

EXPONENTIAL FUNCTIONS REVIEW

Graph the following functions, identify if the function represents exponential growth or decay and state the Domain and Range for the entire function, not just for the portion you are graphing!

1. $y = 3 \cdot 2^x$ $3 \cdot 2^{-2} = 3 \cdot \frac{1}{4} = \frac{3}{4}$

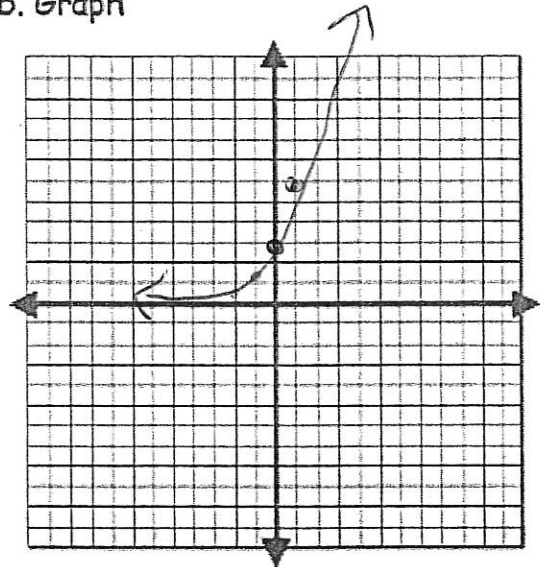
a. Table $3 \cdot 2^{-1} = 3 \cdot \frac{1}{2} = \frac{3}{2}$ b. Graph

| | | | | | | |
|---|---------------|---------------|---|---|----|----|
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | $\frac{3}{4}$ | $\frac{3}{2}$ | 3 | 6 | 12 | 24 |

c. Growth or Decay (circle one)

d. Domain: $(-\infty, \infty)$

e. Range: $(0, \infty)$



2. $y = 4 \cdot (1/2)^x$

a. Table

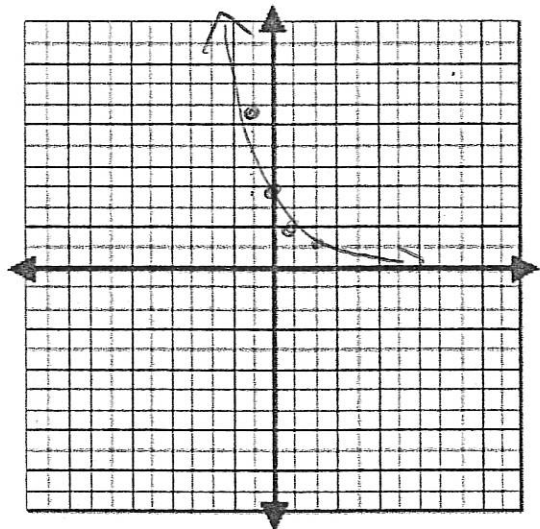
| | | | | | | |
|---|----|----|----|---|---|---|
| x | -3 | -2 | -1 | 0 | 1 | 2 |
| y | 32 | 16 | 8 | 4 | 2 | 1 |

c. Growth or Decay (circle one)

d. Domain: $(-\infty, \infty)$

e. Range: $(0, \infty)$

b. Graph





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3. Each of these situations and tables can be represented by either a linear function or an exponential function. Decide whether it is linear or exponential. Then, write a $y = \dots$ equation and use it to answer the questions posed.

a. The bacteria population doubles every day. There were originally 2 bacteria. Write equation for finding the number of bacteria after t days.

Circle one: Linear or Exponential? $y = \underline{2(2)^x}$

How many bacteria will there be after two weeks?

$t = \# \text{ days}$
 $\therefore t = 14$ $2(2)^{14} =$

When will there be 1000 bacteria?

$$1000 = 2(2)^x$$

Calc Intersect
 $y_1 = 1000$
 $y_2 = 2(2)^x$

b.

| | | | | | | | | |
|---|---|----|----|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |

Circle one: Linear or Exponential? $y = \underline{y = 5x + 5}$

c. You buy a house for \$290,000 in 2018. Its value increases at a rate of 6% per year.

