Lesson #3 Geometric Sequences Practice:

Key

The formula for the nth term of a GEOMETRIC SEQUENCE:

$$a_n = a_1 \cdot r^{(n-1)}$$

1) Find the next 3 terms of each geometric sequence. Then, write the general term, a_n.

a) -15, 30, -60,... $a_n = -15(-2)^{(n-1)}$ $a_n =$ ______

$a_1 = -15$

$r = -2$

-15, 30, -60, 120, -240, 480

b) $10, 2, \frac{2}{5}, \ldots$ $\frac{2}{25}, \frac{2}{125}, \frac{1}{300}$ $a_n = 10\left(\frac{1}{5}\right)^{(n-1)}$

$a_1 = 10$

$r = \frac{1}{5}$

2) Find the specified terms of each geometric sequence and write the rule for the nth term

a) $a_1 = 7, \quad r = -3 \quad a_7 =$ 5103

$a_n = a_1 \cdot r^{(n-1)}$

$a_7 = 7 \cdot (-3)^{7-1}$

$a_7 = 7(-3)^6$

Rule: $a_n : 7(-3)^{n-1}$

b) $a_1 = 144, \quad r = \frac{1}{4} \quad a_5 =$ 9/16

$a_n = a_1 \left(\frac{1}{4}\right)^{n-1}$

$a_n = 144\left(\frac{1}{4}\right)^{n-1}$

$a_5 = 144\left(\frac{1}{4}\right)^4 =$

Rule: $a_n = (144)\left(\frac{1}{4}\right)^{(n-1)}$

$a_5 = 144\left(\frac{1}{4}\right)^4$

$= \frac{3 \cdot 3}{4 \cdot 4} = 9/16 \quad 12 \cdot 12 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$

c) $a_9 = 6561, \quad r = 3 \quad a_3 =$ 9

$(9, 6561) \quad r = 3$

$a_n = a_1 \cdot r^{(n-1)}$

$6561 = a_1 (3)^8$

$1 = a_1$

$a_3 = 9$

Rule: $a_n = 1(3)^{n-1}$

d) $a_7 = 3645, \quad r = 3 \quad a_2 =$ 15

$(7, 3645) \quad r = 3$

$a_n = a_1 \cdot r^{(n-1)}$

$3645 = \cancel{a7}\,(3)^{(\cancel{7}-1)}$

$3645 = a_1 (3)^6$

$3645 = 729 a_1$

$\frac{3645}{729}$ $\quad 5 = a_1$

Rule: $a_n = 5(3)^{n-1}$

3) Find a_1 and r for each geometric sequence. Then write the rule for the nth term.

a) $a_2 = 10 \quad a_6 = 160 \quad a_1 =$ ______ $r =$ ______

$a_2 = a_1 \cdot r^{n-1}$

$10 = a_1 \cdot r$

$\frac{10}{r} = a_1$

$160 = a_1 \cdot r^5$

$160 = \frac{10}{r} \cdot r^5$

$160 = 10r^4$

$16 = r^4$

$\pm 2 = r$

$a_1 = \frac{10}{\pm 2} = \pm 5$

Rule: $a_n = 5(2)^{n-1}$

or $a_n = -5(-2)^{n-1}$

b) $a_3 = 45 \quad a_5 = 405 \quad a_1 =$ ______ $r =$ ______

$45 = a_1 \cdot r^2$

$a_1 = \frac{45}{r^2}$

$405 = a_1 \cdot r^4$

$405 = \frac{45}{r^2} \cdot r^4$

$405 = 45r^2$

$9 = r^2$

$r = \pm 3$

$a_1 = \frac{45}{9}$

$a_1 = 5$

Rule: $a_n = 5(3)^{(n-1)}$

or $a_n = 5(-3)^{(n-1)}$

c) $a_5 = 16 \quad a_8 = 2 \quad a_1 =$ ______ $r =$ ______

$16 = a_1 \cdot r^4$

$a_1 = \frac{16}{r^4}$

$2 = a_1 \cdot r^7$

$2 = \frac{16}{r^4} \cdot r^7$

$a_1 = \frac{16}{r^4}$

$2 = 16r^3$

$\frac{1}{8} = r^3$

$r = \frac{1}{2}$

$a_1 = 16 \div \frac{1}{16}$

Rule: $a_n = 256\left(\frac{1}{2}\right)^{(n-1)}$

d) $a_7 = 6 \quad a_{11} = \frac{3}{8} \quad a_1 =$ ______ $r =$ ______

$6 = a_1 r^6$

$a_1 = \frac{6}{r^6}$

$\frac{3}{8} = a_1 r^{10}$

$\frac{3}{8} = \frac{6}{r^6} \cdot r^{10}$

$\frac{3}{8} = 6r^4$

$\frac{3}{8} \cdot \frac{1}{6} = r^4$

$\frac{1}{16} = r^4$

$r = \pm\frac{1}{2}$

$a_1 = 6 \div \frac{1}{2^6}$

$6 \div \frac{1}{64} = a_1$

$384 = a_1$

Rule: $a_n = 384(\pm\frac{1}{2})^{n-1}$

Arithmetic Sequences – Quick Check/Warm Up after L1 Name: Key

A sequence is arithmetic if there is a common difference.

