Honors Algebra 2 Name ____________________

6.3 Perform Function Operations and Composition

Operations of Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g.

Given $f(x) = x^2 + 3x - 5$ and $g(x) = 2x^2 + x - 9$, perform the indicated operations.

1. $(f+g)(x) = f(x) + g(x)$

just ADD the functions

$= (x^2+3x-5) + (2x^2+x-9)$

$\boxed{(f+g)(x) = 3x^2+4x-14}$

2. $(g-f)(x) = g(x) - f(x)$

just SUBTRACT the functions in this order!

$= (2x^2+x-9) - (x^2+3x-5)$

$= 2x^2+x-9$
$\;\; -x^2-3x+5$ $= \boxed{x^2-2x-4}$

3. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

just DIVIDE the functions (Reduce if possible)

$\boxed{\left(\frac{f}{g}\right)(x) = \frac{x^2+3x-5}{2x^2+x-9}}$

4. $(g \cdot f)(x) = g(x) \cdot f(x)$

just MULTIPLY the functions

$= (2x^2+x-9) \cdot (x^2+3x-5)$

$= 2x^4+6x^3-10x^2+x^3+3x^2-5x-9x^2-27x+45$

$\boxed{(g \cdot f)(x) = 2x^4+7x^3-16x^2-32x+45}$

We can also evaluate any operations of two functions by one of two methods:

- by evaluating f and g separately then performing the operation with the outputs
- Or perform the operation with f and g, then evaluate the resulting function

5. $(f-g)(0)$

$f(0) - g(0)$

$0^2+3(0)-5 = -5$

$2(0)^2+0-9 = -9$

$-5-(-9)$

$= -5+9$

$= 4$

$(f-g)(x)$

$(x^2+3x-5)-(2x^2+x-9)$

$= x^2+3x-5-2x^2-x+9$

$= -x^2+2x+4$

$-(0)^2+2(0)+4$

$= 4$

6. $\left(\frac{g}{f}\right)(1)$

$= \frac{g(1)}{f(1)} = \frac{2(1)^2+1-9}{(1)^2+3(1)-5} = \frac{2+1-9}{1+3-5} = \frac{-6}{-1}$

$= 6$

7. $(g+f)(-4)$

$= g(-4) + f(-4)$

$= [2(-4)^2+(-4)-9] + [(-4)^2+3(-4)-5]$

$= [32-4-9] + [16-12-5]$

$= 19 + (-1)$

$= 18$

8. $(g \cdot f)(3)$

$= g(3) \cdot f(3)$

$= (2 \cdot 3^2+3-9) \cdot (3^2+3(3)-5)$

$= (18+3-9) \cdot (9+9-5)$

$= (12)(13)$

$= 156$

Composition of Functions: the process of combining two or more functions with ***substitution*** to create a new function. The **composition** of a function g with a function f is $g(f(x))$, which can also be written as $[g \circ f](x)$. This is read aloud as "g of f of x".

→ put f(x) INTO g(x)

To Evaluate $g(f(x))$:

1. Find the $f(x)$ value (the output of function f).
2. Put the $f(x)$ value into $g(x)$ to find $g(f(x))$.

** The ***OUTPUT*** of $f(x)$ becomes the ***INPUT*** of $g(x)$. **

input output x y

Given $f = \{(1,2)\ (3,4)\ (5,4)\}$ and $g = \{(2,3)\ (4,3)\ (6,1)\}$, evaluate the composition.

9. $[f \circ g](6)$ find $g(6) = 1$ $= f(g(6))$ then find $f(1) = 2$ $= f(1) = 2$ $[f \circ g](6) = 2$	10. $g(f(3))$ $f(3) = 4$ $g(4) = 3$

Given $f(x) = x + 7$, $g(x) = x^2 - 3x + 6$, $h(x) = 4x - 1$, evaluate or perform the composition.

11. $[h \circ g](2) = h(g(2))$ $g(2) = 4 - 6 + 6 = 4$ $= h(4)$ $= 4(4) - 1$ $= 16 - 1$ $= 15$	12. $h(f(-10))$ $f(-10) = -10 + 7 = -3$ $= h(-3)$ $= 4(-3) - 1$ $= -12 - 1$ $= -13$
13. $h(g(x))$ $g(x) = x^2 - 3x + 6$ $= h(x^2 - 3x + 6)$ $= 4(x^2 - 3x + 6) - 1$ $= 4x^2 - 12x + 24 - 1$ $= 4x^2 - 12x + 23$	14. $[g \circ h](x) = g(h(x))$ $= g(4x - 1)$ $= (4x-1)^2 - 3(4x-1) + 6$ $= 16x^2 - 8x + 1 - 12x + 3 + 6$ $= 16x^2 - 20x + 10$

15. $h(f(g(x))) = h(f(x^2 - 3x + 6))$

$= h(x^2 - 3x + 6 + 7)$

$= h(x^2 - 3x + 13)$

$= 4(x^2 - 3x + 13) - 1$ $= 4x^2 - 12x + 51$