Linear or Exponential?

a) Model $y = \underline{290,000(1.06)^t}$ $t = \# \text{ yrs since 2018.}$

b.) What is the Value of house after 10 years?

$$290,000(1.06)^{10}$$

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D. You buy a computer for \$3,000. Its value depreciates by 8% every month.

Linear or Exponential?

Model: $y = \underline{3,000 (.92)^t}$

How much is your computer worth after 3 years?

$t = \# \text{ months}$

$3 \text{ yrs} = 36 \text{ months} \quad y = 3000 (.92)^{36}$

When will the value of your computer decrease to \$1000? (GC)

$1000 = 3000 (.92)^t$ calc int.

E. In a laboratory, they were testing a certain bacteria. They started with 50 bacteria and they noticed it triples every 30 minutes.

Linear or Exponential?

Model: $y = \underline{50 (3)^t}$ where $t = \# \text{ of 30 minute intervals.}$

How many will exist after 2 hours?

$2 \text{ hrs} \rightarrow t = 4$

$y = 50 (3)^4$

How long will it take for the bacteria population to reach 5000 bacteria?

(GC)

$5000 = 50 (3)^t$

calc: intersect.

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#4 Simplify -

a) $(4y-5)(2y^2-5y+7)$

| | | | |
|------|----------|----------|--------|
| | $2y^2$ | $-5y$ | $+7$ |
| $4y$ | $8y^3$ | $-40y^2$ | $+28y$ |
| -5 | $-10y^2$ | $+25y$ | -35 |

$$8y^3 - 50y^2 + 53y - 35$$

b) $(4m^2 - 2m + 3) - (3m^2 - 5m + 3)$

$$= m^2 + 3m + 0$$

$$= m^2 + 3m$$

c) $3y^2(2x^2 - 5yx + 9)$

$$6x^2y^2 - 15xy^3 + 27y^2$$

d) $(2x - 5)(3x + 8)$

$$6x^2 + 16x - 15x - 40$$

$$6x^2 + x - 40$$

e) $(x-6)^2$

$$(x-6)(x-6)$$

$$x^2 - 12x + 36$$

f) $(5x+4)(5x-4)$

$$25x^2 - 16$$

g) $\frac{-18x^2 + 21x}{-3x}$

$$6x - 7$$

h) $\frac{20x^4 + 15x^3 - 5x^2}{5x^2}$

$$4x^2 + 3x - 1$$

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EXPONENTS REVIEW

5. Simplify the expressions below. NO NEGATIVE EXPONENTS!

| | |
|--|--|
| <p>1. $2x^3 \cdot 3x^5$</p> <p>$6x^8$</p> | <p>2. $(y^2)^4$</p> <p>y^8</p> |
| <p>3. $(-3x^3)^2$</p> <p>$9x^6$</p> | <p>4. 4^0</p> <p>1</p> |
| <p>5. $(-3a^3)^3 \cdot (4a)^0$</p> <p>$(-27a^9)(1)$ $= -27a^9$</p> | <p>6. $(-a^3b^5)^2 (a^4b)^2$</p> <p>$a^6b^{10} a^8b^2$ $= a^{14}b^{12}$</p> |
| <p>7. $(3x)^{-2}$</p> <p>$\frac{1}{(3x)^2} = \frac{1}{9x^2}$</p> | <p>8. $\left(\frac{4}{7}\right)^{-2}$</p> <p>$= \frac{4^{-2}}{7^{-2}} = \frac{49}{16}$</p> |
| <p>9. $5x^{-2}$</p> <p>$= \frac{5}{x^2}$</p> | <p>10. x^0y^{-4}</p> <p>$\frac{1}{y^4}$</p> |
| <p>11. $\frac{6x^{-2}y^2}{3y^{-4}}$</p> <p>$= \frac{2y^6}{x^2}$</p> | <p>12. $(-5x^{-3}y^5)^2$</p> <p>$25x^{-6}y^{10} = \frac{25y^{10}}{x^6}$</p> |
| <p>17. $\frac{15x^3z^{-5}}{25y^{-4}}$</p> <p>$= \frac{3x^3y^4}{5z^5}$</p> | <p>18. $\frac{m^{-5}}{m^7 \cdot m^{-4}}$</p> <p>$= \frac{m^{-5}}{m^3}$ $= \frac{1}{m^8}$</p> |

