# Practice Problems

## Objectives: Evaluating trig function inverses, simplifying trig expressions, verifying, solving trig equations.

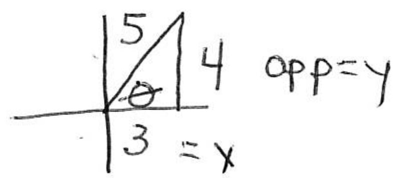
1. Find the exact value of each expression. Do not use a calculator. Remember the quadrant rules for inverses - COS Q1 and Q2, SIN Q1 and Q4.

a) $Tan^{-1}1$ $\pi/4$ b) $Cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $5\pi/6$ c) $Sec^{-1}\sqrt{2}$ $\pi/4$

$\sin^{-1}, \tan^{-1}$ $\frac{-\pi}{2} < x < \frac{\pi}{2}$ $\cos^{-1}$ $0 < x < \pi$

2. Find the exact value, if any, of each composite function.

a) $Sin^{-1}\left(\sin\frac{3\pi}{8}\right)$ $3\pi/8$ b) $\tan\left(Sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $-\sqrt{3}$ c) $\sin\left(Cot^{-1}\frac{3}{4}\right)$ $\frac{4}{5}$




3. **Simplify** each expression.

a. $\frac{\sin x \cos x}{1-\cos^2 x}$

$\frac{\sin x \cos x}{\sin^2 x}$

$= \frac{\cos x}{\sin x}$

3 a.) $\cot x$

b. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

$\sin^2 + 2\sin x\cos x + \cos^2 x$
$+ \sin^2 x - 2\sin x \cos x + \cos^2 x$
$= 2\sin^2 x + 2\cos^2 x$
$= 2(\sin^2 x + \cos^2 x)$

b.) $2$

c. $\frac{\tan^2 x}{\sec x+1}+1$

$\frac{\sec^2 x - 1}{\sec x + 1} + 1$

$= \sec x - 1 + 1$

3c.) $\sec x$

d. $\sec x - \sin x \tan x$

$\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$

$= \frac{\cos^2}{\cos x}$

3d.) $\cos x$

4. **Solve.** Give your answers in degrees $\left(0° \le \theta < 360°\right)$ and radians $\left(0 \le \theta < 2\pi\right)$ without using a calculator.

a. $\cot\theta = -\frac{\sqrt{3}}{3}$

$\therefore \frac{\pi}{3}$ family

$\frac{2\pi}{3}$ $\frac{5\pi}{3}$

4a.)____________________

b. $\sec\theta = \sqrt{2}$

$\frac{1}{\cos\theta} = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\pi/4$ fam

$\pi/4$ $7\pi/4$

4b.)____________________

5. Verify each identity.

a) $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

$\tan\theta \cdot \frac{1}{\tan\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$ ✓

b) $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\csc\theta$

$\frac{(1-\cos\theta)}{(1-\cos\theta)}$ $\frac{(\sin\theta)}{\sin\theta}$

LCD

$\frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)}$

$= \frac{2 - 2\cos\theta}{\sin\theta(1-\cos\theta)} = \frac{2(1-\cos\theta)}{\sin(1-\cos\theta)}$ ✓

$= \frac{2}{\sin\theta} = 2\csc\theta$

c) $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}$

$\frac{1-\sin\theta}{1-\sin\theta}$ Conj

$= \frac{\cos^3\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos^3\theta(1-\sin\theta)}{\cos^2\theta}$

$= \cos\theta(1-\sin\theta)$

$\frac{1-\sin\theta}{\sec\theta} = \frac{1}{\sec\theta} \cdot (1-\sin\theta)$

d) $\frac{\tan x - \sin x\cos x}{\sin^2 x} = \tan x$

Break

$\frac{\tan x}{\sin^2 x} - \frac{\sin x\cos x}{\sin^2 x}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x}$

$\frac{1}{\sin x\cos x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} = \frac{\sin^2 x}{\sin x\cos x}$

