Final Review

13. **I can evaluate trig values given one value and other information.**

a. Given $\sin\theta = \frac{3}{4}$ and $\cos\theta < 0$, evaluate $\tan\theta$ and $\sec\theta$.

b. Given $\tan\theta = \frac{7}{4}$ and $\sec\theta < 0$, evaluate $\sin\theta$ and $\cos\theta$.

c. Given $\sin\theta = \frac{3}{5}$ and θ is in Quadrant II, evaluate $\cos\theta$ and $\sec\theta$.

d. Given $\tan\theta = \frac{-5}{3}$ and θ is in Quadrant IV, evaluate $\sin\theta$ and $\sec\theta$.

Unit 2 Practice Problems

1. Convert the angle measure from degrees to radians or from radians to degrees. (calc)

$115^\circ \cdot \frac{\pi}{180} = \boxed{\frac{23\pi}{36}}$

a. 115° ______

$\frac{13\pi}{2} \cdot \frac{180}{\pi} = 13(90) = \boxed{1170^\circ}$

b. $\frac{13\pi}{2}$ ______ $\frac{4\pi}{2} \cdot 3 =$

3. Determine two coterminal angles (one positive and one negative) for each angle. Give your answer in radians. (calc)

a. $\theta = \frac{\pi}{12}$ $\frac{25\pi}{12}$ & $\frac{-23\pi}{12}$

± 360 $\pm 2\pi$ $\pm \frac{24\pi}{12}$

b. $\theta = -435^\circ$ $+360$ -75° & 285°

$= -2\pi +$

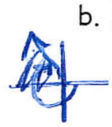
4. Find the reference angle and determine which quadrant θ lies. (calc)

a. θ= 203° 23°

$203-180=23^\circ$

Quadrant: Q3

b. θ= -245° 65°



Quadrant: Q2

c. $\theta = \frac{2\pi}{3}$ $\pi/3$

Quadrant: Q2

d. $\theta = -\frac{13\pi}{3}$ $\pi/3$

Quadrant: Q4

5. Find the radian measure of the central angle of a circle if the radius = 14.5 centimeters and the arc length = 25 centimeters. (calc)

$s = \theta r$

$25 = 14.5\theta$

$\theta = 25/14.5 =$

1.724 radians

6. Find the length of the arc on a circle of radius r intercepted by a central θ.(calc)

a. radius = 15 inches, central angle θ= 180° ______

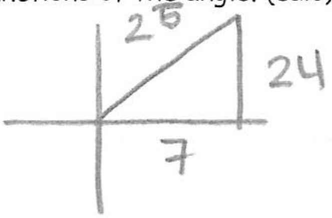
$s = 15(\pi) = 15\pi$ inches ≈ 47 inches

b. radius = 20 centimeters, central angle $\theta = \frac{\pi}{4}$ radians ______

$s = 20 \cdot \frac{\pi}{4} = 5\pi \approx 15.7$ cm

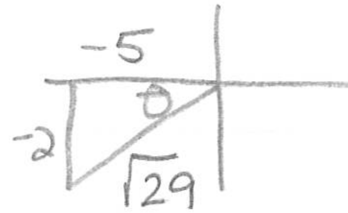
7. The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle. (calc)

a. (7, 24)



$x^2+y^2=r^2$

b. (-5,-2)



sin a $24/25$ csc a $25/24$

cos a $7/25$ sec a $25/7$

tan a $24/7$ cot a $7/24$

sin a $-2\sqrt{29}/29$ csc a $-\frac{\sqrt{29}}{2}$

cos a $-5\sqrt{29}/29$ sec a $-\sqrt{29}/5$

tan a $2/5$ cot a $5/2$

8. State the quadrant in which θ lies. (no calc)

a. $\sin\theta > 0$ and $\cos\theta > 0$ 1

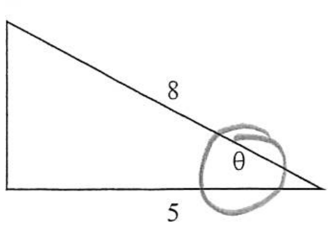
b. $\sec\theta > 0$ and $\cot\theta < 0$ ~~4~~ 2

9. A carousel with a 50-foot diameter makes 4 revolutions per minute. What is the angular velocity in radians per hour? What is the linear velocity in inches per hour? (calc)

Angular Velocity: 1508 rad/hr

Linear Velocity: 452,389.34 in/hr.

