3.5 Notes Applications SOLVE USING LOGS Date: Key 2016

TYPE 1 : FINANCIAL

Ex 1: Compound Interest $A = P\left(1+\frac{r}{n}\right)^{nt}$

A = ________ P = ________ r = ________

n = ________ t = ________

a) A credit union pays 6% interest on its savings account, compounded monthly. If you deposit \$1000, how long must you leave the money in the account before the balance reaches \$1500?

$$1500 = 1000\left(1+\frac{.06}{12}\right)^{12t}$$

$$1.5 = \left(1+\frac{.06}{12}\right)^{12t}$$

$$\frac{\text{LN } 1.5}{12 \text{ LN } 1.005} = \frac{12t \text{ LN } (1.005)}{\text{LN } 1.005}$$

$$t = 6.775 \text{ yrs}$$

Ex. 2 The formula $A = Pe^{rt}$ is used when an amount of money P is invested at a rate of r and compounded continuously for t years.

A = ________ P = ________ r = ________ t = ________

a) Suppose you invest \$500 in an account at 6% interest compounded continuously. How long will it take the account to double in value?

$A_0 = 500$

$r = .06$

$A = 1000$ (doubled)

$1000 = 500e^{.06t}$

$2 = e^{.06t}$

$\ln 2 = .06t$

$\ln 2 \div .06 = t$

$t \approx 11.6$ yrs

EX 4) A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once a population drops below 100, the situation is irreversible and the species will become extinct. At this rate, when will this happen (will the population reach 100)?

$A = A_0 e^{kt}$ DECAY - so k will be negative

A = Ao = t = k =

(0, 1400)
(5, 1000)

where t = 0 represents 5 yrs ago.

k = -.06729 ≈ -.0673

Model:

$A = 1400e^{-.0673t}$

Solution to question:

39 yrs from 5 yrs ago

∴ 34 yrs from now.

TYPE 3 - HALF LIFE or Percentage of substance left:

Radium-226, a common isotope of radium, has a half-life of 1620 years. Professor Korbel has a 120 gram sample of radium-226 in his laboratory.

a.) Find the constant of proportionality for radium-226, otherwise known as k!

Since this is a half-life problem, we know that: $A = A_0/2$

$$y = y_0 e^{kt}$$

$$60 = 120e^{k \cdot 1620}$$

$$\frac{60}{120} = e^{1620k}$$

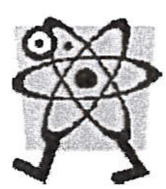
$$\ln\frac{1}{2} = \ln e^{1620k} = 1620k \ln e$$

$$\ln\frac{1}{2} = 1620k$$

$$\frac{\ln\frac{1}{2}}{1620} = k$$

$$k \approx -0.000427868629975$$

It should make sense that k is a negative value, since this is an example of **decay.**



TYPE 4: **Newton's Law of Cooling is used to the temperature of an object as it cools or decreases exponentially over time. The formula is**

$$T = C + (T_0 - C)e^{kt}.$$

T = temp of heated object at time t

C = constant temp of surrounding medium

T_0 = initial temp of heated object

t = time

k = negative constant associated with cooling

always negative

5. A cake removed from an oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

1) Find a model for the temperature of the cake, T, after t minutes.

Sub in what you know, solve for k: T = C = To = t =

$T_0 = 210$
$C = 70$
$t = 30$
$T = 140$

$140 = 70 + \boxed{(210 - 70)e^{30k}}$

$70 = 140e^{30k}$

$\frac{1}{2} = e^{30k}$

$\ln\frac{1}{2} = 30k$

$k = -.0231$

1B) Create the exponential model for this problem:

Leave T and t as variables. $T = 70 + 140e^{-.0231t}$

2) What is the temperature of the cake after 40 minutes?

sub in $t = 40$

125.6°

3) When will the temperature of the cake be 90°F?

$90 = 70 + 140e^{-.0231t}$

$\frac{20}{140} = e^{-.0231t}$

$\ln\frac{1}{7} = -.0231t$

$\ln\left(\frac{1}{7}\right) \div -.0231 = t$

~~35~~ minutes.
84

$t = 84$ mins.

2. Name: ______________________________ M☺re Exponential Growth and Decay

1) Low interest rates, easy credit, and strong demand from new immigrants have driven up the average sales price (the price increases exponentially) of new one-family houses in the United States. In 1995, the average sales price was \$158,700 and by 2000 it had increased to \$207,200. When will the average sales price of a new one-family house reach \$300,000?

$207200 = 158700e^{5K}$

$K \approx .0533$

$300{,}000 = 158700e^{.0533t}$

$t \approx 11.95$ yrs

$+ \; 1995$

2007

2) A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once a population drops below 100, the situation is irreversible and the species will become extinct. At this rate, when will this happen (will the population reach 100)?

$1000 = 1400e^{5K}$

$\ln\left(\frac{5}{7}\right) = 5K$

$\ln\left(\frac{5}{7}\right) \div 5 = K$

$K = \approx -.0673$

$100 = 1400e^{-.0673t}$

$t \approx 39.21$ yrs

3) In ceramics class, you have decided to make a model of an ellipse for your mother on Mother's Day. It is removed from a heating oven with an initial temperature of 450° F and left sitting in a room that has a temperature of 70°. After 5 minutes, it has cooled to 300°.

Newton's Law of Cooling can be used to model the temperature as a function of time. Big I's Law of Cooling is $T = C + (T_0 - C)e^{kt}$, in which T_0 is the initial temperature, C is the constant temperature of the surrounding environment, t is the time in minutes, and T is the final temperature.

a) What is the temperature of the model after 20 minutes?

(1) find K

$300 = 70 + (450-70)e^{5K}$

$230 = 380e^{5K}$

$K \approx -.1004$

$T = 121.22°$ F

b) When will the temperature of the model be 80°?

$80 = 70 + 380e^{-.1004t}$

$10 = 380e^{-.1004t}$

$\frac{1}{38} = e^{-.1004t}$

$\ln\left(\frac{1}{38}\right) = -.1004t$

$t = 36.23$ min