$\tan x$

e) $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x$

$\dfrac{(1+\sin)}{(1+\sin)}$ $\dfrac{(1-\sin)}{(1-\sin)}$ LCD

$\dfrac{1+\sin x + 1-\sin x}{1-\sin^2 x}$

$= \dfrac{2}{\cos^2 x}$

$= 2 \cdot \dfrac{1}{\cos^2 x} = 2\sec^2 x$

f) $\dfrac{1+\sin x}{\cos x}+\dfrac{\cos x}{1+\sin x}=2\sec x$

$\dfrac{(1+\sin)}{(1+\sin)}$ $\dfrac{(\cos x)}{(\cos x)}$ LCD

$\dfrac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$

$\dfrac{1+2\sin x+1}{\cos x(1+\sin x)} = \dfrac{2+2\sin x}{\cos(1+\sin x)}$

$= \dfrac{2(1+\sin x)}{\cos(1+\sin x)}$

$= 2\cdot\dfrac{1}{\cos x}$

$= 2\sec x$ ✓

g) $\dfrac{1+\sin x}{\cos x}=\sec x+\tan x$

BREAK INTO 2

$\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$\sec x + \tan x =$ ✓

h). $\dfrac{\sin x}{1+\cos x}=\dfrac{1-\cos x}{\sin x}$ CONJ

$\dfrac{1-\cos x}{1-\cos x}$

DO NOT CROSS MULTIPLY

$\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$

$\dfrac{\sin x-\sin x\cos x}{\sin^2 x}$

$\dfrac{\sin x(1-\cos x)}{\sin^2 x}$

$= \dfrac{1-\cos x}{\sin x}$ ✓

6. Solve each equation on the interval $0 \le \theta < 2\pi$ .

Strategies: linear equation, plus or minus square root, "u", GCF, factoring, replacement, square both sides

a) $2\sin^2\theta-3\sin\theta+1=0$ ____________________

$(2\sin\theta-1)(\sin\theta-1)=0$

$\sin\theta=\dfrac{1}{2}$ $\sin\theta=1$

$\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{2}$

$\pi/6$ fam

b.) $3\sin x-2=5\sin x-1$ ____________________

$-2\sin x = 1$

$\sin x = -\dfrac{1}{2}$

$\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

c.) $5\sin x = 3\sin x + \sqrt{3}$ $\frac{\pi}{3}, \frac{2\pi}{3}$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\pi/3$ fam

d). tan3x = 1 $\frac{\pi}{12}, \frac{5\pi}{12}$

let $u = 3x$

$\tan u = 1$

$u = \frac{\pi}{4}$ $u = \frac{5\pi}{4}$

$3x = \pi/4$ $3x = 5\pi/4$

e.) $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$ ________________

let $u = \frac{1}{2}x$

$\sin u = \frac{\sqrt{3}}{2}$ $\pi/3, \frac{2\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3}$ $\frac{x}{2} = \frac{2\pi}{3}$

$x = 2\pi/3$ $x = 4\pi/3$

f) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\sin x = 1$

$\pi/6, 5\pi/6$ $\pi/2$

g.) $\tan x \sin^2 x = 3\tan x$

GCF

$\tan x \sin^2 x - 3\tan x = 0$

$\tan x(\sin^2 x - 3) = 0$

$\tan x = 0$ $\sin^2 x = 3$

$\frac{y}{x} = 0$ $0, \pi$ $\sin x = \pm\sqrt{3}$

outside domain

h.) $2\cos^2 x + 3\sin x = 0$

Replace

$2(1 - \sin^2 x) + 3\sin x = 0$

$2 - 2\sin^2 x + 3\sin x = 0$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = 2$

$\varnothing$

$\sin x = -\frac{1}{2}$

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

i). sinx - cosx = 1

SQUARE BOTH CHECK SOL.