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SOLVING QUADRATICS:

A. Solve by factoring and the ZERO PRODUCT PROPERTY:

1.) $x^2 - 64 = 0$

$$(x+8)(x-8) = 0$$

$$x = -8$$

$$x = 8$$

2.) $8x^2 - 2x - 18 = -15$

$$8x^2 - 2x - 3 = 0$$

$$(4x-3)(2x+1) = 0$$

$$4x-3=0 \quad 2x+1=0$$

$$x = 3/4 \quad x = -1/2$$

3.) $x^2 + 3x = 40$

$$x^2 + 3x - 40 = 0$$

$$(x+8)(x-5) = 0$$

$$x = -8$$

$$x = 5$$

4.) $2x^2 + 3x + 1 = 0$

$$(2x+1)(x+1) = 0$$

$$x = -\frac{1}{2} \quad x = -1$$

5.) $4x^2 - 8x = 3$

$$4x^2 - 8x - 3 = 0$$

$$(\quad)(\quad) = 0$$

not factorable

6.) $3x^2 + 6x - 42 = 0$

GCF

$$x^2 + 2x - 7 = 0$$

Does not factor

Solve by square roots. Be sure to take plus/minus the square root after isolating the quantity squared!

7.) $4x^2 - 3 = 78$

$$4x^2 = 81$$

$$x^2 = \frac{81}{4}$$

$$x = \pm \sqrt{\frac{81}{4}}$$

$$x = \pm \frac{9}{2}$$

8.) $(4x-3)^2 + 7 = 39$

$$(4x-3)^2 = 32$$

$$4x-3 = \pm \sqrt{32}$$

$$x = \frac{3 \pm \sqrt{32}}{4} = \frac{3 \pm 4\sqrt{2}}{4}$$

For #13-15, write the expression for the discriminant. Use this to find the number of real solutions for each equation. Be sure to set equal to zero first!

9.) $2x^2 - 3x + 1 = 0$

$$b^2 - 4ac$$

$$(-3)^2 - 4(2)(1)$$

$$9 - 8 = 1$$

2 solutions

10.) $x^2 + 4x = -7$

$$b^2 - 4ac$$

$$x^2 + 4x + 7 = 0$$

$$4^2 - 4(1)(7)$$

$$16 - 28 = -12 \rightarrow \text{no real sol.}$$

11.) $x^2 + 9 = 6x$

$$x^2 - 6x + 9 = 0$$

$$36 - 36 = 0$$

one soln

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Solve using the quadratic formula. $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

12.) $2x^2 - 6 = -x$

$2x^2 + x - 6 = 0$
 $a = 2, b = 1, c = -6$

$b^2 - 4ac$
 $= 1 - 4(2)(-6)$
 $= 1 + 48$
 $= 49$

$\sqrt{49} = 7$

$x = \frac{-1 \pm 7}{4} \rightarrow \frac{6}{4} = \boxed{\frac{3}{2}}$
 $\searrow \frac{-8}{4} = \boxed{-2}$

13.) $3x^2 - 2x - 5 = 0$

$a = 3, b = -2, c = -5$

$b^2 - 4ac = 4 - 4(3)(-5)$
 $= 4 + 60$
 $= 64$

$\sqrt{64} = 8$

$x = \frac{4 \pm 8}{6} \rightarrow \boxed{2}$
 $\searrow \boxed{\frac{-2}{3}}$

SOLVE. Recall that the vertex is the min/max of the curve, and that the x coordinate of the vertex can be found using $x = -b/2a$.

14. A manufacturer of fax machines find that the cost (in dollars) generated by manufacturing x units per week is given by the function $C(x) = 0.15x^2 - 39x + 4500$. How many units should be manufactured to minimize the cost?

min \rightarrow vertex (x, c)
 $x = \frac{-b}{2a} = \frac{39}{.3} = 130$ units

15. Given the projectile motion model, $h(t) = -16t^2 + 8t + 4$,

$h(t) = -16t^2 + V_0t + h_0$

A. What is the initial height?

4 ft

B. What is the velocity of the object in feet per second?

8 ft/sec.

