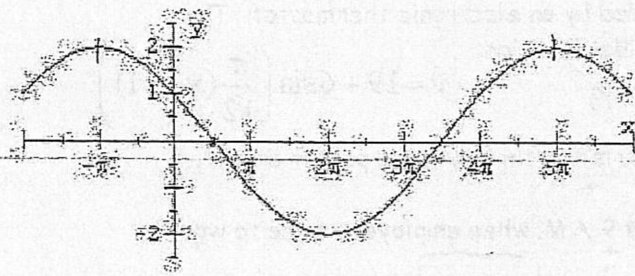
13

5. Write the equation of the graphs represented below:

a.



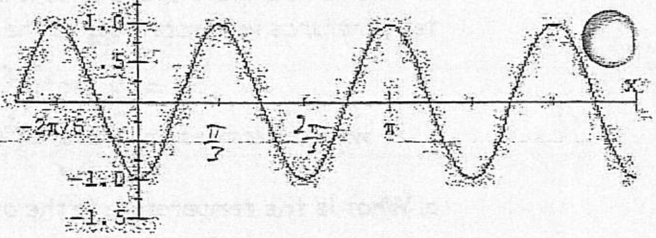
Period = 6π $\quad K = \frac{-2+2}{2} = 0$

$\frac{2\pi}{b} = 6\pi$ $\quad a = 2$

$b = \frac{1}{3}$

$$\boxed{\begin{array}{l} y = 2\cos\frac{1}{3}(x+\pi) \\ y = -2\sin\frac{1}{3}\left(x - \frac{\pi}{2}\right) \end{array}}$$

b.



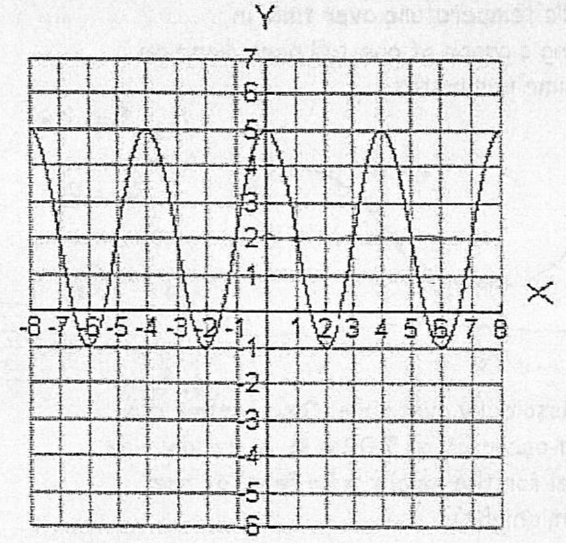
Period = $\frac{2\pi}{3}$ $\quad K = \frac{1+(-1)}{2} = 0$

$\frac{2\pi}{b} = \frac{2\pi}{3}$ $\quad a = 1$

$b = 3$

$$\boxed{\begin{array}{l} y = -\cos 3x \\ y = \sin 3\left(x - \frac{\pi}{6}\right) \end{array}}$$

c.



Period = 4 $\quad K = \frac{-1+5}{2} = 2$

$a = 3$

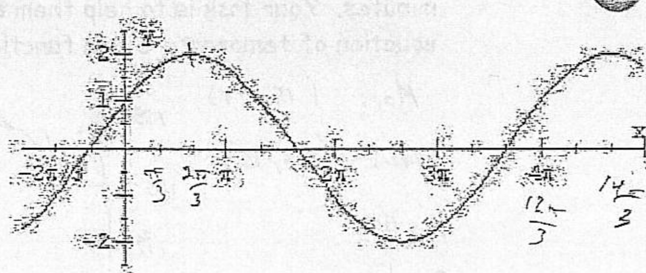
$\frac{2\pi}{b} = 4$

$b = \frac{\pi}{2}$

$$\boxed{\begin{array}{l} y = 3\cos\frac{\pi}{2}x + 2 \\ y = -3\sin\frac{\pi}{2}(x-1) + 2 \end{array}}$$

8

d.



