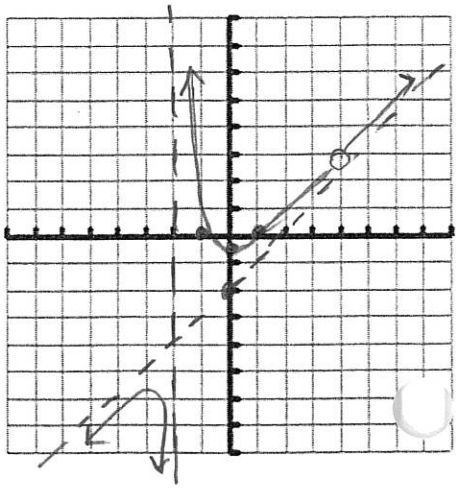
Hole(s) None Vertical Asymptotes $x=0$

X-intercepts $(3,0)$ Horizontal Asymptotes $y=2$

Y-intercepts None Slant Asymptotes None

c. $G(x)=\dfrac{(x^2-1)(x-4)}{x^2-2x-8} = \dfrac{(x^2-1)(x-4)}{(x-4)(x+2)}$

$$x+2 \overline{)\, x^2+0x-1} \quad x-2$$
$$-x^2+2x$$
$$-2x-1$$
$$+2x+4$$



Domain $\{x \mid x \neq 4, -2\}$

Hole(s) $(4, 2.5)$ Vertical Asymptotes $x=-2$

X-intercepts $(-1,0)$ $(1,0)$ Horizontal Asymptotes None

Y-intercepts $(0, -\frac{1}{2})$ Slant Asymptotes $y=x-2$