FINAL REVIEW - PRECALC - Name: Key

UNIT 5 - Chapter 4.1 - 4.4

Unit 5 Practice Problems

1. Convert the angle measure from degrees to radians or from radians to degrees. (calc) π rad $= 180°$

a. $115° \cdot \frac{\pi}{180} = \frac{23\pi}{6}$

b. $\frac{13\pi}{2} \cdot \frac{180}{\pi} = 1170°$

3. Determine two coterminal angles (one positive and one negative) for each angle. Give your answer in radians. (calc) $\pm 360K$ $\pm 2\pi K$

a. $\theta = \frac{\pi}{12}$ $\frac{25\pi}{12}$ & $\frac{-23\pi}{12}$

b. $\theta = -435°$ $-75°$ & $285°$

+360 +360

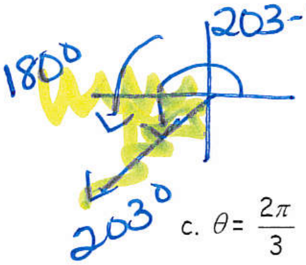
(plus again to get pos. ∡).

positive acute ∡ formed by terminal side of ∡ and closest x axis.

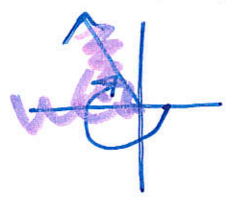
4. Find the reference angle and determine which quadrant θ lies. (calc)

a. $\theta = 203°$ $23°$

$203 - 180 = 23$



Quadrant: 3



b. $\theta = -245°$ $65°$

Quadrant: Q 2

c. $\theta = \frac{2\pi}{3}$ $\pi/3$

Quadrant: Q 2

d. $\theta = -\frac{13\pi}{3}$ $\pi/3$

Quadrant: Q4

5. Find the radian measure of the central angle of a circle if the radius = 14.5 centimeters and the arc length = 25 centimeters. (calc)

arc length = radius • radians

$25 = 14.5\,\theta$

$\frac{25}{14.5} = \theta$

$\theta = 1.724$ (rads)

1.724 (radians)

6. Find the length of the arc on a circle of radius r intercepted by a central θ.(calc)

a. radius = 15 inches, central angle θ = 180° ≈ 47 inches

θ must be in rads.

$\theta = \pi$

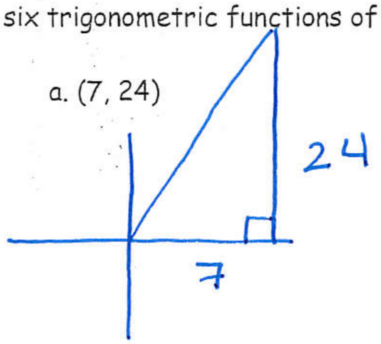
$s = 15(\pi) = 15\pi$ inches ≈ 47 inches

b. radius = 20 centimeters, central angle $\theta = \frac{\pi}{4}$ radians 15.7 cm

$s = 20 \cdot \frac{\pi}{4} = 5\pi \approx 15.7$ cm

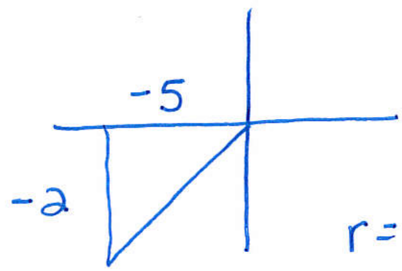
7. The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle. (calc). (MAKE A TRIANGLE!)

a. (7, 24)



$x^2 + y^2 = r^2$ $r = 25$

b. (-5,-2)



radius always positive!

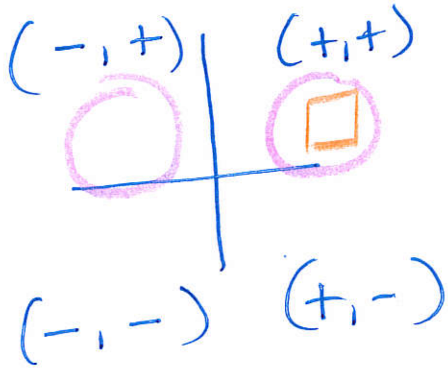
$r = \sqrt{29}$

sin α $24/25$	csc α $25/24$	sin α $-2\sqrt{29}/29$	csc α $-\sqrt{29}/2$
cos α $7/25$	sec α $25/7$	cos α $-5\sqrt{29}/29$	sec α $-\sqrt{29}/5$
tan α $24/7$	cot α $7/24$	tan α $2/5$	cot α $5/2$

Know your Trig Fns & Identities.

8. State the quadrant in which θ lies. (no calc)

a. sin θ > 0 and cos θ > 0 Q1



b. sec θ > 0 and cot θ < 0 Q2

∴ cos θ > 0 ∴ tan < 0

radius = 1 radian

9. A carousel with a 50-foot diameter makes 4 revolutions per minute. What is the angular velocity in radians per hour? What is the linear velocity in inches per hour? (calc)

Have $\rightarrow$ $r = 25$ feet

Get to

$$\frac{4\text{rev}}{1\text{min}} \cdot \frac{60\text{min}}{\text{hr}} \cdot \frac{2\pi \text{ rads}}{1\text{ rev}} = \frac{480\pi \text{ rad}}{\text{hr}}$$

Angular Velocity: 1508 rad/hr

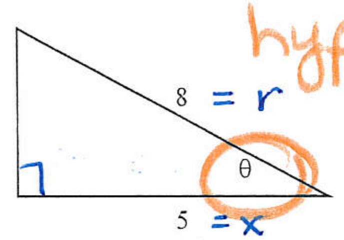
Linear Velocity: 452,398.34 in/hr

Have radius Get to

$$\frac{480\pi \text{ rads}}{\text{hr}} \cdot \frac{25\text{ ft}}{1\text{ rad}} = 1200\pi \cdot \frac{\text{inches}}{\text{hour}}$$

10. Find the 6 trig functions for θ in the triangle below. Assume the triangle is a right triangle.

opp $= y = \sqrt{39}$



$x^2 + y^2 = r^2$

$a^2 + b^2 = c^2$

$5^2 + b^2 = 8^2$

$25 + b^2 = 64$

$b^2 = 39$

$b = \sqrt{39}$

pos. only.

so

$\cos\theta = \frac{x}{r} = \boxed{\frac{\text{adj}}{\text{hyp}}}$

$\cos\theta = \frac{5}{8}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{39}}{8}$

$\tan\theta = \frac{\sqrt{39}}{5} \cdot \frac{\text{opp}}{\text{adj}}$

$\sec\theta = \frac{h}{a} = \frac{1}{\cos\theta} = \frac{8}{5}$

$\csc\theta = \frac{h}{o} = \frac{1}{\sin\theta} = \frac{8\sqrt{39}}{39}$

$\cot\theta = \frac{o}{a} = \frac{1}{\tan\theta} = \frac{5\sqrt{39}}{39}$

$y = a[\cos[b(x-c)]] + d$ (old p. 10)

