Lesson #8 Intro to Trigonometric Identities

My Cps

- similar to proofs

* $[\sin(x)]^2 = \sin^2 x$ → more common b/c easier to work

I. Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Cross mult → $\csc\theta\sin\theta = 1$

II. Quotient Identities

$\sin\theta$ in unit circle $= y$

$\cos\theta = x$

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

III. Pythagorean Identities

- in Unit Circle $x^2 + y^2 = r^2$

a) $x^2 + y^2 = r^2$

in unit circle $r = 1$ $\sin\theta = y$ $\cos\theta = x$

$(\cos\theta)^2 + (\sin\theta)^2 = (1)^2$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

or $\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

- $\cos^2\theta - \sin^2\theta = -1$ Are these?

* not $1 - \sin\theta = \cos\theta$

$1 - \cos\theta = \sin\theta$

} all work!

b) $\frac{\cos^2\theta + \sin^2\theta = 1}{\cos^2\theta}$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$\tan^2\theta = \sec^2\theta - 1$

c) $\frac{\cos^2\theta + \sin^2\theta = 1}{\sin^2\theta}$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

$\cot^2\theta = \csc^2\theta - 1$

$a^2 + b^2 = c^2$

$\sqrt{3^2 + 4^2} = \sqrt{5^2}$

$3 + 4 \neq 5$

have them put this in lower corner of a- backpg.

* ① $\frac{x+y}{y}$ vs. $\frac{xy}{y}$

② $\frac{xy}{x} = \_\_\_ \cdot \_\_\_$

③ $\frac{x+y}{x} = \_\_\_ + \_\_\_$

Strategies for Simplifying Trig Identities

Simplify all the following trigonometric expression into a SINGLE trig function or number

Strategy #1: Write all trig functions in terms of sines and cosines (look for Pythagorean identities)

a) $\cos x \csc x \tan x$

$\cos x \left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$= 1$

b) $\frac{\csc\theta}{\cot\theta}$

$\frac{\frac{1}{\sin\theta}}{\frac{\cos\theta}{\sin\theta}} = \frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \sec\theta$

c) $\frac{\cot^2 x}{1-\sin^2 x}$

$\frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{\cos^2 x}{1}} \cdot \frac{1}{\cos^2 x} =$

$\frac{1}{\sin^2 x} = \csc^2 x$

Strategy #2: Find a common denominator

a) $\tan\beta \sin\beta + \cos\beta$

$\frac{\sin\beta}{\cos\beta} \cdot \sin\beta + \cos\beta$

$\frac{\sin^2\beta}{\cos\beta} + \frac{\cos^2\beta}{\cos\beta} = \frac{\sin^2\beta + \cos^2\beta}{\cos\beta}$

$\frac{1}{\cos\beta} = \sec\beta$

b) $\sin\theta + \cos\theta\cot\theta$

$\sin\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} = \frac{1}{\sin\theta}$

$\csc\theta$

c) $\frac{\sec x - \cos x}{\sec x}$ $\frac{x-y}{x}$

$\frac{\sec x}{\sec x} - \frac{\cos x}{\sec x}$

$1 - \frac{\cos x}{1} \cdot \frac{\cos x}{1}$

$1 - \cos^2 x$

$\sin^2 x$

Strategy #3: Factoring (or FOIL'ing)

$\sec^2 x - 1$

a) $\csc^2 x \cot x - \cot x$

$\cot x\left(\csc^2 x - 1\right)$

$\cot x\left(\cot^2 x\right)$

$\cot^3 x$

b) $(1+\tan x)^2 - 2\sin x \sec x$

$1 + 2\tan x + \tan^2 x - 2\sin x \sec x$

$1 + 2\tan x + \tan^2 x - 2\sin x \cdot \frac{1}{\cos x}$

$1 + 2\tan x + \tan^2 x - 2\tan x$

$1 + \tan^2 x$

$\sec^2 x$

c) ~~$\cos x - \cos x \sin^2 x$~~

$\frac{\cos^2\beta}{1-\sin\beta}$ * use diff 2 squares

$\frac{1-\sin^2\beta}{1-\sin\beta}$

$\frac{(1-\sin\beta)(1+\sin\beta)}{(1-\sin\beta)}$

$1+\sin\beta$

try indep

d) $\sin^2 x + \sin^2 x \tan^2 x$

$\sin^2(1 + \tan^2 x)$

$\sin^2 x (\sec^2 x)$

$\sin^2 x \left(\frac{1}{\cos^2 x}\right) = \boxed{\tan^2 x}$

$x^4 + 2x^2y^2 + y^4$ $4x^2 + 12x + 9$

e) $\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta$ $(2x+3)^2$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$1 \cdot 1$

(1)

Independent Practice

Simplify each trigonometric expression

1) $\cos\alpha + \sin\alpha \tan\alpha$

$\cos\alpha + \sin\alpha\left(\frac{\sin\alpha}{\cos\alpha}\right)$

$\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}$

$\frac{\cos^2\alpha}{\cos\alpha} + \frac{\sin^2\alpha}{\cos\alpha} = \frac{1}{\cos\alpha} = \boxed{\sec\alpha}$

2) $\cos x \cot x + \sin x$

$\cos x\left(\frac{\cos x}{\sin x}\right) + \sin x$

$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$\frac{1}{\sin x} = \boxed{\csc x}$

3) $\sin x \cot x$

$\sin x\left(\frac{\cos x}{\sin x}\right)$

$\boxed{\cos x}$

4) $\frac{\sec x}{\csc x}$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1}$

$= \boxed{\tan x}$

5) $\sin x - \sin x \cos^2 x$

$\sin x(1 - \cos^2 x)$

$\sin x(\sin^2 x)$

$\boxed{\sin^3 x}$

6) $\sin^3 x + \sin\cos x \cos^2 x$

$\sin(\sin^2 x + \cos^2 x)$

$\boxed{\sin x}$

7) $\frac{\csc x - \sin x}{\csc x}$

$\frac{\csc x}{\csc x} - \frac{\sin x}{\csc x}$

$1 - \frac{\sin x}{1} \cdot \frac{\sin x}{1}$

$1 - \sin^2 x$

$\boxed{\cos^2 x}$

8) $\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$

$\frac{\sin x(1+\sin x)}{\cos x(1+\sin x)} + \frac{\cos x(\cos x)}{(1+\sin x)(\cos x)}$

$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)} = \frac{\sin x + 1}{\cos x(1+\sin x)} = \frac{1}{\cos x}$

$= \boxed{\sec x}$

Use an identity to find the value of each expression or to simplify completely. Do NOT use a calculator

9) $\tan 6.5 \cot 6.5$

$\tan 6.5 \cdot \frac{1}{\tan 6.5} = 1$

10) $\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8}$ $= 1$

11) $1 - \cos^2 \frac{5\pi}{7}$

$\sin^2 \frac{5\pi}{7}$

12) $\cot^2 \frac{8}{9}\left(1 - \cos^2 \frac{8}{9}\right)$

$\frac{\cos^2 \frac{8}{9}}{\sin^2 \frac{8}{9}} \cdot \sin^2 \frac{8}{9}$

$\cos^2 \frac{8}{9}$