10. Find the 6 trig functions for θ in the triangle below. Assume the triangle is a right triangle.



$\sqrt{39}$

$x^2+y^2=r^2$

$a^2+b^2=c^2$

$25+b^2=64$

$b^2=39$

$b=\sqrt{39}$

$\cos\theta = 5/8$

$\sin\theta = \sqrt{39}/8$

$\tan\theta = \sqrt{39}/5$

$\sec\theta = 8/5$

$\csc\theta = 8\sqrt{39}/39$

$\cot\theta = 5\sqrt{39}/39$

EVEN MORE PRACTICE - 4.1 – 4.4

41. A circle has a radius of 7 inches. Find the **length of the arc** intercepted by a central angle of $240°$.

41.)______________

42. The circular blade on a saw rotates at 2400 revolutions per minute.

a. Find the **angular speed** in radians per second.

42a.)______________

b. The blade has a radius of 4 inches. Find the **linear speed** of a blade tip in inches per second.

42b.)______________

43. A satellite in circular orbit 1125 km above a planet makes one complete revolution every 120 minutes. Assuming that he planet is a sphere of radius 6400 km, find the linear speed of the satellite in **kilometers per minute.** Round your answer to the nearest whole number.

43.)______________

44. A truck is moving at a rate of 90 km per hour and the diameter of its wheels is 1.25 meters. Find the angular speed of the wheels in **radians per minute.**

44.)______________

45. **Evaluate** (if possible) the six trigonometric functions if $\theta = -\frac{2\pi}{3}$.

45.

$\sin\left(-\frac{2\pi}{3}\right) =$ $\qquad$ $\csc\left(-\frac{2\pi}{3}\right) =$

$\cos\left(-\frac{2\pi}{3}\right) =$ $\qquad$ $\sec\left(-\frac{2\pi}{3}\right) =$

$\tan\left(-\frac{2\pi}{3}\right) =$ $\qquad$ $\cot\left(-\frac{2\pi}{3}\right) =$

46. **Evaluate** the trigonometric function.

a. $\sin\left(-\frac{3\pi}{4}\right)$ $\qquad$ b. $\csc\left(\frac{7\pi}{6}\right)$ $\qquad$ c. $\tan\left(\frac{5\pi}{3}\right)$

46a.)______________

46b.)______________

46c.)______________

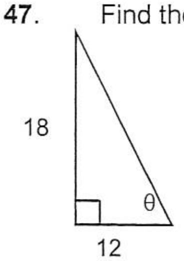
d. $\sec(-4\pi)$ e. $\cot\left(\frac{5\pi}{2}\right)$ f. $\cos\left(\frac{13\pi}{4}\right)$

46d.)_______________

46e.)_______________

46f.)_______________

47. Find the **exact values** of the **six trigonometric functions** of the angle θ shown in the figure.



$\sin(\theta)=$ $\csc(\theta)=$

47. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

53. Use the given value and the trigonometric identities to **find the remaining trigonometric functions** of the angle.

$\sin(\theta)=$ $\csc(\theta)=$

$\cos\theta = -\frac{3}{7}, \ \sin\theta < 0$

53. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

54. The point is on the terminal side of an angle in standard position. **Determine** the **exact values** of the six trigonometric functions of the angle.

$(8, -15)$

$\sin(\theta)=$ $\csc(\theta)=$

54. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

KEY TO EVEN MORE PRACTICE:

41. $\frac{28\pi}{3}$ inches

42a. 80π rad/sec

42b. ≈ 1005 in/sec

43. 394 km/min

44. 2400 rad/min

45.
$\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\quad \csc\left(-\frac{2\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\quad \sec\left(-\frac{2\pi}{3}\right) = -2$

$\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$ $\quad \cot\left(-\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$

46a. $-\frac{\sqrt{2}}{2}$

46b. -2

46c. $-\sqrt{3}$

46d. 1

46e. 0

46f. $-\frac{\sqrt{2}}{2}$

47.
$\sin(\theta) = \frac{3\sqrt{13}}{13}$ $\quad \csc(\theta) = \frac{\sqrt{13}}{3}$

$\cos(\theta) = \frac{2\sqrt{13}}{13}$ $\quad \sec(\theta) = \frac{\sqrt{13}}{2}$