Period = 4π $\quad K = \frac{2+(-2)}{2} = 0$

$\frac{2\pi}{b} = 4\pi$ $\quad a = 2$

$b = \frac{1}{2}$ $\quad (0, \pi)$

$$\boxed{\begin{array}{l} y = 2\cos\frac{1}{2}\left(x - \frac{2\pi}{3}\right) \\ y = 2\sin\frac{1}{2}\left(x + \frac{\pi}{3}\right) \end{array}}$$

6. The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function:

$$y = 6 \sin\left[\frac{\pi}{12}(x-11)\right] + 19 \qquad y = 19 + 6\sin\left(\frac{\pi}{12}(x-11)\right)$$

where y is the temperature (°C) and x is the time in hours past midnight.

a. What is the temperature in the office at 9 A.M. when employees come to work?

$\boxed{16\,°C}$

b. What are the maximum and minimum temperatures in the office?

Max: 25° C

Min: 13° C

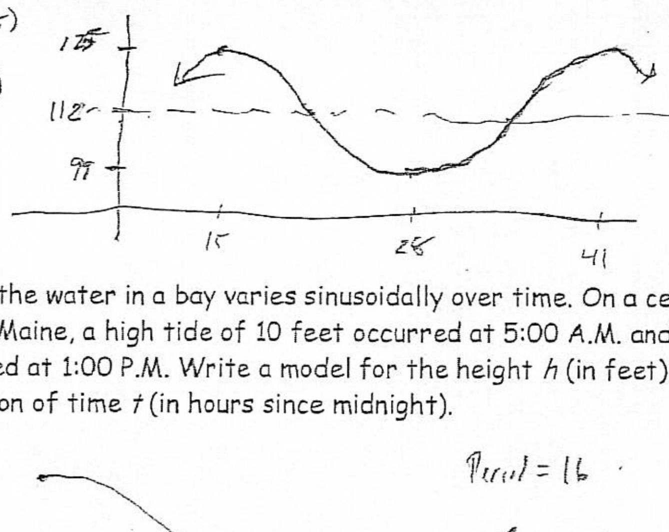
7. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125 ° occurs 15 minutes after they start their examination. A minimum temperature of 99 ° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period and an equation of temperature as a function over time in minutes.

Max: (15, 125)

Min: (28, 99)

K = 112

a = 13



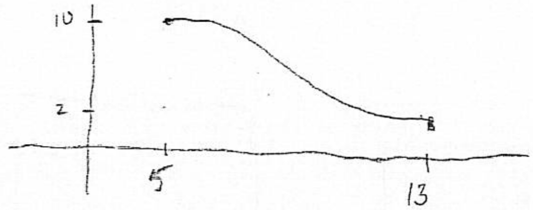
Period = 26

$\frac{2\pi}{b} = 26$

$b = \pi/13$

$$\boxed{y = 13\cos\frac{\pi}{13}(x-15) + 112}$$

8. The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).



Period = 16

$\frac{2\pi}{b} = 16$

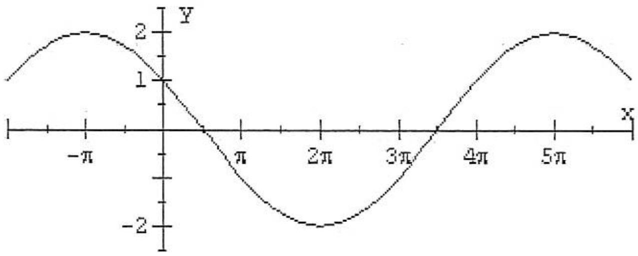
$b = \pi/8$

$K = \frac{2+10}{2} = 6$

$a = 4$

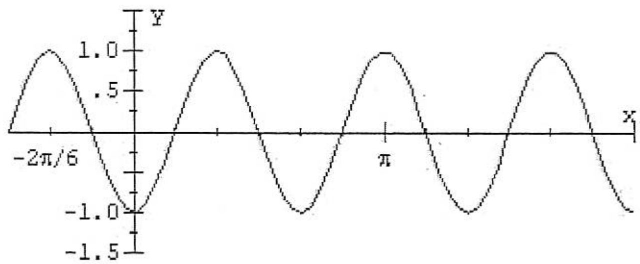
$$\boxed{y = 4\cos \pi/8\,(x-5) + 6}$$

5. Write the equation of the graphs represented below. Where possible, write a sine and cosine function.



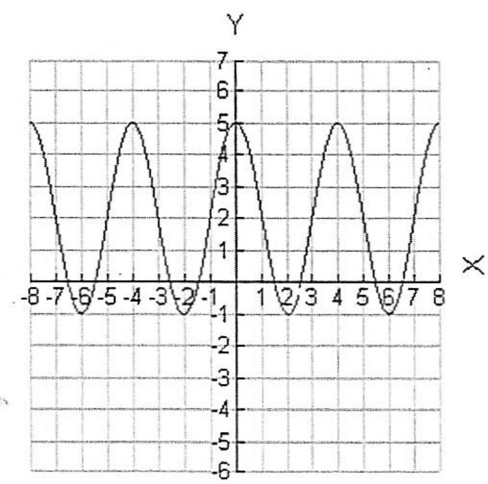
$y = 2\cos\left[\frac{1}{3}(x+\pi)\right]$

