FINAL REVIEW - PRECALC - Name: ______________________________
UNIT 5 - Chapter 4.1 - 4.4

Pre-Calculus FINAL EXAM REVIEW 2017
Chapter 4A
Learning Objectives

Section 4.1

1. I can sketch a positive or negative rotation and find co-terminal angles and the reference angle.

NOTES:

Determine the quadrant in which each angle lies and find a positive and a negative coterminal angle.

a. $\theta = \frac{-5\pi}{6}$ b. $\theta = \frac{7\pi}{4}$ c. $\theta = 2.5$

d. $\theta = \frac{11\pi}{3}$ e. $\theta = \frac{-13\pi}{4}$ f. $\theta = 420^o$

Determine the quadrant in which each angle lies and find its reference angle).

g. $\theta = \frac{5\pi}{6}$ h. $\theta = \frac{5\pi}{4}$ i. $\theta = 3$

2. I can convert between degrees/radians

NOTES:

Convert the following angle measures from degrees to radians in terms of pi.

a. 150° b. 70°

Convert the following angle measures from radians to degrees.

d. $\frac{5\pi}{7}$ e. $\frac{12\pi}{5}$ f. -5.5

]

3. I can define radians in terms of arc length and radius and solve for unknowns.
ARC LENGTH = RADIUS (RADIANS)

Find the length of the arc intercepted by a central angle with the given radius.

a. $\theta = \frac{5\pi}{6}$ $r = 3$ *inches* b. $\theta = 173^o$ $r = 12$ *feet*

4. I can solve problems involving angular and linear speed using unit analysis.

NOTES:

a. The cylindrical roller on highway roller has a 48 inch diameter and makes .7 revolutions per second. Find the angular speed of the roller in radians per second and find the linear speed of the roller.

b. The tire on a car has a radius of 16 inches and is spinning at a rate of 4 revolutions per second. Find the angular speed of the roller in radians per second and find the linear speed in mph.

Section 4.2

5. I can identify the angles (degree and radian) and the (x, y) coordinate on the unit circle.

a. Draw a unit circle and complete the important points – degree, radian, and (x, y) points.

b. Evaluate exactly $\cos\frac{\pi}{3} + \tan\frac{2\pi}{3} + \sin\frac{5\pi}{6}$.

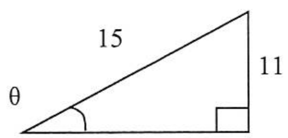
c. **Evaluate:** $\sin\frac{\pi}{2} \bullet \sec\frac{5\pi}{6} - \cot(-\frac{3\pi}{4})$

Section 4.3

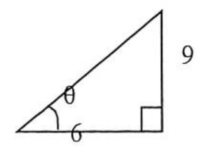
8. I can use a triangle and 2 given sides to evaluate the six trig functions.
NOTES: (SOH – CAH – TOA)

Find the exact values of the six trigonometric functions of the angle θ.

a.



b.



10. I can use inverse trig functions to find Θ in both radians and degrees by memory or with a calculator.

Evaluate exact values for θ when possible, otherwise use a calculator. Give both the degree & radian measure. Assume θ is in the first quadrant.

a. $\sin(\theta) = \frac{\sqrt{3}}{2}$ b. $\cos(\theta) = \frac{1}{2}$ c. $\tan(\theta) = \frac{\sqrt{3}}{3}$

d. $\sin(\theta) = \frac{\sqrt{2}}{3}$ e. $\cos(\theta) = \frac{1}{4}$ f. $\tan(\theta) = \frac{17}{2}$

11. I can evaluate trig functions at a given angle by memory or with a calculator.

Find exact values for θ when possible, otherwise use a calculator. Assume θ is in the first quadrant.

a. $\csc(120^\circ)$ c. $\tan\left(\frac{5\pi}{6}\right)$

Section 4.4:

12. I can determine the six trig functions exact value given a point on the terminal side of an angle in standard position.

a. Given the point (5, -7) on the terminal side of an angle, determine the six trig functions.

13. I can evaluate trig values given one value and other information.

a. Given $\sin\theta = \frac{3}{4}$ and $\cos\theta < 0$, evaluate $\tan\theta$ and $\sec\theta$.

b. Given $\tan\theta = \frac{7}{4}$ and $\sec\theta < 0$, evaluate $\sin\theta$ and $\cos\theta$.

c. Given $\sin\theta = \frac{3}{5}$ and θ is in Quadrant II, evaluate $\cos\theta$ and $\sec\theta$.

d. Given $\tan\theta = \frac{-5}{3}$ and θ is in Quadrant IV, evaluate $\sin\theta$ and $\sec\theta$.

