Key

Inverses Homework

Concept – Verifying that 2 Functions are Inverses of Each Other

Directions – Find f(g(x)) and g(f(x)) and verify that they are inverses of each other (both f(g(x)) and g(f(x)) equal x).

1. $f(x)=\sqrt{x-4}$
   $g(x)=x^2+4$

$f(g(x))$	$g(f(x))$
$f(x^2+4)$	$g(\sqrt{x-4})$
$=\sqrt{x^2+4-4}$	$=(\sqrt{x-4})^2+4$
$=\sqrt{x^2}$	$=x-4+4$
$=x$ ☺	$=x$ ☺

2. $f(x)=\dfrac{x+3}{x-2}$
   $g(x)=\dfrac{2x+3}{x-1}$

$f(g(x))$

$f\left(\dfrac{2x+3}{x-1}\right)$

$=\dfrac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}\cdot\dfrac{(x-1)}{(x-1)}$

$=\dfrac{2x+3+3(x-1)}{2x+3-2(x-1)}$

$=\dfrac{5x}{x-1}\div\dfrac{5}{x-1}$

$\dfrac{5x}{x-1}\cdot\dfrac{x-1}{5}=x$ ✓

$g(f(x))$

$g\left(\dfrac{x+3}{x-2}\right)$

$=\dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1}\cdot\dfrac{(x-2)}{(x-2)}$

$=\dfrac{5x}{x-2}\div\dfrac{5}{x-2}$

$=\dfrac{5x}{x-2}\cdot\dfrac{x-2}{5}$

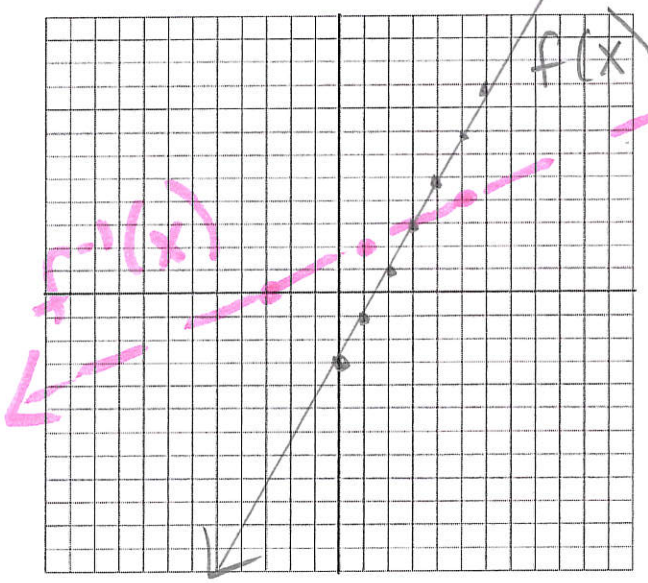
$=x$ ☺

Concept – Use Graphs of Functions To Determine Whether a Function has an Inverse

Directions – Plot f(x) and $f^{-1}(x)$ on the coordinate plane.

$f(x)=2x-3$

$f^{-1}(x)=\dfrac{x+3}{2}=\dfrac{1}{2}x+\dfrac{3}{2}$



or

x	y
0	-3
2	1
4	5

so $f^{-1}(x)$

x	y
-3	0
1	2
5	4

Describe the relationship between the graphs of f(x) and $f^{-1}(x)$.

reflection over line $y=x$

Find the domain and range of f(x) and $f^{-1}(x)$.

$(-\infty,\infty)$

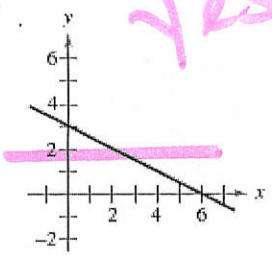
What do you observe?

Points ⇒ (x & y) values interchanged

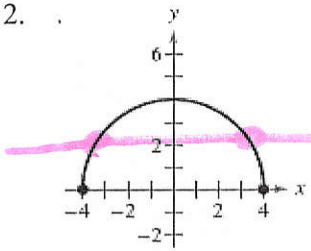
Inverses Homework

Horizontal Line Test and One-to-One Functions

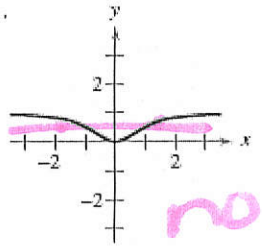
Part 1 – Does this function have an inverse function? (I.E. is it One-to-One?)

1. 

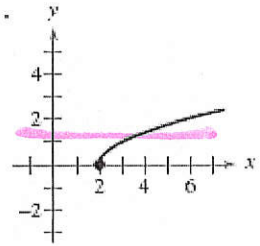
yes!

2. 

no

3. 

no

4. 

yes

Part 2 – Use a graphing calculator to graph the function and determine if it is One-To-One (I.E. it has an inverse function)

5. $g(x)=10$


no

6. $h(x)=-2x\sqrt{16-x^2}$


no
we'd have to restrict domain

7. $f(x)=|x+4|-|x-4|$ (hint – abs() is under MATH->NUM->#1 abs()

Finding Inverses Algebraically

Directions – Find the inverse of the given function algebraically. Then verify that $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$.

1. $f(x)=6x-8$

$y=6x-8$

$x=6y-8$

$\frac{x+8}{6}=y$

$f^{-1}(x)=\frac{x+8}{6}$

$f(g(x))$

$f\left(\frac{x+8}{6}\right)$

$=6\left(\frac{x+8}{6}\right)-8$

$=(x)$ ✓

$g(f(x))$

$g(6x-8)$

$=\frac{6x-8+8}{6}$

$=(x)$ ✓

2. $f(x)=(x-2)^2$

$y=(x-2)^2$

$x=(y-2)^2$

$\pm\sqrt{x}=y-2$

$f^{-1}(x)=\pm\sqrt{x}+2$

$f(g(x))$

$f(\pm\sqrt{x}+2)$

$=(\pm\sqrt{x}+2-2)^2$

$=(\pm\sqrt{x})^2$

$=x$ ✓

$g(f(x))$

$=g(x-2)^2$

$=\pm\sqrt{(x-2)^2}+2$

$=x-2+2$

$=-x+2+2$ only (+)

*. $f(x)=\frac{x+1}{x-2}$

$y=\frac{x+1}{x-2}$

$x=\frac{y+1}{y-2}$ $(y-2)$

$(y-2)$

$xy-2x=y+1$

$xy-y=2x+1$

$y(x-1)=2x+1$

$y=\frac{2x+1}{x-1}$

$f^{-1}(x)=\frac{2x+1}{x-1}$