Precalculus Name Key

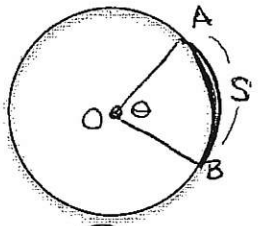
4.1 Notes: Radian and Degree Measure- *Day 3* *(Arc Length & DMS)*

Arc length: this is essentially a review from Geometry!

In Geometry, you learned that arc length was just a fraction of the circle's circumference. You found the length of an arc from the measure of the central angle that intercepts the arc in a circle with a specified radius. The angle measure you used was in degrees. Let's take that formula from Geometry and rearrange some factors to discover a formula for arc length using the angle measure in radians!

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{degree measure of arc}}{360^\circ}$$

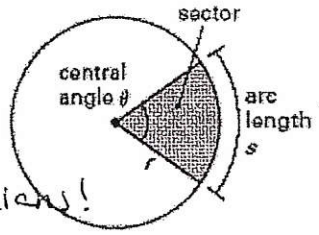
$$\frac{S}{2\pi r} = \frac{\theta}{360}$$



$m\overset{\frown}{AB} = \theta$

ARC LENGTH

The arc length s with radius r and central angle θ (measured in radians) is as follows.



2π rad $= 360^\circ$

$$\frac{S}{2\pi r} = \frac{\theta}{2\pi}$$

$$S = \frac{\theta}{2\pi} \cdot 2\pi r$$

Formula:

$$S = r\theta$$

* θ must be in radians!

Examples:

1. If r = 7 in. and the central angle of a circle is 215°, then find the length of the arc that subtends the circle.

$\theta = 215^\circ$

$\theta = 215^\circ \cdot \frac{\pi}{180^\circ}$ $\quad \theta = \frac{43\pi}{36}$

$S = r\theta$

$S = 7\left(\frac{43\pi}{36}\right)$

$S = \frac{301\pi}{36} \approx 26.267$ in

2. Suppose the central angle $\theta = \frac{5\pi}{6}$ and the radius of the circle is 5 ft. Find the length of the arc that meets this angle.

$S = r\theta$

$S = 5\left(\frac{5\pi}{6}\right)$

$S = \frac{25\pi}{6}$

$S = \frac{25\pi}{6} \approx 13.090$ ft

3. Find the arc length with given radius and angle: r = 10 m. $\theta = \frac{2\pi}{7}$

$S = r\theta$

$S = 10 \cdot \frac{2\pi}{7}$

$S = \frac{20\pi}{7} \approx 8.976$ m

4. Find the arc length with given radius and angle: r = 5 cm. $\theta = 75^\circ$

$\theta = 75^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{12}$

$S = r\theta$

$S = 5\left(\frac{5\pi}{12}\right) = \frac{25\pi}{12} \approx 6.545$ cm

5. Find the radius of the circle with given arc length and angle: $s = 3.5$ in. $\theta = 60°$

$\theta = 60° \cdot \frac{\pi}{180°} = \frac{\pi}{3}$ $\quad S = r\theta$

$3.5 = r \cdot \frac{\pi}{3}$

$\frac{3.5}{(\pi/3)} = r$

$r \approx 3.342$ inches

6. Find the central angle with given radius and arc length: $r = 4$ ft. $s = 6.25$ ft.

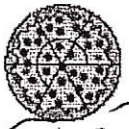
$S = r\theta$

$6.25 = 4\theta$

$6.25/4 = \theta$

$\theta = 1.5625$ radians

7. A pizza is cut into six pieces. Each piece represents an equivalent sector of the pizza. Find the measure of the angle formed by four pieces of pizza. Give the answer in both degrees and radians.



$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$ radians (each piece)

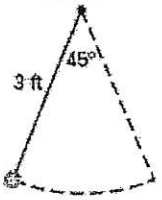
$\theta = \frac{360}{6} = 60°$

4 pieces

$4(\pi/3) = \frac{4\pi}{3}$ RADS

$4(60°) = 240°$

8. The pendulum of a grand father clock is 3 feet long and swings back and forth creating a 45° angle. Find the length of the arc created by the pendulum after one swing.



$\theta = 45 \cdot \frac{\pi}{120°} = \frac{\pi}{4}$

$S = r\theta$

$S = 3\left(\frac{\pi}{4}\right)$

$S = \frac{3\pi}{4}$ RADS

≈ 2.356 ft

Conversions with Degrees, Minutes & Seconds

$1° = 60'$ (minutes)
$1' = 60''$ (seconds)

Symbol for degree: °
Symbol for minute: '
Symbol for second: "

There is another way to state the size of an angle, one that subdivides a degree using a system different than the decimal number. The words minute and second used in this context have no immediate connection to how those words are usually used as amounts of time. 2nd Apps → Angle Menu to find seconds " → Alpha +

- *In a full circle there are 360 degrees.*
- *Each degree is split up into 60 parts, each part being 1/60 of a degree. These parts are called minutes.*
- *Each minute is split up into 60 parts, each part being 1/60 of a minute. These parts are called seconds.*

Convert from D°M'S" (Degrees/Minutes/Seconds) to decimal form:

9. 29°45'27"

$29 + \frac{45}{60} + \frac{27}{3600}$

$29.7575°$

10. −408°16'25"

$-\left(408 + \frac{16}{60} + \frac{25}{3600}\right)$

$-408.2736°$

Convert the angle measure to D°M'S" form.

11. 280.6°

$.6 \div 1/60 = 36$ min

$280° 36' 0"$

12. −345.12°

$.12(60) = 7.2$ min

$.2(60) = 12$ sec

$-345° 7' 12"$