$y = a[\sin[b(x-c)]] + d$

UNIT 6 Practice Problems – Trig Graphing and Modeling

1. Graph the following trigonometric functions. (no calc)

a. $y = 3\cos\left(2x - \frac{\pi}{4}\right) - 1$ Factored form: ______________

a = ________ b = ________

c = ________ d = ________

Amp.________ Per________

P.S.________ V.S.________

DOMAIN:

RANGE:

see old p. 10.

b. $y = -3\text{Sin}[\frac{\pi}{2}(x+2)] + 10$

a = ________ b = ________

c = ________ d = ________

Amp.________ Per________

P.S.________ V.S.________

DOMAIN

RANGE

"a"
- pos cos → starts at max
- neg cos → starts at min
- neg sin → starts at mid, goes ↓
- pos cos → starts at mid, goes ↑

amplitude = |a|
= distance from midline to max or min. (How big the wave is).

Period = $\frac{2\pi}{b}$

increment = $\frac{\text{period}}{4}$

d = vertical shift
= midline.

c = phase (horizontal shift)
x − 3 → 3 to Right.

Practice Problems

1. Graph the following trigonometric functions. (no calc)

a. $y=3\cos\left(2x-\frac{\pi}{4}\right)-1$ Factored form: $y = 3\cos\left[2\left(x-\frac{\pi}{8}\right)\right]-1$

incr $=\pi/2$

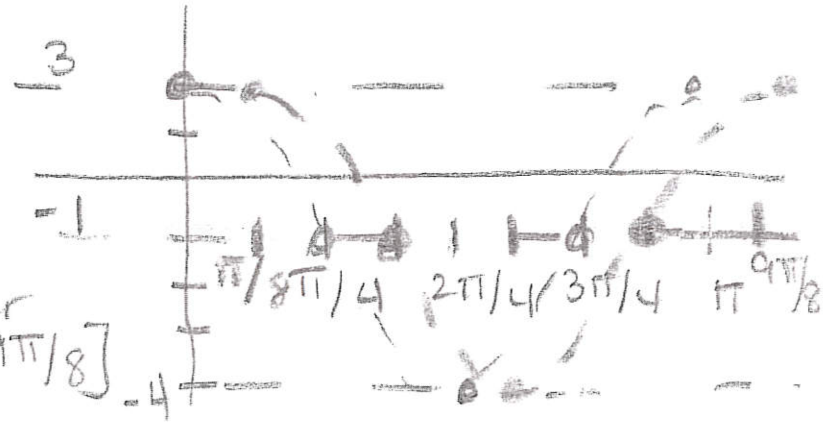
a = 3 b = 2

c = $\pi/8$ d = -1

Amp. 3 Per $2\pi/b = \pi$

P.S. $\pi/8$ R V.S. $\downarrow 1$

DOMAIN: $(-\infty, \infty)$ or 1 per $= [\pi/8, 9\pi/8]$



RANGE:

$[-4, 2]$

b. $y=-3\text{Sin}[\frac{\pi}{2}(x+2)]+10$

a = -3 b = $\pi/2$

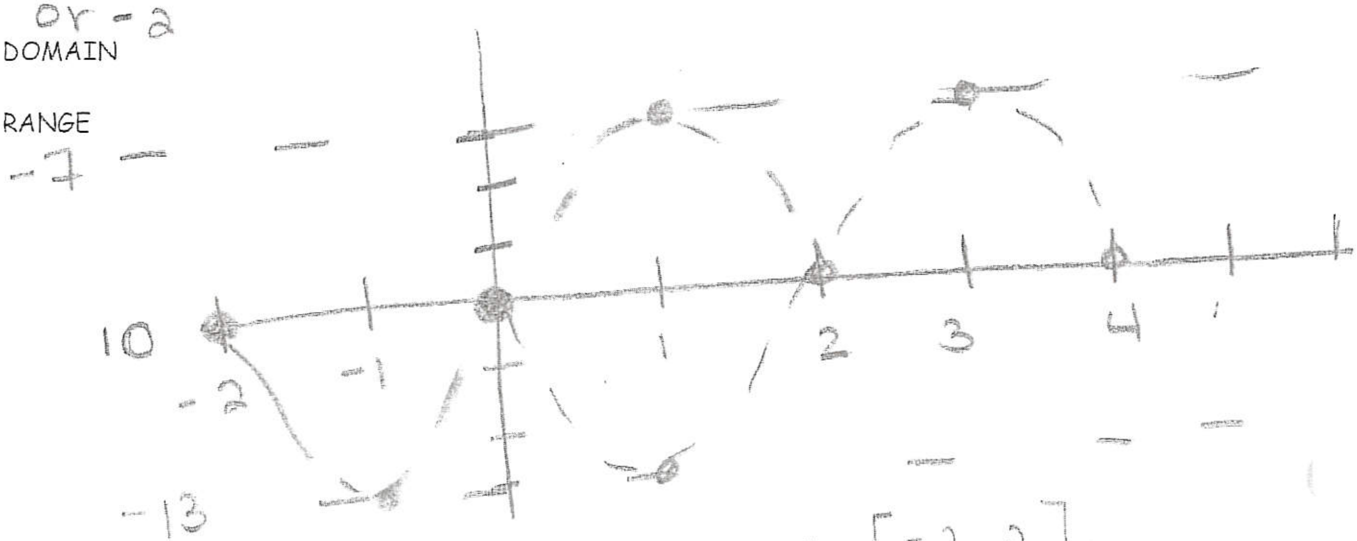
c = -2 d = 10

Amp. 3 Per $2\pi \div \pi/2 = 4$ ∴ incr $= 4 \div 4 = 1$

P.S. 2 L or -2 V.S. $\uparrow 10$

DOMAIN

RANGE



D $(-\infty, \infty)$ or 1 per $[-2, 2]$

R : $= [-13, 7]$

Radians

radius = amplitude !

so $|a| = 15$

3. A ferris wheel has a $\boxed{\text{diameter of 30 meters.}}$ The center of the wheel is 18 feet off the ground. It makes two revolutions every MINUTE (60 seconds).

∴ Vertical Shift is 18

see below

A. Sketch the trig graph of one complete cycle, assuming the rider gets on at the lowest point.

Use revolutions to get period. ∴ $d=18$

$$\frac{2 \text{ revolutions}}{60 \text{ seconds}} = \frac{1 \text{ rev}}{30 \text{ seconds}} = \text{period} = 30$$

$$\text{so } \frac{2\pi}{b} = 30$$

$$2\pi = 30b$$

$$\frac{\pi}{15} = b$$

Ferris Wheel-7

Get on at min ....