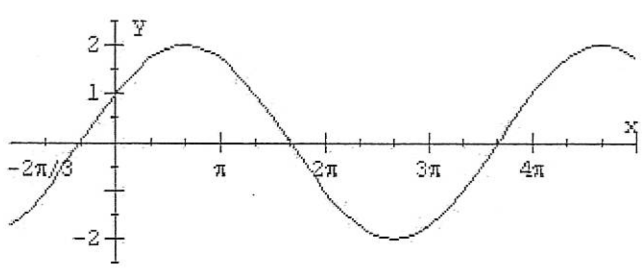
$y = -2\sin\left[\frac{1}{3}\left(x-\frac{\pi}{2}\right)\right]$

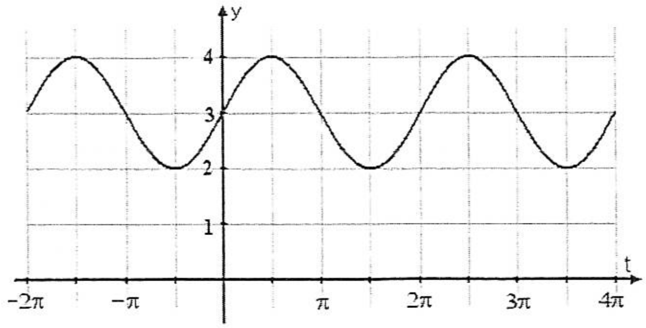


$y = -\cos(3x)$

$y = \sin\left[3\left(x-\frac{\pi}{6}\right)\right]$







6. The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function:

$$y = 19 + 6\sin\left(\frac{\pi}{12}(x-11)\right)$$

where y is the temperature (ºC) and x is the time in hours past midnight.

a.) What is the temperature in the office at 9 A.M. when employees come to work?

b.) What are the maximum and minimum temperatures in the office?

c). By how much do the max and min temperatures vary?

7. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125 ° occurs 15 minutes after they start their examination. A minimum temperature of 99 ° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period and an equation of temperature as a function over time in minutes.

8. The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).

Practice Problems

Objectives: Evaluating trig function inverses, simplifying trig expressions, verifying, solving trig equations.

1. Find the exact value of each expression. Do not use a calculator. Remember the quadrant rules for inverses – COS Q1 and Q2, SIN Q1 and Q4.

a) $\text{Tan}^{-1}1$ $\pi/4$ b) $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $5\pi/6$ c) $\text{Sec}^{-1}\sqrt{2}$ $\pi/4$

$\sin^{-1}, \tan^{-1}$ $\frac{-\pi}{2} < x < \frac{\pi}{2}$ $\cos^{-1}$ $0 < x < \pi$

2. Find the exact value, if any, of each composite function.

a) $\text{Sin}^{-1}\left(\sin\frac{3\pi}{8}\right)$ $3\pi/8$ b) $\tan\left(\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $-\sqrt{3}$ c) $\sin\left(\text{Cot}^{-1}\frac{3}{4}\right)$ $\frac{4}{5}$

3. **Simplify** each expression.

a. $\dfrac{\sin x\cos x}{1-\cos^2 x}$

$\dfrac{\sin x \cos x}{\sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

3 a.) $\cot x$

b. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

$\sin^2 + 2\sin x\cos x + \cos^2 x$
$+ \sin^2 x - 2\sin x \cos x + \cos^2 x$
$= 2\sin^2 x + 2\cos^2 x$
$= 2(\sin^2 x + \cos^2 x)$

b.) 2

c. $\dfrac{\tan^2 x}{\sec x+1}+1$

$\dfrac{\sec^2 x - 1}{\sec x + 1} + 1$

$= \sec x - 1 + 1$

3c.) $\sec x$

d. $\sec x-\sin x\tan x$

$\dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{\cos^2}{\cos x}$

3d.) $\cos x$

4. **Solve.** Give your answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$ without using a calculator.