$\tan(\theta) = \frac{3}{2}$ $\quad \cot(\theta) = \frac{2}{3}$

53.
$\sin(\theta) = -\frac{2\sqrt{10}}{7}$ $\quad \csc(\theta) = -\frac{7\sqrt{10}}{20}$

$\cos(\theta) = -\frac{3}{7}$ $\quad \sec(\theta) = -\frac{7}{3}$

$\tan(\theta) = \frac{2\sqrt{10}}{3}$ $\quad \cot(\theta) = \frac{3\sqrt{10}}{20}$

54.
$\sin(\theta) = -\frac{15}{17}$ $\quad \csc(\theta) = -\frac{17}{15}$

$\cos(\theta) = \frac{8}{17}$ $\quad \sec(\theta) = \frac{17}{8}$

$\tan(\theta) = -\frac{15}{8}$ $\quad \cot(\theta) = -\frac{8}{15}$

Pre-Calculus UNIT 6
Chapter 4.5-4.7
Learning Objectives

Section 4.5

1. I can sketch a graph of a sine or cosine function that has been stretched horizontally/vertically, translated horizontally/vertically, and/or reflected. I can also state the domain and range of the function.

Please remember, Sine & Cosine graphs should have 5 key points labeled on a period
Remember to write in factored form if needed!

MODELS: ______________________________

Steps:

RECALL:

a. Sketch $y = -3\sin(2x - 2\pi)$

b. Sketch $y = 2\sin\left(\frac{1}{2}x + \pi\right) - 2$

3. I can write the equation of the trig graph based on its graph, given a max and min, or given a set of data. I can express the equation as a sine function and a cosine function.

a. Find an equation of a sine wave with a peak of 12 and a minimum of 6, starts its cycle at 3π and completes one full cycle every 4π units.

4. I can use sine and cosine functions to model real life data. I can use models to make predictions.

a. The water level in a city water storage tank oscillates in a simple harmonic motion. The water level varies depending on the time of day and the corresponding demand of the people. The low point of the water in the tank, 22 feet, occurs at 8am and 8pm when demand is highest. The high points occur at 2am and 2pm with a water level of 58 feet. Create a sinusoidal function that models the data and use it to predict the water height at 4pm.

5. I can state the domain and range

Section 4.7

8. I can evaluate inverse trig functions from memory or by using my calculator. I understand the restrictions on each trig function. . .

NOTES: SIN COS

a. $\arctan\left(\frac{\sqrt{3}}{3}\right)$

b. $\arcsin\left(-\frac{1}{2}\right)$

c. $\arcsin(2)$

d. $\arctan(-1)$

9. I can use properties of inverse trig functions to evaluate expressions.

a. sin(arcsin 1)

b. cos(arcos .3)

c. $\arctan(\tan \pi)$

10. I can find the exact value or an algebraic expression for a trig expression by using the "triangle technique."

a. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

b. $\sin\left(\arctan\left(\frac{5}{6}\right)\right)$

c. $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$

Practice Problems

1. Graph the following trigonometric functions. (no calc)

a. $y = 3\cos\left(2x - \frac{\pi}{4}\right) - 1$ Factored form: $y = 3\cos\left[2\left(x - \frac{\pi}{8}\right)\right] - 1$

incr = π/2

a = 3 b = 2

c = π/8 d = −1

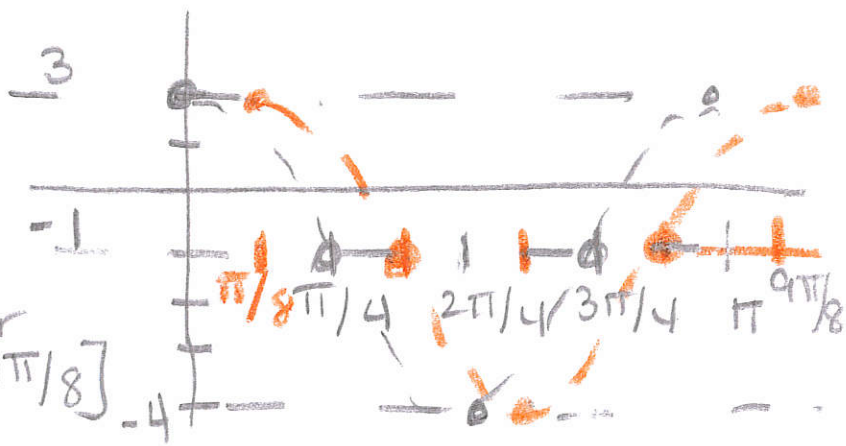
Amp. 3 Per 2π/b = π

P.S. π/8 R V.S. ↓ 1

DOMAIN: (−∞, ∞) or 1 per = [π/8, 9π/8]