∴ neg cos, no p.s.

B. Find the cosine equation of the graph.

$\boxed{|a| = 15}$ $\boxed{b = \frac{\pi}{15}}$ $\boxed{d=18}$ $\boxed{c=0}$

$y =$

$$y = -15\cos\left(\frac{\pi}{15}x\right) + 18$$

C. What is the height of the rider 52 seconds after he gets on the ride?

Sub in 52. OR CALC VALUE $X=52$

$$y = -15\cos\left(\frac{\pi}{15}\cdot 52\right) + 18 = 19.57 \text{ feet}$$

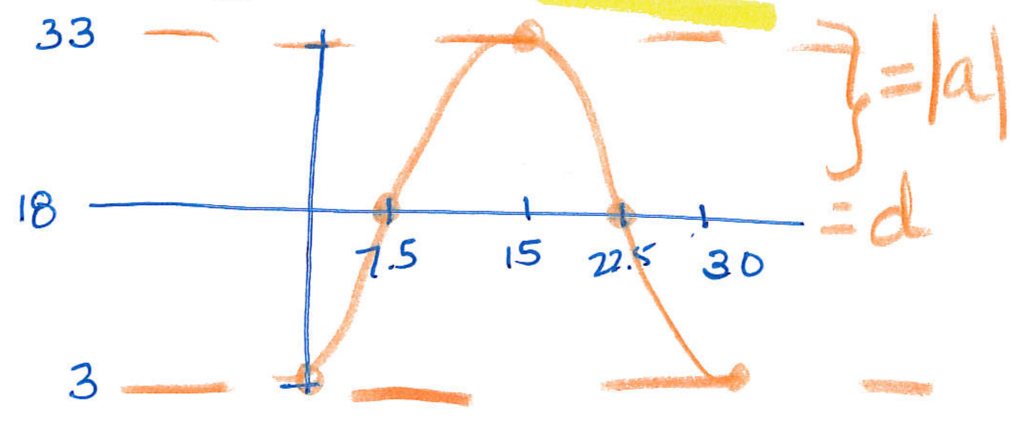
***D. At what times is the rider 20 meters above the ground? Calc Intersect

$y_1 = 20$

$y_2 = -15\cos\left[\frac{\pi}{15}(x)\right] + 18$

CALC INT

A. Sketch



$d = 135 \Rightarrow r = 67.5 \Rightarrow |a| = 67.5$

4. One of the largest Ferris wheels ever built is in the British Airways London Eye which was completed in 2000. The diameter is 135 m and passengers get on at the bottom 4 m above the ground. The wheel rotates once every three minutes.

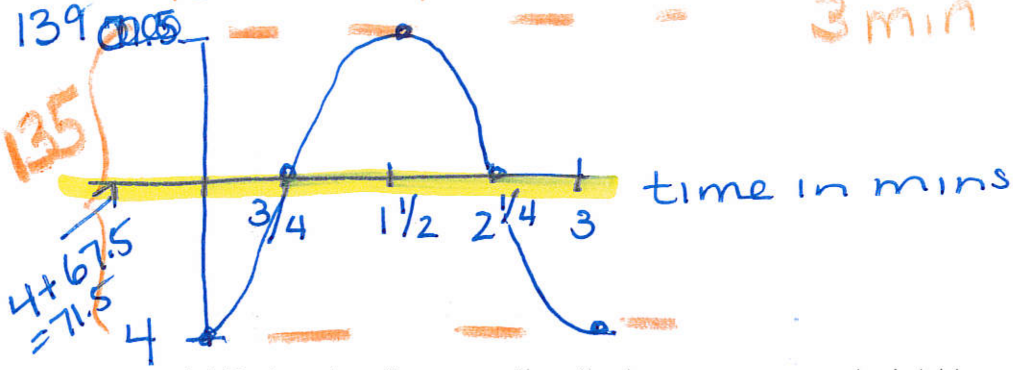
a) Draw a graph which represents the height of a passenger in meters as a function of time in minutes.

lowest pt = 4

$\frac{1 \text{ rev}}{3 \text{ min}}$ so $\frac{2\pi}{b} = 3$

$3b = 2\pi$

$\boxed{b = \frac{2\pi}{3}}$



b) Determine the equation that expresses your height h as a function of elapsed time t

$a = |67.5|$ $\quad b = \frac{2\pi}{3}$ $\quad c = 0$ $\quad d = 71.5$

$h(t) = -67.5\cos\left[\frac{2\pi}{3}(x)\right] + 71.5$

c) How high is a passenger 5 minutes after the wheel starts rotating?

sub in 5 or calc value 5

$h(5) = -67.5\cos\left(\frac{2\pi}{3}\cdot 5\right) + 71.5$

$= 4\text{ m}$

*d) How many seconds after the wheel starts rotating is a passenger 85 m above the ground for the first time. Answer to the nearest tenth.