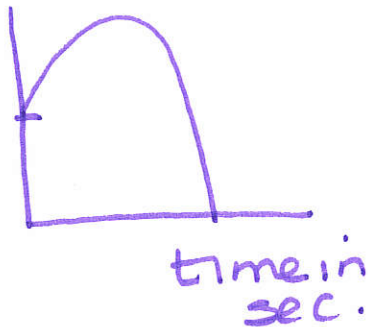
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16. Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. The height of the cliff is 480 feet and his initial upward velocity is 16 ft/sec.

Model: $h(t) = -16t^2 + 16t + 480$

SKETCH:

ht in
ft.
480



How long does it take Jason to reach his maximum height?

max = (t, h)

$t = \frac{-b}{2a} = \frac{-16}{-32} = \frac{1}{2}$ second

What is the maximum height?

sub in $t = \frac{1}{2}$ for t

$h\left(\frac{1}{2}\right) = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 480$
 $= -16\left(\frac{1}{4}\right) + 8 + 480$

When will Jason hit the water?

$= -4 + 8 + 480 = 484$
ft.

$h(t) = 0$

$0 = \frac{-16t^2}{-16} + \frac{16t}{-16} + \frac{480}{-16}$

$0 = t^2 - t - 30$
 $0 = (t - 6)(t + 5)$
 $t = 6$ ~~$t = -5$~~

6 seconds

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17. SIMPLIFYING RADICAL EXPRESSIONS

Recall: Break into a perfect square (cube) house and a non-perfect square (cube) house!

Multiply and divide anything - insides with insides, outsides with outsides.

Add and subtract only like radicals.

$$\begin{aligned} \text{a) } \sqrt{54} &= \sqrt{9} \sqrt{6} \\ &= 3\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{b) } -2\sqrt{48x^3} \\ &= -2\sqrt{16x^2} \sqrt{3x} \\ &= -2 \cdot 4x \sqrt{3x} \\ &= \boxed{-8x\sqrt{3x}} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{49x^3} \\ &= \sqrt{49x^2} \sqrt{x} \\ &= \boxed{7x\sqrt{x}} \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt{40ab^2} \\ &= \sqrt{4b^2} \cdot \sqrt{10a} \\ &= \boxed{2b\sqrt{10a}} \end{aligned}$$

$$\begin{aligned} \text{e) } \sqrt{18y^3z} \\ &= \sqrt{9y^2} \cdot \sqrt{2yz} \\ &= \boxed{3y\sqrt{2yz}} \end{aligned}$$

$$\begin{aligned} \text{f) } \sqrt{100a^2b^4c} \\ &= \sqrt{100a^2b^4} \sqrt{c} \\ &= \boxed{10ab^2\sqrt{c}} \end{aligned}$$

$$\begin{aligned} \text{g. } \sqrt{75m^3n^5} \\ &= \sqrt{25m^2n^4} \sqrt{3mn} \\ &= \boxed{5mn^2\sqrt{3mn}} \end{aligned}$$

$$\begin{aligned} \text{h. } \sqrt{50x^5y^3} \\ &= \sqrt{25x^4y^2} \sqrt{2xy} \\ &= \boxed{5x^2y\sqrt{2xy}} \end{aligned}$$

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i) $7x\sqrt{15x} \cdot \sqrt{6x}$

$$= 7x \sqrt{90x^2}$$

$$= 7x \sqrt{9x^2} \sqrt{10}$$

$$= 7x \cdot 3x \cdot \sqrt{10} = 21x^2\sqrt{10}$$

j) $(5 + 3\sqrt{2}) + (-3 + 4\sqrt{2})$

$$2 + 7\sqrt{2}$$

k) $2\sqrt{27} + 5\sqrt{3}$

$$= 2\sqrt{9}\sqrt{3} + 5\sqrt{3}$$

$$= 6\sqrt{3} + 5\sqrt{3}$$

$$= 11\sqrt{3}$$

l) $2\sqrt{20} - \sqrt{500}$

$$= 2\sqrt{4}\sqrt{5} - \sqrt{100}\sqrt{5}$$

$$= 4\sqrt{5} - 10\sqrt{5}$$

$$= -6\sqrt{5}$$

18. Rewrite the following expressions with rational denominators. Be sure to completely simplify your answers.

$$\frac{4}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{4\sqrt{10}}{10}$$