RANGE:

[−4, 2]



b. $y = -3\mathrm{Sin}[\frac{\pi}{2}(x+2)] + 10$

a = −3 b = π/2

c = −2 d = 10

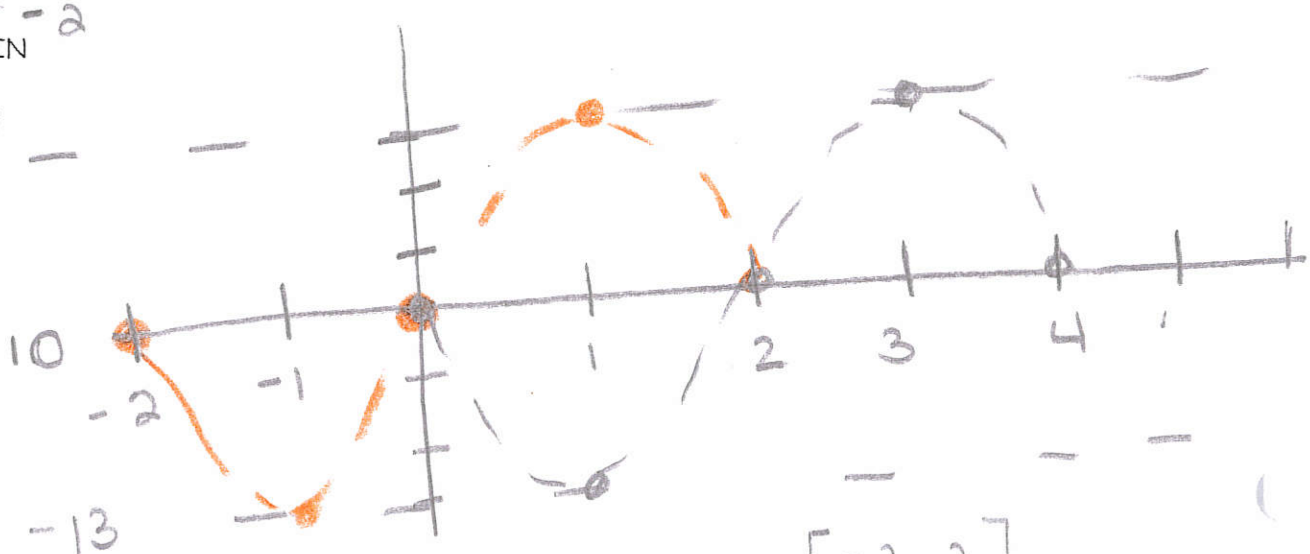
Amp. 3 Per 2π ÷ π/2 = 4 ∴ incr = 4 ÷ 4 = 1

P.S. 2 L or −2 V.S. ↑ 10

DOMAIN

RANGE

-7



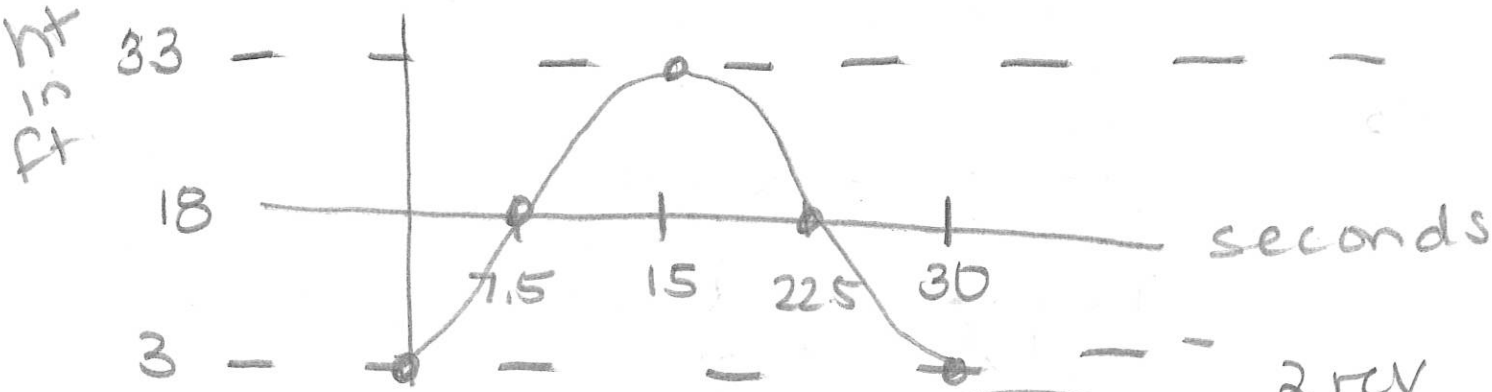
D (−∞, ∞) or 1 per [−2, 2]