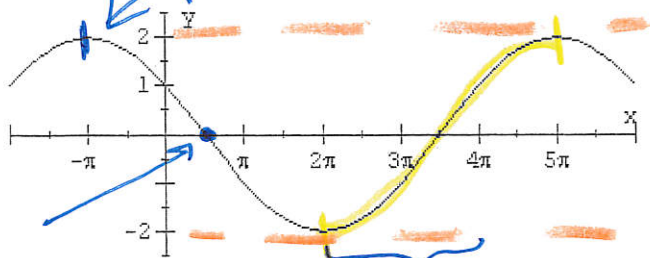
$y_2 = 85$

$y_1 = -67.5\cos\left[\frac{2\pi}{3}(x)\right] + 71.5$

calc intersect

5. Write the equation of the graphs represented below. Where possible, write a sine and cosine function.

pos. cos



neg sine

$\boxed{|a| = 2}$

$d = 0$

$\frac{1}{2}$ per $= 3\pi$ so per $= 6\pi = \frac{2\pi}{b}$

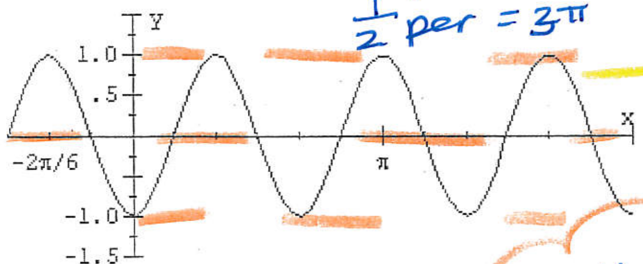
$6\pi b = 2\pi$

$\boxed{b = 3}$

$y = 2\cos[3(x+\pi)]$

or $y = -2\cos[3(x-2\pi)]$

or $y = -2\sin[3(x-\frac{\pi}{2})]$



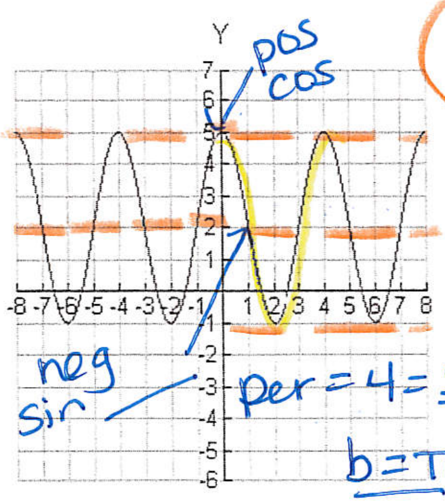
$y = -\cos(3x)$

$y = \sin 3(x - \frac{\pi}{6})$

$|a| = 3$

$d = 2$

pos cos



neg sin

per $= 4 = \frac{2\pi}{b}$

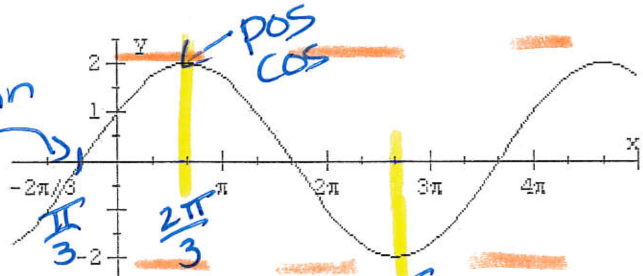
$b = \frac{\pi}{2}$

so $y = 3\cos(\frac{\pi}{2}x) + 2$

$y = -\sin[\frac{\pi}{2}(x+1)] + 2$

pos cos

pos sin



$\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{8\pi}{3}$

$\boxed{|a| = 2}$

$\boxed{d = 0}$

$\frac{1}{2}$ per $= \frac{6\pi}{3} = 2\pi$

$\therefore$ per $= 4\pi = \frac{2\pi}{b}$

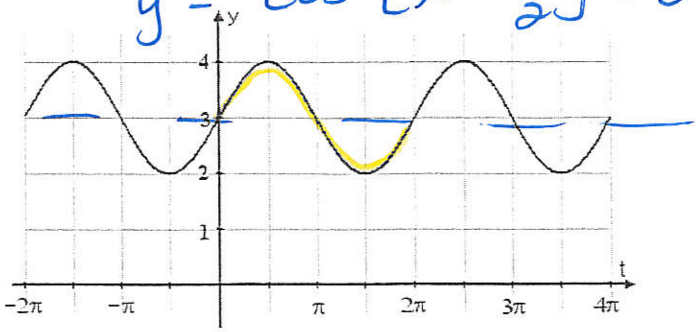
$\boxed{b = \frac{1}{2}}$

$y = 2\cos(\frac{1}{2}(x - \frac{2\pi}{3}))$

$y = 2\sin(\frac{1}{2}(x + \frac{\pi}{3}))$

$y = \sin x + 3$

$y = \cos\left[x - \frac{\pi}{2}\right] + 3$



$|a| = 1$

$d = 3$

$per = 2\pi = \frac{2\pi}{b}$

$b = 1$

6. The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function:

$$y = 19 + 6\sin\left(\frac{\pi}{12}(x - 11)\right)$$

where y is the temperature (°C) and x is the time in hours past midnight.

$y = 6\sin\left[\frac{\pi}{12}(x-11)\right] + 19$

a.) What is the temperature in the office at 9 A.M. when employees come to work?

sub in 9 for x

$y = 6\sin\left[\frac{\pi}{12}(9-11)\right] + 19$

$y = 6\sin\frac{-\pi}{6} + 19 = 6\left(-\frac{1}{2}\right) + 19$

$= -3 + 19$

$\boxed{= 16°}$

b.) What are the maximum and minimum temperatures in the office?

max = mid + $|a|$ = 19 + 6 = 25°

mid = d = 19

min = mid − $|a|$ = 19 − 6 = 13°

c). By how much do the max and min temperatures vary?

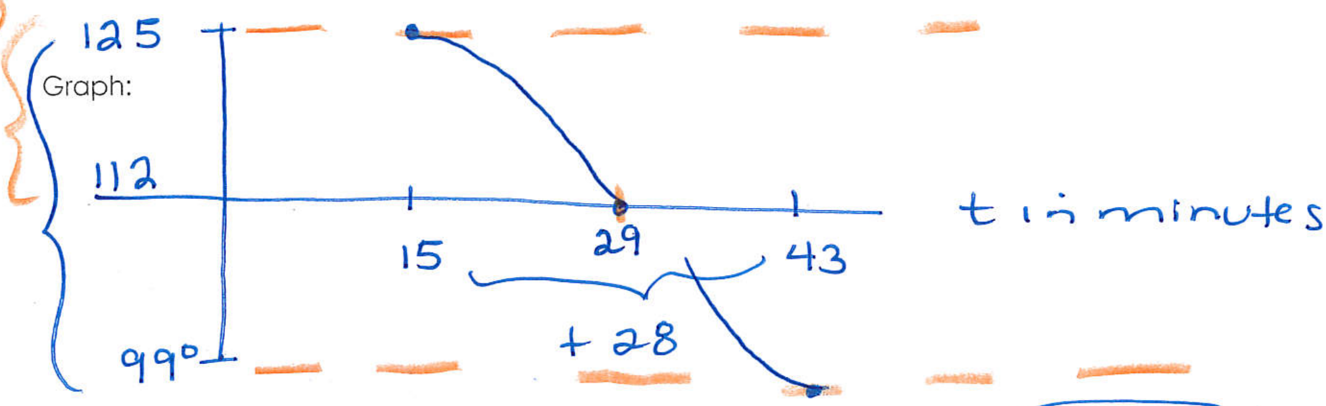
= 12 degrees

7. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125 ° occurs 15 minutes after they start their examination. A minimum temperature of 99 ° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period and an equation of temperature as a function over time in minutes.

$|a| = 13$

125
+99
224
÷ 2
= 112

Graph:



Model: $\frac{1}{2}$ per = 28 per = 56 = $\frac{2\pi}{b}$

$\boxed{b = \frac{\pi}{28}}$ $d = 112$ $|a| = 13$

$\boxed{\text{pos cos} \Rightarrow c = 15}$

$$y = 13\cos\left[\frac{\pi}{28}(x - 15)\right] + 112$$

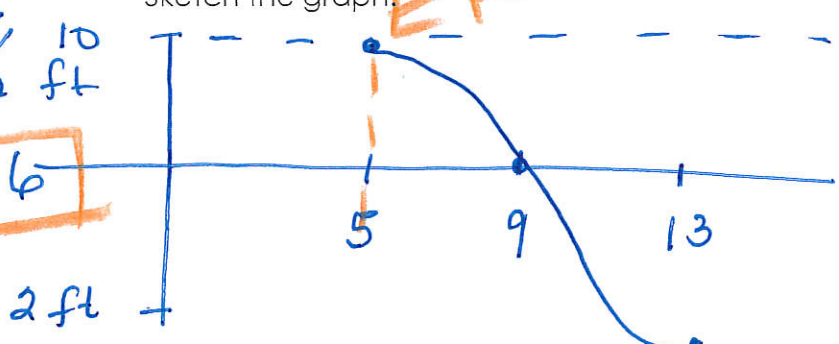
8. The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height *h* (in feet) of the water as a function of time *t* (in hours since midnight).