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$-2\sin x\cos x = 0$

$\sin x = 0$ $\cos x = 0$

$\cancel{0}, \pi/2, \pi, \cancel{3\pi/2}$

check for extraneous!

j.) $2\sec^2 x = 4$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\sqrt{2}/2$

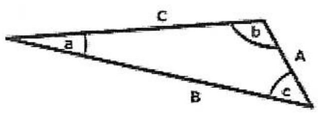
$\pi/4$ fam

# UNIT 8

TOPIC 1: Law of Sines, Law of Cosines, Heron's Formula, Area formula, ambiguous case, angles of elevation and depression, applications of trig.

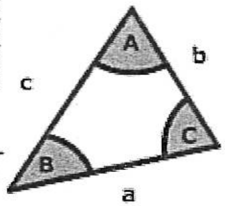
**ASA SAA SSA – check ambiguous case!** **SAS or SSS**

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$




$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$$

Law of Cosines




$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$

$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$

$c^2 = a^2 + b^2 - 2ab \cdot \cos(c)$



## Heron's Formula for area of a triangle given SSS

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \dfrac{a+b+c}{2}$

## Area of a triangle given SAS

$$A_{\triangle} = \frac{1}{2} ab \sin C$$

Check for ambiguous case when you have SSA!
SSS – find angles in order, largest to smallest or smallest to largest.

1. Solve each triangle using the Law of Sines or the Law of Cosines. It may help to draw a picture. (Hint: Remember the ambiguous case!).

a. $B = 10°$, $C = 20°$, $c = 33$ ASA

A = 150°

b = 16.75

a = 48.24

$\frac{\sin 20}{33} = \frac{\sin 10}{b}$

$\frac{33 \sin 10}{\sin 20} = b$

$\frac{\sin 20}{33} = \frac{\sin 150}{a}$ $a = \frac{33 \sin 150}{\sin 20}$

b. $B = 150°$, $a = 10$, $b = 3$

A = ____________

C = ____________

c = ____________

$\frac{\sin 150}{3} = \frac{\sin A}{10}$

$\sin A = \frac{10 \sin 150}{3}$

$\sin A = 1.67$

$\boxed{\text{no } \triangle}$

SSS - lgst 1st

SAS

c. a = 2.5, b = 5.0, c = 4.5

A = 29.9

B = $86.2^\circ$

C = 63.9

$A = 180 - (86.2 + 63.9)$ ✓

$5^2 = 2.5^2 + 4.5^2 - 2(2.5)(4.5)\cos B$

$25 = 6.25 + 20.25 - 22.5\cos B$

$.0667 = \cos B$

$\boxed{B = 86.2^\circ}$

$\frac{\sin 86.2}{5} = \frac{\sin C}{4.5} \Rightarrow \sin C = .8980$

$C = 63.9^\circ$

d. B = 110°, a = 4, c = 4

A = ______

C = ______

b = 6.1

***

$b^2 = 4^2 + 4^2 - 2(4)(4)\cos 110$

$b^2 = 32 - 16\cos 110$

$b^2 = 37.47$

$b = 6.1$

Hmm...

$\frac{\sin 110}{6.1} = \frac{\sin A}{4}$

$\sin A = .6162$

$A = 38$

OR $180 - 110 = 70$

(Since isosceles) $\div 2$

35

e. B = 25°, a = 6.2, b = 4

A = 40.9 / 139.1

C = 114.1 / 15.9

c = 8.64 / 2.6

$\frac{\sin 25}{4} = \frac{\sin A}{6.2}$

$\frac{6.2\sin 25}{4} = \sin A$

$= A$

2. Find the area of each given triangle.

Use formulas

a. B = 60°, a = 4, c = 8 ______ b. a = 15, b = 8, c = 10 ______

$\frac{\sin 25}{4} = \frac{\sin A}{6.2}$

$\frac{6.2\sin 25}{4} = \sin A$

$\boxed{A_1 = 40.9}$ $\boxed{C_1 = 114.10}$

$\frac{\sin 25}{4} = \frac{\sin 114.1}{c_1}$

$c_1 = 8.64$

Is there a Δ 2?