$$= \frac{2\sqrt{10}}{5}$$

$$\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$\frac{4}{\sqrt{8}} = \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\frac{9}{\sqrt{48}} = \frac{9}{16\sqrt{3}} = \frac{9}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{9\sqrt{3}}{12} = \frac{3\sqrt{3}}{4}$$

$$\frac{-4}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-4\sqrt{2}}{6} = \frac{-2\sqrt{2}}{3}$$

$$\frac{\sqrt{3}}{\sqrt{8}} = \frac{\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{4}$$

$$\left(\frac{\sqrt{5}+3}{\sqrt{10}} \right) \left(\frac{\sqrt{10}}{\sqrt{10}} \right)$$

$$= \frac{\sqrt{50} + 3\sqrt{10}}{10} = \frac{\sqrt{25}\sqrt{2} + 3\sqrt{10}}{10} = \frac{5\sqrt{2} + 3\sqrt{10}}{10}$$

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Rational Expressions REVIEW

19. Simplify (remember to factor when necessary).

A. $\frac{18x^6}{27x^4}$

$= \frac{2x^2}{3}$

B. $\frac{x^2 + 6x + 8}{3x + 12}$

$\frac{(x+2)(x+4)}{3(x+4)}$
 $= \frac{x+2}{3}$

C. $\frac{x^2 - 7x + 12}{x^2 + 2x - 15}$

$= \frac{(x-3)(x-4)}{(x+5)(x-3)}$
 $= \frac{x-4}{x+5}$

Multiply or divide (remember to factor when necessary).

D. $\frac{x+3}{x^2 - 4x + 4} \cdot \frac{x^2 - x - 2}{x^2 + 4x + 3}$

$\frac{(x+3)}{(x-2)(x-2)} \cdot \frac{(x-2)(x+1)}{(x+1)(x+3)}$
 $= \frac{1}{x-2}$

E. $\frac{x^2 - x - 12}{3x + 9} \div \frac{x^2 + x - 20}{x + 5}$

$\frac{(x-4)(x+3)}{3(x+3)} \cdot \frac{(x+5)}{(x+5)(x-4)}$
 $= \frac{1}{3}$

F. $\frac{15x^2}{45x^3} \div \frac{5x^6}{9x^4}$

$\frac{15x^2}{45x^3} \cdot \frac{9x^4}{5x^6}$
 $\frac{\cancel{15} \cdot \cancel{9} \cdot x^2 \cdot x^4}{\cancel{45} \cdot \cancel{5} \cdot x^3 \cdot x^6}$
 $= \frac{3}{5x^3}$

G. $\frac{6}{x^2 - 9x + 20} \cdot \frac{5x - 25}{3x - 6}$

$= \frac{\cancel{6} \cdot 2}{(x-4)(x-5)} \cdot \frac{5(x-5)}{\cancel{3}(x-2)}$
 $= \frac{10}{(x-4)(x-2)}$

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14. $\frac{6x-12}{4x^2} \cdot \frac{3x^3}{2x-4}$

$$\frac{6(x-2)}{4 \cdot x \cdot x} \cdot \frac{3 \cdot x \cdot x \cdot x}{2(x-2)}$$

$$= \frac{6 \cdot 3 \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot \cancel{(x-2)}}{4 \cdot 2 \cdot x \cdot x \cdot \cancel{(x-2)}}$$

$$= \frac{\boxed{9x}}{4}$$

15. $\frac{3x-21}{x^2-3x-28} \cdot \frac{5x+20}{2x+8}$

$$\frac{3(x-7)}{(x-7)(x+4)} \cdot \frac{5(x+4)}{2(x+4)}$$

$$= \frac{15}{2(x+4)}$$

16. $\frac{x^2-5x-6}{2x+6} \div \frac{x^2-3x-4}{4x+12}$



• reciprocal.

$$\frac{x^2-5x-6}{2x+6} \cdot \frac{4x+12}{x^2-3x-4}$$

$$= \frac{(x-6)(\cancel{x+1})}{2(\cancel{x+3})} \cdot \frac{4(\cancel{x+3})}{(x-4)(\cancel{x+1})}$$

$$= \frac{2(x-6)}{(x-4)}$$

Happy Summer!