= 13 hours since midnight

Sketch the graph: ← pos cos

$d = 10 + 2 / 2$

$\boxed{d = 6}$



$\frac{1}{2}$ per = 8

per = 16 = $\frac{2\pi}{b}$

$\boxed{b = \frac{\pi}{8}}$

Model: pos cos → $\boxed{c = 5}$

$a = |4|$

$$y = 4\cos\left[\frac{\pi}{8}(x - 5)\right] + 6$$

Inverses

C	S, C, T
	S, T

$\sin^{-1}$ $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\cos^{-1}$ $[0, \pi]$

UNIT 7: Evaluating trig function inverses, simplifying trig expressions, verifying, solving trig equations.

$\tan^{-1}$ $(-\frac{\pi}{2}, \frac{\pi}{2})$

1. Find the exact value of each expression. Do not use a calculator. Remember the quadrant rules for inverses - COS Q1 and Q2, SIN Q1 and Q4.

a) $\mathrm{Tan}^{-1}1$ $\pi/4$ b) $\mathrm{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$ c) $\mathrm{Sec}^{-1}\sqrt{2}$ $\pi/4$

Use inverse rules (vs. find all θ, $0 \le \theta \le 2\pi$)

2. Find the exact value, if any, of each composite function.

a) $\mathrm{Sin}^{-1}\left(\sin\frac{3\pi}{8}\right)$ $\frac{3\pi}{8}$

Q1

b) $\tan\left(\mathrm{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $-\sqrt{3}$

$\tan\left(-\frac{\pi}{3}\right)$

$= -\sqrt{3}$

c) $\sin\left(\mathrm{Cot}^{-1}\frac{3}{4}\right)$ $\frac{4}{5}$

make a △

$\cot = \frac{x}{y}$

5, 4, 3

$\sin\theta = \frac{y}{r} = \frac{4}{5}$

3. **Simplify** each expression.

a. $\frac{\sin x\cos x}{1-\cos^2 x}$

replacement!

$\frac{\sin x\cos x}{\sin^2 x} = \frac{\cos x}{\sin x}$

3 a.) $= \cot x$

b. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

careful!

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x$

$= \sin^2 x + \cos^2 x + \sin^2 x + \cos^2 x$

$= 1 + 1$

b.) $= 2$

c. $\frac{\tan^2 x}{\sec x+1}+1$

replacement

$\frac{\sec^2 x - 1}{\sec x + 1} + 1$

$\frac{\cancel{(\sec x+1)}(\sec x-1)}{\cancel{(\sec x+1)}} + 1$

3c.)

$= \sec x$

d. $\sec x-\sin x\tan x$

$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x}$

$\frac{1-\sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x}$

3d.) $= \cos x$

Radians are fine!

4. **Solve.** Give your answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$ without using a calculator.

a. $\cot\theta = -\frac{\sqrt{3}}{3}$ $\therefore \tan = -\sqrt{3}$

$\frac{\pi}{3}$ fam

$\frac{2\pi}{3}$ $\frac{5\pi}{3}$

b. $\sec\theta = \sqrt{2}$

so $\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

4a.) $\frac{2\pi}{3}, \frac{5\pi}{3}$

4b.) $\frac{\pi}{4}, \frac{7\pi}{4}$

5. Verify each identity. → One side only!

a) $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

$\tan\theta \cdot \frac{1}{\tan\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta = \cos^2\theta$ ✓

b) $\left(\frac{1-\cos}{1-\cos}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right) + \frac{\sin\theta}{1-\cos\theta}\left(\frac{\sin}{\sin}\right) = 2\csc\theta$ 2 fracs →

create common den

$\frac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1-\cos\theta)}$

$\frac{2 - 2\cos\theta}{\sin\theta(1-\cos\theta)} = \frac{2\cancel{(1-\cos\theta)}}{\sin\theta\cancel{(1-\cos\theta)}}$

$= 2\csc\theta$

c) $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}$ conjugate

$\stackrel{?}{=} \frac{\cos^3\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$

$\stackrel{?}{=} \frac{\cos^2\theta\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$= \frac{\cos^2\theta\cos\theta(1-\sin\theta)}{\cos^2\theta}$

$= \cos\theta(1-\sin\theta)$

$= \frac{1}{\sec\theta}(1-\sin\theta) = \frac{1-\sin\theta}{\sec\theta}$

d) $\frac{\tan x - \sin x\cos x}{\sin^2 x} = \tan x$

Split it! OR rewrite $\sin^2$ as $1-\cos^2$, then factor

$\tan x$

$\frac{\sin x}{\cos x} \div \sin^2 x - \sin x\cos x \div \sin^2 x$

$\frac{\sin x}{\cos x \sin^2 x} - \frac{\sin x\cos x}{\sin^2 x}$

$\frac{1}{\cos x\sin x} - \frac{\cos x \cdot \cos x}{\sin x\cos x}$

$= \frac{1-\cos^2 x}{\sin x\cos x} = \sin^2/\sin\cos = \sin/\cos = \tan x$

Adding fractions ⇒ LCD

e) $\frac{(1+\sin)}{(1+\sin)}\left(\frac{1}{1-\sin x}\right)+\frac{1}{1+\sin x}\left(\frac{1-\sin}{1-\sin}\right)=2\sec^2 x$

$$\frac{1+\sin+1-\sin}{1-\sin^2 x}$$

$$\frac{2}{1-\sin^2 x}=\frac{2}{\cos^2 x}=2\sec^2 x \checkmark$$

f) LCD $\frac{(1+\sin)}{(1+\sin)}\left(\frac{1+\sin x}{\cos x}\right)+\frac{\cos x}{1+\sin x}\frac{(\cos x)}{(\cos x)}=2\sec x$

$$\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}=\frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}}$$

$$=2\sec x$$

g) $\frac{1+\sin x}{\cos x}=\sec x+\tan x$

Split it

$$\frac{1}{\cos x}+\frac{\sin x}{\cos x}$$

$$\sec x+\tan x=\sec x+\tan x \checkmark$$

conjugate

h). $\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$

DO NOT CROSS MULTIPLY

$$\frac{(1-\cos x)(\sin x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{(1-\cos x)\sin x}{1-\cos^2 x}$$