a. $\cot\theta = -\frac{\sqrt{3}}{3}$

$\therefore \frac{\pi}{3}$ family

$\frac{2\pi}{3}$ $\frac{5\pi}{3}$

4a.)____________________

b. $\sec\theta = \sqrt{2}$

$\frac{1}{\cos\theta} = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\pi/4$ fam

$\pi/4$ $7\pi/4$

4b.)____________________

5. Verify each identity.

a) $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

$\tan\theta \cdot \frac{1}{\tan\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$ ✓

b) $\frac{(1-\cos\theta)}{(1-\cos\theta)}\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta}\frac{(\sin\theta)}{\sin\theta} = 2\csc\theta$ LCD

$\frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)}$

$= \frac{2 - 2\cos\theta}{\sin\theta(1-\cos\theta)} = \frac{2(1-\cos\theta)}{\sin(1-\cos\theta)}$ ✓

$= \frac{2}{\sin\theta} = 2\csc\theta$

c) $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}\frac{1-\sin\theta}{1-\sin\theta}$ Conj

$= \frac{\cos^3\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos^3\theta(1-\sin\theta)}{\cos^2\theta}$

$= \cos\theta(1-\sin\theta)$

$\frac{1-\sin\theta}{\sec\theta} = \frac{1}{\sec\theta} \cdot (1-\sin\theta)$

d) $\frac{\tan x - \sin x\cos x}{\sin^2 x} = \tan x$ Break

$\frac{\tan x}{\sin^2 x} - \frac{\sin x\cos x}{\sin^2 x}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x}$

$\frac{1}{\sin x\cos x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} = \frac{\sin^2 x}{\sin x\cos x}$ $\tan x$

e) $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$ LCD

f) $\dfrac{1+\sin x}{\cos x}+\dfrac{\cos x}{1+\sin x}=2\sec x$ LCD

g) $\dfrac{1+\sin x}{\cos x}=\sec x+\tan x$ CONJ

h). $\dfrac{\sin x}{1+\cos x}=\dfrac{1-\cos x}{\sin x}$ CONJ

DO NOT CROSS MULTIPLY

6. Solve each equation on the interval $0 \le \theta < 2\pi$.

Strategies: linear equation, plus or minus square root, "u", GCF, factoring, replacement, square both sides

a) $2\sin^2\theta-3\sin\theta+1=0$ ____________________

$(2\sin\theta-1)(\sin\theta-1)=0$

$\sin\theta=\frac{1}{2}$ $\quad \sin\theta=1$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

$\pi/6$ fam

b.) $3\sin x-2=5\sin x-1$ ____________________

$-2\sin x=1$

$\sin x=-\frac{1}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

c.) $5\sin x = 3\sin x + \sqrt{3}$ $\frac{\pi}{3}, \frac{2\pi}{3}$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\pi/3$ fam

d). tan3x = 1 $\frac{\pi}{12}, \frac{5\pi}{12}$

let $u = 3x$

$\tan u = 1$

$u = \frac{\pi}{4}$ $u = \frac{5\pi}{4}$

$3x = \pi/4$ $3x = 5\pi/4$

e.) $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$ ________________

let $u = \frac{1}{2}x$

$\sin u = \frac{\sqrt{3}}{2}$ $\pi/3, \frac{2\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3}$ $\frac{x}{2} = \frac{2\pi}{3}$

$x = 2\pi/3$ $x = 4\pi/3$

f) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\sin x = 1$

$\pi/6, 5\pi/6$ $\pi/2$

g.) $\tan x \sin^2 x = 3\tan x$

GCF

$\tan x \sin^2 x - 3\tan x = 0$

$\tan x(\sin^2 x - 3) = 0$

$\tan x = 0$ $\sin^2 x = 3$

$\frac{y}{x} = 0$ $0, \pi$ $\sin x = \pm\sqrt{3}$

outside domain

h.) $2\cos^2 x + 3\sin x = 0$

Replace

$2(1-\sin^2 x) + 3\sin x = 0$

$2 - 2\sin^2 x + 3\sin x = 0$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{2}$ $\sin x = 2$ $\emptyset$

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

i). sinx - cosx = 1

SQUARE BOTH

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$-2\sin x\cos x = 0$

$\sin x = 0$ $\cos x = 0$

$\cancel{0}, \pi/2, \pi, \cancel{3\pi/2}$

j.) $2\sec^2 x = 4$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$\pi/4$ fam