R := [−13, 7]

3. A ferris wheel has a diameter of 30 meters. The center of the wheel is 18 feet off the ground. It makes two revolutions every MINUTE (60 seconds).

$d = 30$

A. Sketch the trig graph of one complete cycle, assuming the rider gets on at the lowest point.



$d = 18$
$a = \frac{d}{2} = 15$

$\frac{2 \text{ rev}}{1 \text{ min}}$
$= 1 \text{ rev}/30 \text{ sec}$

B. Find the cosine equation of the graph.

$30 \text{ sec} = \frac{2\pi}{b}$
$30b = 2\pi$
$b = \pi/15$

$$y = -3\cos\left[\frac{\pi}{15}t\right] + 15$$

C. What is the height of the rider 52 seconds after he gets on the ride?

$$y = -3\cos\left[\frac{\pi}{15} \cdot 52\right] + 15$$
$$= -3\cos\left[52\pi/15\right] + 15 = \underline{19.57}$$

***D. At what times is the rider 20 meters above the graund?

$$y_1 = -3\cos\left(\frac{\pi}{15}t\right) + 15$$

$$y_2 = 20$$

Calc intersect

8.13 sec
21.86 sec
38.13 sec
51.86 sec

4. One of the largest ferris wheels ever built is in the British Airways London Eye which was completed in 2000. The diameter is 135 m and passengers get on at the bottom 4 m above the ground. The wheel rotates once every three minutes.

a) Draw a graph which represents the height of a passenger in metres as a function of time in minutes.

$r = 67.5$ $\quad \therefore |a| = 67.5$ $\quad$ min $= 4$

mid $= 4 + 67.5 = 71.5 = d$

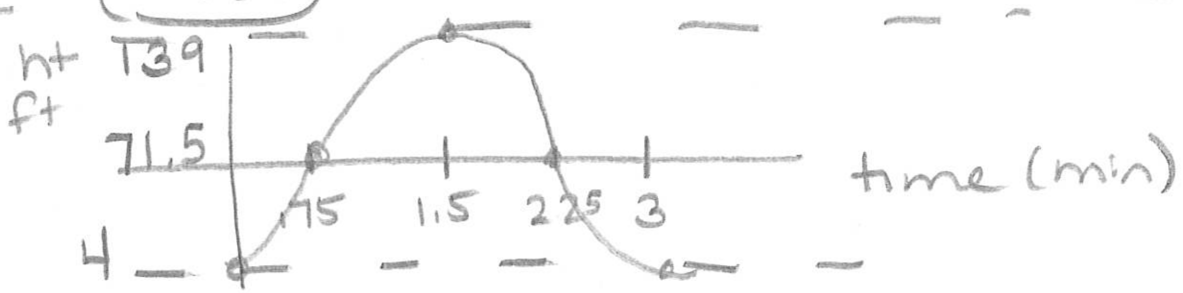
1 rev = 3 min $\quad$ $-\cos$

SO

$\frac{2\pi}{b} = 3$

$2\pi = 3b$

$\frac{2\pi}{3} = b$



b) Determine the equation that expresses your height h as a function of elapsed time t

$y =$

$h(t) = -67.5\cos\left[\frac{2\pi}{3}x\right] + 71.5$

c) How high is a passenger 5 minutes after the wheel starts rotating?

$h(5) = -67.5\cos\left[\frac{2\pi}{3} \cdot 5\right] + 71.5$

4m

*d) How many seconds after the wheel starts rotating is a passenger 85 m above the ground for the first time. Answer to the nearest tenth.

$y_1 =$

$y_2 = 85$

Calc intersect.