$$\frac{(1-\cos x)(\sin x)}{\sin^2 x}=\frac{1-\cos x}{\sin x} \checkmark$$

6. Solve each equation on the interval $0 \le \theta < 2\pi$.

Strategies: linear equation, plus or minus square root, "u", GCF, factoring, replacement, square both sides

a) $2\sin^2\theta-3\sin\theta+1=0$ ____________________

Think $2x^2-3x+1=0$

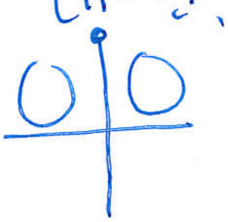
$(2x-1)(x-1)=0$

$2x-1=0 \quad x-1=0$

$x=\frac{1}{2} \quad x=1$

$\sin x=\frac{1}{2} \quad \sin x=1$

$(1,0)$



$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

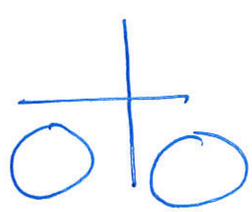
b.) $3\sin x-2=5\sin x-1$ ____________________

Think $3x-2=5x-1$

$-2x=1$

$x=-\frac{1}{2}$

$\sin x=-\frac{1}{2}$



$\frac{7\pi}{6}, \frac{11\pi}{6}$

c.) $5\sin x = 3\sin x + \sqrt{3}$ ______

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

d). tan3x = 1 $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$

$\frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} = 3\pi/4$ $\quad \frac{5\pi}{12} + \frac{8\pi}{12}$ $= \frac{9\pi}{12} = \frac{13\pi}{12}$

let $u = 3x$

$\tan u = 1$

$u = \frac{\pi}{4} + 2\pi k$

$3x = \frac{\pi}{4} + \frac{2\pi}{k} \Rightarrow x = \frac{\pi}{12} + \frac{2\pi}{3}$

$u = \frac{5\pi}{4} + 2\pi k$

$3x = \frac{5\pi}{4} + 2\pi k$

$x = \frac{5\pi}{12} + \frac{2\pi}{3}$

e.) $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$ $\frac{2\pi}{3}, \frac{4\pi}{3}$

let $u = \frac{x}{2}$

$\sin u = \frac{\sqrt{3}}{2}$

$u = \frac{\pi}{3} + 2\pi k$ $\qquad u = \frac{2\pi}{3} + 2\pi k$

$\frac{x}{2} = \frac{\pi}{3} + 2\pi k$ $\qquad \frac{x}{2} = \frac{2\pi}{3} + 2\pi k$

$x = \frac{2\pi}{3} + 4\pi$ $\qquad x = \frac{4\pi}{3} + 4\pi$

f) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\quad \sin x = 1$

$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

g.) $\tan x \sin^2 x = 3\tan x$

set = 0, GCF

CAN NOT divide!

$\tan x \sin^2 x - 3\tan x = 0$

$\tan x(\sin^2 x - 3) = 0$

$\tan x = 0$ $\quad \sin^2 x = 3$

$\tan x = 0$ $\quad \sin x = \pm\sqrt{3}$ } outside of interval.

$\frac{y}{x} = 0 \Rightarrow y = 0$

$0, \pi$

h.) $2\cos^2 x + 3\sin x = 0$

replace

$2(\cos^2 x) + 3\sin x = 0$

$2(1 - \sin^2 x) + 3\sin x = 0$

$-2\sin^2 x + 3\sin x + 2 = 0$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2)$

$\sin x = -\frac{1}{2}$ $\quad \emptyset$

$\boxed{7\pi/6, \frac{11\pi}{6}}$

i). sinx - cosx = 1

SQUARE

$(\sin x - \cos x)^2 = 1^2$

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$1 - 2\sin x\cos x = 1$

$-2\sin x\cos x = 0$

$\left.\begin{array}{l}\sin x = 0\\ \cos x = 0\end{array}\right\}$ $0, \pi/2, \pi, 3\pi/2$

j.) $2\sec^2 x = 4$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

$\cos x = \pm\sqrt{2}/2$

$\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

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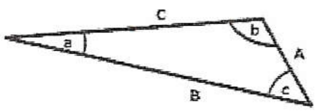
UNIT 8

TOPIC 1: Law of Sines, Law of Cosines, Heron's Formula, Area formula, ambiguous case, angles of elevation and depression, applications of trig.

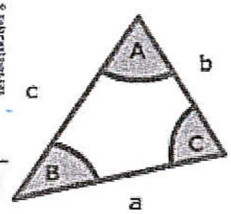
ASA SAA SSA – check ambiguous case! SAS or SSS

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$



$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$$

Law of Cosines



$a^2 = b^2 + c^2 - 2bc\cdot\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cdot\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cdot\cos(c)$

Heron's Formula for area of a triangle given SSS

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

Area of a triangle given SAS

$$A_{\triangle} = \frac{1}{2}ab\sin C$$

Check for ambiguous case when you have SSA!
SSS – find angles in order, largest to smallest or smallest to largest.

1. Solve each triangle using the Law of Sines or the Law of Cosines. It may help to draw a picture. (Hint: Remember the ambiguous case!).

a. B = 10°, C = 20°, c = 33

A = 150°

b = 16.75

a = 48.24

$$\frac{\sin 20}{33} = \frac{\sin B}{10}$$

b. B = 150°, a = 10, b = 3 SSA

A = ______

C = ______

c = ______

$$\frac{\sin 150}{3} = \frac{\sin A}{10}$$

$\sin A = 1.67$

no $\triangle$ exists

SSS → lgst ∡ 1st

SAS → LOC to find b, Then, smallest remaining ∡ 1st

c. a = 2.5, b = 5.0, c = 4.5

A = 29.9

B = 86.2

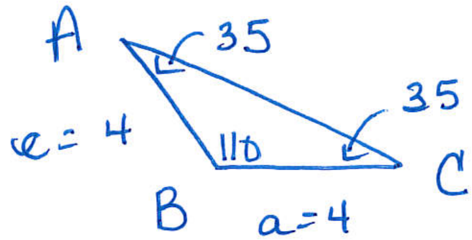
C = 63.9

d. B = 110°, a = 4, c = 4

A = 35°

C = 35°

b = ≈6.6



Isosceles

OR

$b^2 = 4^2 + 4^2 - 2(4)(4)\cos 110$

$b = 6.6$

Then, $\frac{\sin 110}{6.6} = \frac{\sin A}{4}$

$A \approx 34.72$

$\frac{\sin 110}{b} = \frac{\sin 35}{4}$

e. B = 25°, a = 6.2, b = 4

SSA:

