Lesson #8 Intro to Trigonometric Identities

Key

I. Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

II. Quotient Identities

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$

III. Pythagorean Identities

$a^2+b^2=c^2 \rightarrow x^2+y^2=r^2$

In unit ⊙ $x^2+y^2=1$

a) Since $\cos\theta = x$

$\sin\theta = y$

$\cos^2\theta + \sin^2\theta = 1$

b) Now, play with it!

$\rightarrow \frac{\cos^2\theta+\sin^2\theta=1}{\cos^2\theta}$

$\frac{\cos^2+\sin^2=1}{\sin^2\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

So

$\cos^2\theta = 1-\sin^2\theta$

$\sin^2\theta = 1-\cos^2\theta$

$1+\tan^2\theta = \sec^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\therefore \tan^2\theta = \sec^2\theta - 1$

Simplify all the following trigonometric expression into a SINGLE trig function or number

Strategy #1: Write all trig functions in terms of sines and cosines (look for Pythagorean identities)

a) $\cos x \csc x \tan x$

$\cos x \cdot \csc x \cdot \tan x$

$= \cos x \cdot \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x}$

$= 1$

b) $\frac{\csc\theta}{\cot\theta}$

$= \frac{1}{\sin\theta} \div \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \sec\theta$

c) $\frac{\cot^2 x}{1-\sin^2 x}$

$= \frac{\cot^2 x}{\cos^2 x}$

$= \frac{\cos^2 x}{\sin^2 x} \div \frac{\cos^2 x}{1}$

$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$

$\frac{1}{\sin^2 x} = \csc^2 x$

Strategy #2: Find a common denominator

a) $\tan\beta \sin\beta + \cos\beta$

$\frac{\sin B \cdot \sin B}{\cos B} + \cos B$

$= \frac{\sin^2 B}{\cos B} + \frac{\cos^2 B}{\cos B}$

$= \frac{1}{\cos B} = \sec B$

b) $\sin\theta + \cos\theta \cot\theta$

$\sin\theta + \cos\theta \cdot \frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta}$

$\csc\theta$

c) $\frac{\sec x - \cos x}{\sec x}$

$\frac{\sec x}{\sec x} - \frac{\cos x}{\sec x}$

$1 - \cos x \div \frac{1}{\cos x}$

$1 - \cos^2 x$

$\sin^2 x$

Strategy #3: Factoring (or FOIL'ing)

GCF

a) $\csc^2 x \cot x - \cot x$

$= \cot x (\csc^2 x - 1)$

$= \cot x (\cot^2 x)$

$= \cot^3 x$

b) $(1+\tan x)^2 - 2\sin x \sec x$

$= 1 + 2\tan x + \tan^2 x - 2\sin x/\cos x$

$= 1 + 2\tan x + \tan^2 x - 2\tan x$

$1 + \tan^2 x$

$= \sec^2 x$

c) $\frac{\cos^2\beta}{1-\sin\beta}$

Rewrite:

$\frac{1-\sin^2 B}{1-\sin B}$

$\frac{(1+\sin B)(1-\sin B)}{1-\sin B}$

$= 1 + \sin B$

GCF

d) $\sin^2 x + \sin^2 x \tan^2 x$

$\sin^2 x (1 + \tan^2 x)$

$\sin^2 x (\sec^2 x)$

$\sin^2 x \left(\frac{1}{\csc^2 x}\right)$

$= \tan^2 x$

e) $\cos^4 \theta + 2\cos^2 \theta \sin^2 \theta + \sin^4 \theta$

Like a Perfect trinomial square:

$(\cos^2 + \sin^2)(\cos^2 + \sin^2)$

$(1)(1)$

$= 1$

Independent Practice

Simplify each trigonometric expression

1) $\cos\alpha + \sin\alpha \tan\alpha$

Rewrite using sin & cos

$\cos\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$\frac{1}{\cos\theta} = \sec\theta$

2) $\cos x \cot x + \sin x$

$\frac{\cos x \cdot \cos x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$= \frac{1}{\sin x}$

$= \csc x$

3) $\sin x \cot x$

$\sin x \cdot \frac{\cos x}{\sin x}$

$= \cos x$

4) $\frac{\sec x}{\csc x}$

$= \frac{1}{\cos x} \div \frac{1}{\sin x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

5) $\sin x - \sin x \cos^2 x$

$= \sin x (1 - \cos^2 x)$

$= \sin x (\sin^2 x)$

$= \sin^3 x$

6) $\sin^3 x + \sin x \cos^2 x$

$\sin x (\sin^2 x + \cos^2 x)$

$= \sin x (1)$

$= \sin x$

7) $\dfrac{\csc x - \sin x}{\csc x}$

$\dfrac{\csc x}{\csc x} - \dfrac{\sin x}{\csc x}$

$= 1 - \sin x \div \dfrac{1}{\sin x}$

$= 1 - \sin^2 x$

$= \cos^2 x$

8) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x}$

Hmm . . .
Create common denominator

$\dfrac{\sin x}{\cos x}\left(\dfrac{1+\sin x}{1+\sin x}\right) + \dfrac{\cos x}{(1+\sin x)}\dfrac{\cos x}{\cos x}$

$= \dfrac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)}$

$= \dfrac{\sin x + 1}{\cos x\,(\sin x + 1)} = \dfrac{1}{\cos x} = \sec x$

Use an identity to find the value of each expression or to simplify completely. Do NOT use a calculator

9) $\tan 6.5 \cot 6.5$

$\tan 6.5 \cdot \dfrac{1}{\tan 6.5} = 1$

10) $\sin^2 \dfrac{\pi}{8} + \cos^2 \dfrac{\pi}{8} = 1$

11) $1 - \cos^2 \dfrac{5\pi}{7} = \sin^2 \dfrac{5\pi}{7}$

12) $\cot^2 \dfrac{8}{9}\left(1 - \cos^2 \dfrac{8}{9}\right)$

$= \cot^2 (\sin^2)$

$= \dfrac{\cos^2}{\sin^2} \cdot \sin^2 = \cos^2 \dfrac{8}{9}$

Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$ $\quad \cos\theta = \dfrac{1}{\sec\theta}$ $\quad \tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$ $\quad \cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$