Section 3.1 & 3.5 Notes Exponential Equations and Modeling to Solve Problems

Key.

Using Exponential Formulas to Solve Problems:

Ex 1: Compound Interest $A = P\left(1+\frac{r}{n}\right)^{nt}$

A = Balance after t yrs P = Initial deposit r = Interest rate as a decimal

n = # times compounded per yr. t = Time in yrs.

a) A credit union pays 6% interest on its savings account, compounded monthly. Suppose you put \$1000 in such an account. How much will you have after 1 year?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1000\left(1+.06/12\right)^{12(1)}$ \$1061.68

b) A bank agrees to pay 5% interest, compounded quarterly, to its customers. If you put \$500 in the account, how much will you have after 1 year?

$A = 500\left(1+\frac{.05}{4}\right)^{4(1)}$ = \$525.47

Ex 2

a) In would appear that the compounding period affects the growth and that if the number of corresponding periods is increased, the amount should increase significantly. Complete the table, showing how \$1000 grows at 6% over a period of 10 years. Then determine if increasing the number of period increases the amount significantly.

Initial Deposit	Compounding	Model	Total After 10 Years
\$1000	Annually	$1000(1+.06)^{10}$	
\$1000	Semi-Annually	$1000\left(1+\frac{.06}{2}\right)^{20}$	
\$1000	Quarterly	$1000(1+.06/4)^{40}$	
\$1000	Monthly	$1000(1+.06/12)^{120}$	
\$1000	Daily	$1000(1+.06/365)^{365(10)}$	
\$1000	Hourly	$1000(1+.06/8760)$^ 87600	
\$1000	By the Minute	$1000(1+.06/525600)$^ 5256000	

b) Let's take a closer look...

Suppose the original amount, P is 1 and the annual rate is 100% or 1 for 1 year of growth compounded k times. Then the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ becomes ____________________

Complete the table below using this new formula.

k	1	10	100	1,000	10,000	1,000,000
A						

The formula $A = Pe^{rt}$ is used when an amount of money P is invested at a rate of r and compounded continuously for t years.

A = Ending balance after t years P = Initial deposit r = % rate as a decimal t = t = #yrs.

e = Natural base = 2.71828128.....

Ex 3

a) How much will an investment of \$2500 earning 7% interest compounded continuously, be worth in 4 years?

$$A = 2500e^{.07(4)}$$

$$A = 2500e^{.28} = \$3307.82$$

b) How much will an investment of \$3000 earning 6% interest compounded continuously be worth in 5 years?

$$A = 3000e^{5(.06)}$$

$$3000e^{.3} \quad \$4049.58$$

c) Suppose you invest \$500 in an account at 6% interest compounded continuously. How long will it take the account to double in value? (HINT: Use calculator to explore or use graphing feature and calc intersect!)

$$1000 = 500e^{.06t}$$

$$2 = e^{.06t}$$

Calc intersect 11.55 years

d) Suppose you invest \$1000 in an account at 5% interest compounded continuously. How long will it take the account to double in value? (Use calculator - calc intersect or repeated operation)

$$2000 = 1000e^{.05t}$$

$$2 = e^{.05t}$$

13.86 years

The formula $A = A_0 e^{kt}$ is used for problems that deal with exponential growth or decay. With exponential growth or decay, quantities grow or decay at a rate directly proportional to their size. For example, populations that are growing exponentially grow very quickly as they get larger because there are more adults to have offspring.

A= Amount at time t A_0 = Initial amount   k = % change as a decimal, + → growth, - → decay   t = time

Generating Exponential Models:

Ex 4.

RADIOACTIVE DECAY

Let y = mass, in grams, or radioactive strontium, whose half-life is 29 years. The amount of strontium present after t years is modeled by the equation y = 10(1/2)^(t/29) = ______________________

A. What is the initial mass? 10 grams

B. How much of the initial mass is present after 80 years?

$y = 10(.5)^{(80/29)}$ 1.4477 g.