A = 40.9	A_2 = 139.1
C = 114.1	C_2 = 15.9
c = 8.64	c_2 = 2.6

$\frac{\sin 25}{4} = \frac{\sin A}{6.2}$

$\sin A = \frac{6.2\sin 25}{4}$

$\sin A = .6550$

$A = 40.9°$

Is there a Δ 2? yes

$180 - 40.9 = 139.1$

$+\ 25$

164.1

$A_2 = 139.1$

$C_2 = 180 - 164.1 = 15.9$

2. Find the area of each given triangle.

a. B = 60°, a = 4, c = 8 ______

SAS

$K = \frac{1}{2} ac \sin B$

$= \frac{1}{2}(4)(8)\sin 60$

$= 13.85 \text{ units}^2$

b. a = 15, b = 8, c = 10 ______

SSS

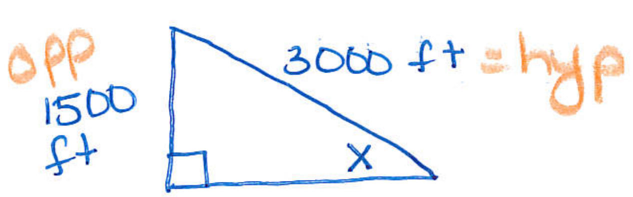
$s = \frac{15+8+10}{2} = \frac{33}{2} = 16.5$

$A = \sqrt{16.5(1.5)(8.5)(6.5)}$

$A = 36.98 \approx 37$ sq. units

3. You are skiing down a mountain with a vertical height of 1500 feet. The distance from the top of the mountain to the base is 3000 feet. What is the angle of elevation from the base to the top of the mountain?

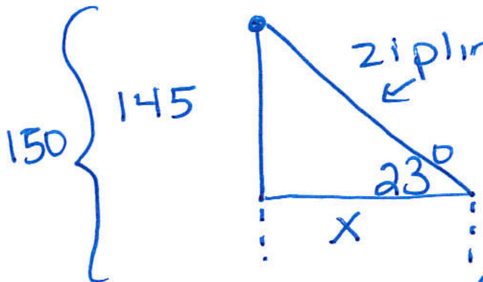
RT Δ → SOH CAH TOA



$\sin x = \frac{1500}{3000} = \frac{1}{2}$

30° or $\frac{\pi}{6}$ rads.

4. A steel cable zip-line is being constructed for a competition on a reality television show. One end of the zip-line is attached to a platform on top of a 150-foot pole. The other end of the zip-line is attached to the top of a 5 foot stake. The angle of elevation ~~of~~ to the platform is 23°.



$\sin 23^\circ = \frac{145}{h}$

$h = \frac{145}{\sin 23^\circ}$

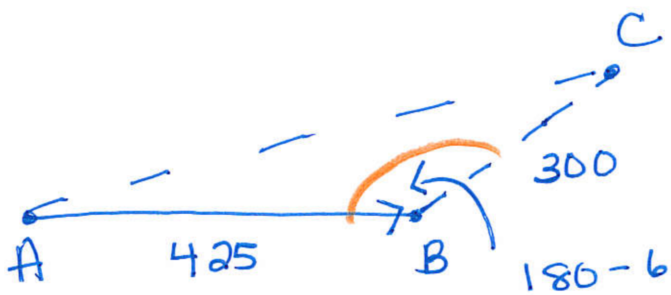
a) How long is the zip-line? 371 feet

b) How far is the stake from the pole?

$\tan 23 = \frac{145}{x}$ $\quad x = \frac{145}{\tan 23}$

341.6 feet

5. To approximate the length of a marsh, a surveyor walks 425 meters from point A to point B. Then the surveyor turns 65° and walks 300 meters to point C. Approximate the length of AC of the marsh.



$180 - 65^\circ = 115^\circ$

SAS

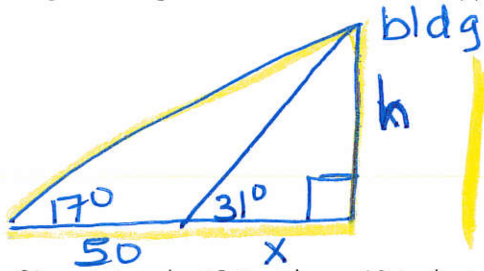
$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 300^2 + 425^2 - 2(300)(425)\cos 115^\circ$

$b = \sqrt{}$

$b = 615$ feet

6. From a certain distance, the angle of elevation to the top of a building is 17°. At a point 50 meters closer to the building, the angle of elevation is 31°. Approximate the height of the building.



LG Δ $\tan 17 = \frac{h}{x+50}$ $\qquad \tan 31^\circ = \frac{h}{x}$

$h = x \tan 31$

$(x+50)(\tan 17) = x \tan 31$

$x \tan 17 + 50 \tan 17 = x \tan 31$

$x(\tan 17 - \tan 31) = -50 \tan 17$

$x = -50 \tan 17 \div (\tan 17 - \tan 31)$

$\boxed{x = 51.8 \text{ meters}}$

7.] Given triangle ABC with a = 10 inches, b = 8 inches, and c = 12 inches, find the measure of angle B.

SSS → lgst 1st

B is smallest

$12^2 = 10^2 + 8^2 - 2(10)(8)\cos C$

$144 = 164 - 160 \cos C$

$\frac{-20}{-160} = \cos C$ $\qquad \boxed{C = 82.8^\circ}$

→ now use LOS to find B

$\frac{\sin 82.8}{12} = \frac{\sin B}{8}$

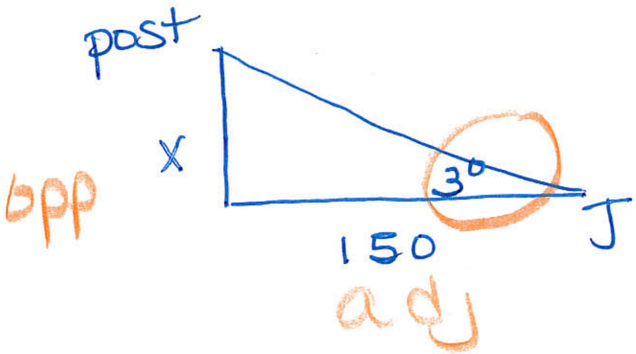
$\sin B = .6614$

$B = 41.4^\circ$

10.] A large flagpole stands at the top of the Smythe Office Building. From the street, the angle of elevation to the top of the flagpole is 55°. The angle of elevation to the bottom of the flagpole is 50°. Find the height of the flagpole.

skip

11. John walks 150 feet away from a fence post on his farm. He measures the angle of elevation from the ground to the top of the post to be 3°. How tall is the fence post?



$$\tan 3° = \frac{x}{150}$$

$$150 \tan 3 = x$$

7.86 feet

12. Find the area of a regular pentagon inscribed in a circle with diameter 14 feet.

$r = 7$

$$\frac{360}{5} = 72°$$

$$A_{penta} = \left[\frac{1}{2}(7)(7)\sin 72\right] \cdot 5$$

$$A_{penta} = 116.5 \text{ ft}^2$$

13. Ms. M has a triangular garden with sides 10, 14 and 16. A bag of organic fertilizer covers 50 square feet. How many bags will she need?

$$s = \frac{1}{2}(10 + 14 + 16) = 20$$

$$K = \sqrt{20(20-10)(20-14)(20-16}$$

$$= \sqrt{4800} \approx \frac{69.3 \text{ square feet}}{50} = 2 \text{ bags}$$

Additional Practice - Key is at end of pkt.

