

Identities A PRACTICE

Verify

1. $\boxed{\csc \theta \sin \theta - \sin^2 \theta} = \cos^2 \theta$

↑
rewrite!
 $\frac{1}{\sin \theta} \cdot \sin \theta - \sin^2 \theta$

$$1 - \sin^2 \theta$$

$$\cos^2 \theta$$

$$= \cos^2 \theta \quad \checkmark$$

2. $\boxed{\sin \theta (\cot \theta + \tan \theta)} = \sec \theta$

↑
rewrite!
 $\sin \theta \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right)$

$$= \cos \theta + \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta}$$

$$= \sec \theta$$

$$= \sec \theta \quad \checkmark$$

$$3. (\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$$

DISTRIBUTE \rightarrow diff. squares!

$$\csc^2 \theta - \cot^2 \theta$$

$$1 - 1 = 0 \quad \checkmark$$

$$4. \csc^4 \theta - \csc^2 \theta = \cot^4 \theta + \cot^2 \theta$$

Re-Arrange!

$$\csc^4 \theta - \cot^4 \theta = \cot^2 \theta + \csc^2 \theta$$

$$(\csc^2 \theta + \cot^2 \theta)(\csc^2 \theta - \cot^2 \theta)$$

$$(\csc^2 \theta + \cot^2 \theta)(1)$$

$$\csc^2 \theta + \cot^2 \theta = \cot^2 \theta + \csc^2 \theta \quad \checkmark$$

$$5. \csc x + \cot x = \frac{\sin x}{1 - \cos x} \cdot \frac{(1 + \cos x)}{(1 + \cos x)}$$

CONJUGATE!

$$= \frac{\sin x + \sin x \cos x}{1 - \cos^2 x}$$

$$= \frac{\sin x + \sin x \cos x}{\sin^2 x}$$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \csc x + \cot x$$

$$6. \quad 8 \csc^2 \theta - 3 \cot^2 \theta = 3 + 5 \csc^2 \theta$$

Rearrange 1st!

$$3 \csc^2 \theta = \boxed{3 \cot^2 \theta + 3}$$

$$\downarrow \quad = 3(\cot^2 \theta + 1)$$

$$\quad \quad = 3(\csc^2 \theta)$$

$$3 \csc^2 \theta = 3 \csc^2 \theta \quad \checkmark$$

$$7. \quad \boxed{\frac{1}{1 - \sec \theta} + \frac{1}{1 + \sec \theta}} = -2 \cot^2 \theta$$

Adding Fractions \rightarrow need common denominator!

$$\left[\frac{1 + \sec \theta}{1 + \sec \theta} \right] \left[\frac{1}{1 - \sec \theta} \right] + \left[\frac{1}{1 + \sec \theta} \right] \left[\frac{1 - \sec \theta}{1 - \sec \theta} \right] =$$

$$= \frac{1 + \sec \theta + 1 - \sec \theta}{(1 + \sec \theta)(1 - \sec \theta)} =$$

$$= \frac{2}{1 - \sec^2 \theta} =$$

$$= \frac{2}{-\tan^2 \theta} = -2 \cot^2 \theta = -2 \cot^2 \theta \quad \checkmark$$