Arithmetic sequences are discrete rather than continuous, but are similar to ______________ fns.

1. Find the indicated term and an explicit equation the general term, a_n.

1, -6, -13, -20,... $a_n =$ $-7n+1$ $a_{17} =$ -118

$d = -7$

$a_1 = -6$

n	1	2	3
a_n	-6	-13	-20

$a_{17} = -7(17) + 1 = -70 - 49 = -119 + 1$

$a_n = a_1 + d(n-1)$

$a_n = -6 - 7(n-1) = -6 - 7n + 7 = -7n + 1$

2. Find a_1 and d for each arithmetic sequence. Then write the explicit formula for the sequence

$a_3 = -5$ $a_6 = 16$ $a_1 =$ -19 $d =$ 7

$(3, -5)$ $(6, 16)$ OR

n	1	2	3	4	5	6
a_n			-5			16

$d = \frac{16 - -5}{6-3} = 7$

$a_n = -19 + 7(n-1)$

$a_n = -19 + 7n - 7 = 7n - 26$

To find a_1: Use $a_n = a_1 + 7(n-1)$ &

pick one point $(3, -5)$

$-5 = a_1 + 7(3-1)$

$-5 = a_1 + 14$ $\boxed{-19 = a_1}$

3. In the arithmetic sequence 17, 22, 27, 32, ..., which term is 232?

$a_n = 5n + 12$

$a_1 = 17$

$d = 5$

$a_n = 232$

find n

$a_n = a_1 + d(n-1)$

$232 = 17 + 5(n-1)$

$232 = 17 + 5n - 5$

$232 = 5n + 12$

$220 = 5n$ $n = \frac{220}{5} = 44^{th}$

4. In the arithmetic sequence $\frac{9}{4}, 2, \frac{7}{4}, \ldots$, which term is $-\frac{17}{4}$?

$2\frac{1}{4},\ 2\ \ 1\frac{3}{4}$

$d = -\frac{1}{4}$

$a_1 = \frac{9}{4}$

Rule:

$a_n = a_1 + d(n-1)$

$a_n = \frac{9}{4} + -\frac{1}{4}(n-1)$

$a_n = \frac{9}{4} - \frac{1}{4}n + \frac{1}{4}$

$a_n = \frac{5}{2} - \frac{1}{4}n$

$-\frac{17}{4} = \frac{10}{4} - \frac{1}{4}n$

$-\frac{27}{4} = -\frac{1}{4}n$

$n = 27$

27th term

5. The last term of an arithmetic sequence is 207, the common difference is 3, and the number of terms is 14. What is the first term of the sequence?

$d = 3$

$n = 14$

$a_{14} = 207$

$\left.\begin{matrix} n = 14 \\ a_{14} = 207 \end{matrix}\right\} (14, 207)$

$a_n = a_1 + d(n-1)$

$207 = a_1 + 3(14-1)$

$207 = a_1 + 3(13)$

$207 = a_1 + 39$

$168 = a_1$

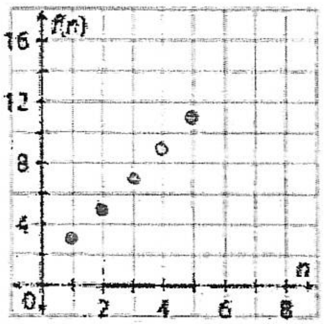
Honors Algebra 2 Name Key Per. ____

12.1 & 12.2 Practice Worksheet #2

Arithmetic & Geometric Sequences (Explicit Rules and Finding n^{th} terms)

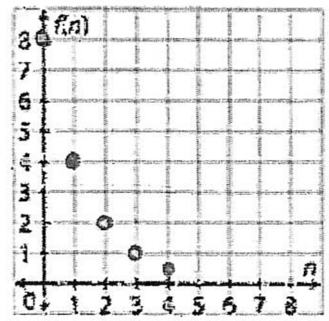
Use the explicit rule to write the first five terms for the sequence. Then graph the sequence.