EVEN MORE PRACTICE - 4.1 – 4.4

41. A circle has a radius of 7 inches. Find the **length of the arc** intercepted by a central angle of $240°$. 41.)__________

42. The circular blade on a saw rotates at 2400 revolutions per minute.

a. Find the **angular speed** in radians per second. 42a.)__________

b. The blade has a radius of 4 inches. Find the **linear speed** of a blade tip in inches per second. 42b.)__________

43. A satellite in circular orbit 1125 km above a planet makes one complete revolution every 120 minutes. Assuming that he planet is a sphere of radius 6400 km, find the linear speed of the satellite in **kilometers per minute**. Round your answer to the nearest whole number. 43.)__________

44. A truck is moving at a rate of 90 km per hour and the diameter of its wheels is 1.25 meters. Find the angular speed of the wheels in **radians per minute**. 44.)__________

45. **Evaluate** (if possible) the six trigonometric functions if $\theta = -\frac{2\pi}{3}$.

$\sin\left(-\frac{2\pi}{3}\right) =$ $\csc\left(-\frac{2\pi}{3}\right) =$

$\cos\left(-\frac{2\pi}{3}\right) =$ $\sec\left(-\frac{2\pi}{3}\right) =$

$\tan\left(-\frac{2\pi}{3}\right) =$ $\cot\left(-\frac{2\pi}{3}\right) =$

46. **Evaluate** the trigonometric function.

a. $\sin\left(-\frac{3\pi}{4}\right)$ b. $\csc\left(\frac{7\pi}{6}\right)$ c. $\tan\left(\frac{5\pi}{3}\right)$

46a.)__________

46b.)__________

46c.)__________

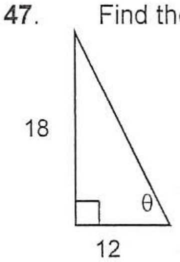
d. $\sec(-4\pi)$ e. $\cot\left(\dfrac{5\pi}{2}\right)$ f. $\cos\left(\dfrac{13\pi}{4}\right)$

46d.)_______________

46e.)_______________

46f.)_______________

47. Find the **exact values** of the **six trigonometric functions** of the angle θ shown in the figure.



$\sin(\theta)=$ $\csc(\theta)=$

47. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

53. Use the given value and the trigonometric identities to **find the remaining trigonometric functions** of the angle.

$\sin(\theta)=$ $\csc(\theta)=$

$\cos\theta = -\dfrac{3}{7},\ \sin\theta < 0$

53. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

54. The point is on the terminal side of an angle in standard position. **Determine** the **exact values** of the six trigonometric functions of the angle.

$(8, -15)$

$\sin(\theta)=$ $\csc(\theta)=$

54. $\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

KEY TO EVEN MORE PRACTICE:

41. $\frac{28\pi}{3}$ inches

42a. 80π rad/sec

42b. ≈ 1005 in/sec

43. 394 km/min

44. 2400 rad/min

45. $\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\csc\left(-\frac{2\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\sec\left(-\frac{2\pi}{3}\right) = -2$

$\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$ $\cot\left(-\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$

46a. $-\frac{\sqrt{2}}{2}$

46b. -2

46c. $-\sqrt{3}$

46d. 1

46e. 0

46f. $-\frac{\sqrt{2}}{2}$

47. $\sin(\theta) = \frac{3\sqrt{13}}{13}$ $\csc(\theta) = \frac{\sqrt{13}}{3}$

$\cos(\theta) = \frac{2\sqrt{13}}{13}$ $\sec(\theta) = \frac{\sqrt{13}}{2}$

$\tan(\theta) = \frac{3}{2}$ $\cot(\theta) = \frac{2}{3}$

53. $\sin(\theta) = -\frac{2\sqrt{10}}{7}$ $\csc(\theta) = -\frac{7\sqrt{10}}{20}$

$\cos(\theta) = -\frac{3}{7}$ $\sec(\theta) = -\frac{7}{3}$

$\tan(\theta) = \frac{2\sqrt{10}}{3}$ $\cot(\theta) = \frac{3\sqrt{10}}{20}$

54. $\sin(\theta) = -\frac{15}{17}$ $\csc(\theta) = -\frac{17}{15}$

$\cos(\theta) = \frac{8}{17}$ $\sec(\theta) = \frac{17}{8}$

$\tan(\theta) = -\frac{15}{8}$ $\cot(\theta) = -\frac{8}{15}$

REVIEW UNIT 6 (Book 4.5 – 4.7)

1. a. Sketch $y = -3\sin(2x - 2\pi)$

b. Sketch $y = 2\sin\left(\frac{1}{2}x + \pi\right) - 2$

3. I can write the equation of the trig graph based on its graph, given a max and min, or given a set of data. I can express the equation as a sine function and a cosine function.

a. Find an equation of a sine wave with a peak of 12 and a minimum of 6, starts its cycle at 3π and completes one full cycle every 4π units.

4. I can use sine and cosine functions to model real life data. I can use models to make predictions.

a. The water level in a city water storage tank oscillates in a simple harmonic motion. The water level varies depending on the time of day and the corresponding demand of the people. The low point of the water in the tank, 22 feet, occurs at 8am and 8pm when demand is highest. The high points occur at 2am and 2pm with a water level of 58 feet. Create a sinusoidal function that models the data and use it to predict the water height at 4pm.

5. I can state the domain and range

Section 4.7

8. I can evaluate inverse trig functions from memory or by using my calculator. I understand the restrictions on each trig function. . .

NOTES: SIN COS

a. $\arctan\left(\frac{\sqrt{3}}{3}\right)$ b. $\arcsin\left(-\frac{1}{2}\right)$

c. $\arcsin(2)$ d. $\arctan(-1)$

9. I can use properties of inverse trig functions to evaluate expressions.

a. sin(arcsin 1)   b. cos(arcos .3)   c. $\arctan(\tan \pi)$

10. I can find the exact value or an algebraic expression for a trig expression by using the "triangle technique."

a. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$ b. $\sin\left(\arctan\left(\frac{5}{6}\right)\right)$ c. $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$