$180 - 40.9 = 139.1 = A_2$

$+ 25 = B$

$164.1 < 180$

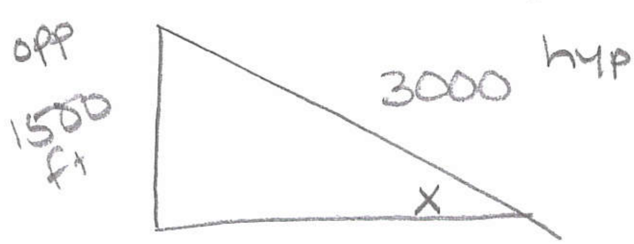
2Δs

$\frac{\sin 25}{4} = \frac{\sin 15.9}{c_2}$

$c_2 = 2.6$

$m\angle C_2 = 15.9^\circ$

3. You are skiing down a mountain with a vertical height of 1500 feet. The distance from the top of the mountain to the base is 3000 feet. What is the angle of elevation from the base to the top of the mountain?

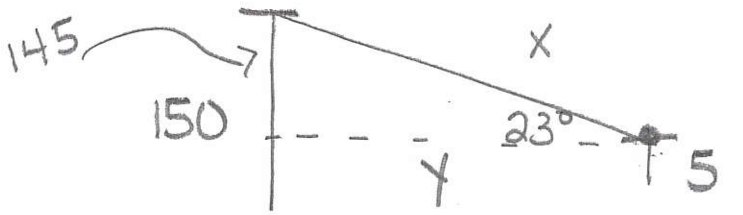



$\sin x = \frac{1500}{3000}$

$\sin x = \frac{1}{2}$

$x = 30^\circ$

4. A steel cable zip-line is being constructed for a competition on a reality television show. One end of the zip-line s attached to a platform on top of a 150-foot pole. The other end of the zip-line is attached to the top of a 5 foot stake. The angle of elevation of the platform is 23°.




$\sin 23° = \frac{150}{x}$

$x = \frac{150}{\sin 23°}$

$x = 383.9$ ft

a) How long is the zip-line?

383.9 ft

b) How far is the stake from the pole?

$\tan 23 = \frac{150}{y}$

$y = 150/\tan 23$

$\boxed{353.4 \text{ ft}}$

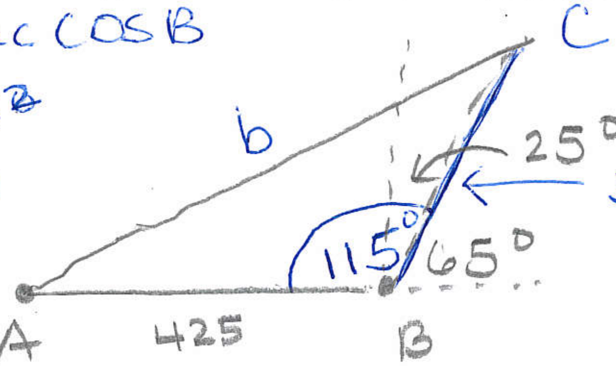
5. To approximate the length of a marsh, a surveyor walks 425 meters from point A to point B. Then the surveyor turns 65° and walks 300 meters to point C. Approximate the length of AC of the marsh.

SAS → LOC

$b^2 = a^2 + c^2 - 2ac\cos B$

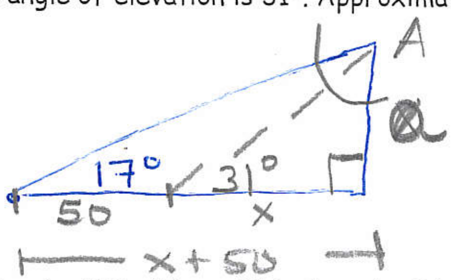
$b^2 = 300^2 + 425^2 - 2(300)(425)\cos 115°$

$b \approx 615$ m




6. From a certain distance, the angle of elevation to the top of a building is 17°. At a point 50 meters closer to the building, the angle of elevation is 31°. Approximate the height of the building.

31 meters




$\tan 17 = \frac{h}{x+50}$ $\tan 31 = \frac{h}{x}$

$x(\tan 31 - \tan 17) = 50\tan 17$

$x = 51.8$ $h = x\tan 31 \approx 31$ m.