C. When will there be no more strontium present?

$0 = 10(.5)^\wedge(t/29)$

when you try calc zero can you get below x axis? ∴ Never → Asymptote

Solutions (4A. 10 grams)   (4B. 1.48 grams)

Ex. 5 POPULATION

The number of fruit flies in an experimental population after t hours is given by Q(t) = 20e^.03t, where t > 0.

A. What was the initial number of fruit flies? $\boxed{20}$

B. How many fruit flies will there be after 3 days?

$3 \text{ days} \cdot \frac{24 \text{ hrs}}{1 \text{ day}} = 72 \text{ hours}$

$Q(72) = 20e^\wedge .03(72)$

$\boxed{173}$

C. When will the fruit fly population reach 800?

$800 = 20e^{.03t}$

Answers A. 20   B. 173 flies

$Y_1 = 20e^{.03t}$

$Y_2 = 80$ → CALC INTERSECT

123 hours

PRACTICE:

1. Solve 20e^.05x = 1500 using a calculator

$\boxed{x = 86.35}$

2. Solve 4^2x = 8^(x + 1) without using a calculator

$2^{4x} = 2^{3x+3}$ $\boxed{x=3}$

$4x = 3x + 3$

3. Your grandparents invest $10,000 at the time of your birth into an account that pays 5% compounded continuously.

A. Write an equation to model this situation.

$A = Pe^{rt}$ $A = 10{,}000e^{.05t}$

B. If you are given the money when you graduate from college at age 22, how much will you collect?

$A = 10{,}000e^{(.05 \cdot 22)} = \$30{,}041.66$

C. How long would you need to leave your money in this account in order for the balance to reach $100,000?

$100{,}000 = 10{,}000e^{(.05t)}$

$10 = e^{(.05t)}$ Calc intersect.

46 years.

4. Let Q represent a mass, in grams, of radioactive radium whose half life is 1600 years. The quantity of radium present after t years is given by Q = 25(1/2)^(t/1600).

A. What was the initial amount of this sample?

25 grams

B. How much will be left after 1000 years?

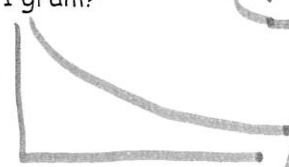
$25(.5)^{(1000/1600)} = 25(.5)^{(\frac{5}{8})}$

16.2 grams

C. How many years will it take for the mass to fall below 1 gram?

$Y_1 = 25(.5)^{t/1600}$

$Y_2 = 1$



7430 years

D. Will there ever be 0 grams of radium remaining?

NO

Asymptote

5. The CT Board of Regents has just approved a 5% tuition increase at all CT state schools. Future tuitions may be found, therefore, using the model T = P(1.05)^t. There t is the time in years, and P is the present cost of tuition. If UCONN tuition is presently $19,000, predict how much it will be 20 years from now. . .

A. Using algebraic substitution $T = 19,000(1.05)^{20} = \$59412.66$

B. Using Calc Value Graph $y_1 =$; calc value $x = 20$

C. Using the table Graph, TABLE → search $x = 20$

When will tuition be $25,000?

$y_1 = 25000$
$y_2 = 19,000(1.05)^t$

5.625 ∴ 5 or 6 years.

6. The present average price of a new car is $30,000. After t years, the car's value is given by the function,
V(t) = 30,000(4/5)^t.

A. What is the yearly percent decrease in value? (Look at the equation) 20%

B. When will the car be worth half of its original value?

$15,000 = 30,000(.8)^t$

CALC Int. 3.1 yrs

C. How much will the car be worth in 7 years?

$y = 30,000(.8)^7 = \$6291.46$

7. The projected population of California for the years 2020 through 2060 can be modeled by P = 36.308e^.0065t where P is the population in MILLIONS and t is the time in years, with t - 20 corresponding to 2020.

∴ t = 2040

A. Graph this for the years 2020 through 2060 on your calculator. Change the domain to reflect this range of years.

(-20, 20)

B. Use the table to determine the year in which the population will exceed 51 million.

x = 53 → y = 51.241 million

if t - 20 ∴ 2020 + 73 = 2093 ?

C. Use graphing to determine the year in which the population will exceed 52 million.

$y_1 =$

$y_2 = 52$

Calc intersect 55.26 years.

∴ 2095