1. $f(n)=3+2(n-1),\ 1\le n\le 5$



$\boxed{3, 5, 7, 9, 11}$

2. $f(n)=8\cdot\left(\frac{1}{2}\right)^n,\ 0\le n\le 4$



$\boxed{8, 4, 2, 1, \frac{1}{2}}$

Write an explicit rule for each sequence.

3. 4, 7, 10, 13, 16, 19, 22

Arithmetic $d=3$

$\boxed{f(n)=4+3(n-1),\ 1\le n\le 7}$

or

$\boxed{f(n)=4+3n,\ 0\le n\le 6}$

4.

n	1	2	3	4	5	6
$f(n)$	11	5	−1	−7	−13	−19

Arithmetic $d=-6$

$\boxed{f(n)=11-6(n-1),\ 1\le n\le 6}$

5.

n	0	1	2	3	4	...
$f(n)$	100	10	1	0.1	0.01	...

Geometric $r=\frac{1}{10}$ or $r=0.1$

$\boxed{f(n)=100\left(\frac{1}{10}\right)^n,\ n\ge 0}$

6. $\frac{1}{4}$, 1, 4, 16, 64, ...

Geometric $r=4$

$\boxed{f(n)=\frac{1}{4}(4)^{n-1},\ n\ge 1}$

or

$\boxed{f(n)=\frac{1}{4}(4)^{n},\ n\ge 0}$

7. 9, −6, 4, $-\frac{8}{3}$, ...

Geometric $r=-\frac{2}{3}$

$\boxed{f(n)=9\left(-\frac{2}{3}\right)^{n-1},\ n\ge 1}$

or

$\boxed{f(n)=9\left(-\frac{2}{3}\right)^{n},\ n\ge 0}$

8.

n	0	1	2	3	4
$f(n)$	−6	1	8	15	22

Arithmetic $d=7$

$\boxed{f(n)=-6+7n,\ 0\le n\le 4}$

Write an explicit rule and then find the indicated term of the sequence.

9. $\frac{1}{2}, 4, \frac{15}{2}, 11, \frac{29}{2}, \ldots$

Find the 82nd term.

Arithmetic

$d = 3.5$ or $\frac{7}{2}$

$\boxed{f(n) = \frac{1}{2} + \frac{7}{2}(n-1),\ n \geq 1}$

$\boxed{\text{82nd term is 284}}$

$f(82) = \frac{1}{2} + \frac{7}{2}(82-1)$

$= \frac{1}{2} + \frac{7}{2}(81)$

$= 284$

or $f(n) = \frac{1}{2} + \frac{7}{2}n,\ n \geq 0$

$f(81) = \frac{1}{2} + \frac{7}{2}(81)$

10. The sixth term of an arithmetic sequence is 87 and the twelfth term is 129. Find the 120th term.

OR: $d = \frac{129-87}{12-6} = 7$

$f(n) = f(1) + d(n-1),\ n \geq 1$

$f(6) = 87$

$f(12) = 129$

$\boxed{f(n) = 52 + 7(n-1),\ n \geq 1}$

$87 = f(1) + d(6-1)$

$129 = f(1) + d(12-1)$

$87 = f(1) + 5d$

$129 = f(1) + 11d$

$87 - 5d = f(1)$

$129 = 87 - 5d + 11d$

$42 = 6d$

$7 = d$

$f(120) = 52 + 7(120-1)$

$= 885$

$87 - 5(7) = f(1)$

$52 = f(1)$

$\boxed{\text{120th term is 885}}$

11. The second term of a geometric sequence -18 and the fifth term is $\frac{2}{3}$. Find the sixth term.

$f(n) = f(1) \cdot r^{n-1},\ n \geq 1$

$f(2) = -18$

$-18 = f(1) \cdot r^{2-1}$

$-18 = f(1) \cdot r$

$\frac{-18}{r} = f(1)$

$f(1) = \frac{-18}{-1/3} = -54$

$f(5) = \frac{2}{3}$

$\frac{2}{3} = f(1) \cdot r^{5-1}$

$\frac{2}{3} = f(1) \cdot r^4$

$\frac{2}{3} = \frac{-18}{r} \cdot r^4$

$\frac{2}{3} = -18r^3$

$-\frac{1}{27} = r^3$

$-\frac{1}{3} = r$

$\boxed{f(n) = -54\left(-\frac{1}{3}\right)^{n-1},\ n \geq 1}$

$\boxed{\text{6th term is } -\frac{2}{9}}$

12. 3, −9, 27, −81, 243, ...

Find the 20th term.

Geometric

$r = -3$

$\boxed{f(n) = 3(-3)^{n-1},\ n \geq 1}$

$f(20) = 3(-3)^{20-1}$

$= -3{,}486{,}784{,}401$

$\boxed{\text{20th term is } -3{,}486{,}784{,}401}$