7.] Given triangle ABC with $a$ = 10 inches, $b$ = 8 inches, and $c$ = 12 inches, find the measure of angle $B$.

SSS b is smallest

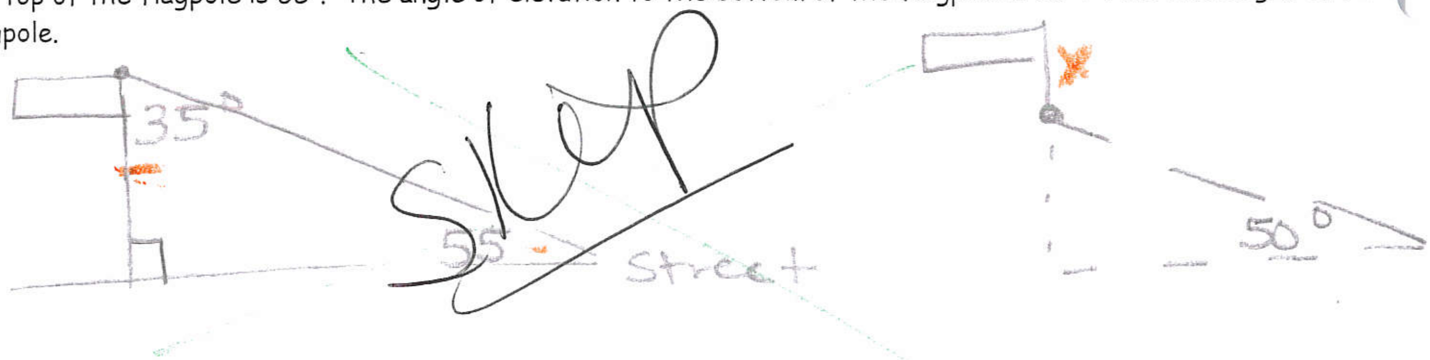
41.4°

$8^2 = 10^2 + 12^2 - 2(10)(12)\cos B$

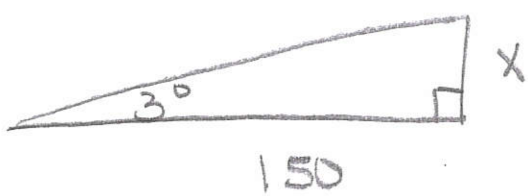
$64 = 244 - 240\cos B$

$\frac{-180}{240} = \cos B$ $\frac{-3}{4} = \cos B$ $B = \cos^{-1}(-.75)$

10.] A large flagpole stands at the top of the Smythe Office Building. From the street, the angle of elevation to the top of the flagpole is 55°. The angle of elevation to the bottom of the flagpole is 50°. Find the height of the flagpole.




11. John walks 150 feet away from a fence post on his farm. He measures the angle of elevation from the ground to the top of the post to be 3°. How tall is the fence post?



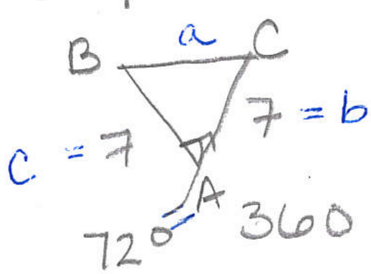

$\tan 3 = \frac{x}{150}$

$150 \tan 3 = x$

$\boxed{7.9 \text{ ft}}$

12. Find the area of a regular pentagon inscribed in a circle with diameter 14 feet.

$r = 7$




$360 \div 5 = 72$

$A = 5 \cdot \left[\frac{1}{2} \, 7 \cdot 7 \sin 72\right]$

$116.5 \text{ ft}^2$

13. Find the area of a triangular garden with sides 10, 14 and 16.

$s = \frac{1}{2}(10 + 14 + 16) = 20$

$K = \sqrt{20(20-10)(20-14)(20-16)}$

$= \sqrt{20(10)(6)(4)}$

$= \sqrt{4800} \approx 69.3$ sq. units