Unit 2 Practice Problems

1. Convert the angle measure from degrees to radians or from radians to degrees. (calc)

a. 115° ____________

b. $\frac{13\pi}{2}$ ____________

3. Determine two coterminal angles (one positive and one negative) for each angle. Give your answer in radians. (calc)

a. $\theta = \frac{\pi}{12}$ __________ & __________

b. $\theta = -435°$ __________ & __________

4. Find the reference angle and determine which quadrant θ lies. (calc)

a. θ= 203° ____________

Quadrant: ______

b. θ= -245° ____________

Quadrant: ______

c. $\theta = \frac{2\pi}{3}$ ____________

Quadrant: ______

d. $\theta = -\frac{13\pi}{3}$ ____________

Quadrant: ______

5. Find the radian measure of the central angle of a circle if the radius = 14.5 centimeters and the arc length = 25 centimeters. (calc)

____

6. Find the length of the arc on a circle of radius r intercepted by a central θ.(calc)

a. radius = 15 inches, central angle θ= 180° ____________

b. radius = 20 centimeters, central angle $\theta = \frac{\pi}{4}$ radians ____________

7. The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle. (calc)

a. (7, 24)

sin α__________ csc α________

cos α_________ sec α________

tan α_________ cot α________

b. (-5,-2)

sin α__________ csc α________

cos α_________ sec α________

tan α_________ cot α________

8. State the quadrant in which θ lies. (no calc)

a. sin $\theta > 0$ and cos $\theta > 0$ ______________

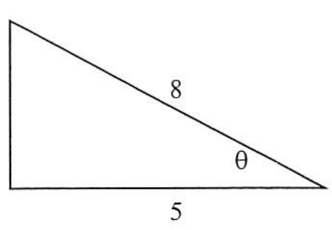
b. sec $\theta > 0$ and cot $\theta < 0$ ____________

9. A carousel with a 50-foot diameter makes 4 revolutions per minute. What is the angular velocity in radians per hour? What is the linear velocity in inches per hour? (calc)

Angular Velocity: _____________

Linear Velocity: ______________

10. Find the 6 trig functions for θ in the triangle below. Assume the triangle is a right triangle.



EVEN MORE PRACTICE - 4.1 – 4.4

41. A circle has a radius of 7 inches. Find the **length of the arc** intercepted by a central angle of $240°$.

41.)_______________

42. The circular blade on a saw rotates at 2400 revolutions per minute.

a. Find the **angular speed** in radians per second.

42a.)_______________

b. The blade has a radius of 4 inches. Find the **linear speed** of a blade tip in inches per second.

42b.)_______________

43. A satellite in circular orbit 1125 km above a planet makes one complete revolution every 120 minutes. Assuming that he planet is a sphere of radius 6400 km, find the linear speed of the satellite in **kilometers per minute**. Round your answer to the nearest whole number.

43.)_______________

44. A truck is moving at a rate of 90 km per hour and the diameter of its wheels is 1.25 meters. Find the angular speed of the wheels in **radians per minute**.

44.)_______________

45. **Evaluate** (if possible) the six trigonometric functions if $\theta = -\frac{2\pi}{3}$.

45.

$\sin\left(-\frac{2\pi}{3}\right) =$ $\quad$ $\csc\left(-\frac{2\pi}{3}\right) =$

$\cos\left(-\frac{2\pi}{3}\right) =$ $\quad$ $\sec\left(-\frac{2\pi}{3}\right) =$

$\tan\left(-\frac{2\pi}{3}\right) =$ $\quad$ $\cot\left(-\frac{2\pi}{3}\right) =$

46. **Evaluate** the trigonometric function.

a. $\sin\left(-\frac{3\pi}{4}\right)$ $\quad$ b. $\csc\left(\frac{7\pi}{6}\right)$ $\quad$ c. $\tan\left(\frac{5\pi}{3}\right)$

46a.)_______________

46b.)_______________

46c.)_______________

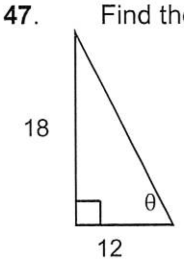
d. $\sec(-4\pi)$ **e.** $\cot\left(\frac{5\pi}{2}\right)$ **f.** $\cos\left(\frac{13\pi}{4}\right)$

46d.)______________

46e.)______________

46f.)______________

47. Find the **exact values** of the **six trigonometric functions** of the angle θ shown in the figure.



47.

$\sin(\theta)=$ $\csc(\theta)=$

$\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

53. Use the given value and the trigonometric identities to **find the remaining trigonometric functions** of the angle.

$\cos\theta = -\frac{3}{7}, \quad \sin\theta < 0$

53.

$\sin(\theta)=$ $\csc(\theta)=$

$\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

54. The point is on the terminal side of an angle in standard position. **Determine** the **exact values** of the six trigonometric functions of the angle.

$(8, -15)$

54.

$\sin(\theta)=$ $\csc(\theta)=$

$\cos(\theta)=$ $\sec(\theta)=$

$\tan(\theta)=$ $\cot(\theta)=$

KEY TO EVEN MORE PRACTICE:

41. $\frac{28\pi}{3}$ inches

42a. 80π rad/sec

42b. ≈ 1005 in/sec

43. 394 km/min

44. 2400 rad/min

45.
$\sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\csc\left(-\frac{2\pi}{3}\right) = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\sec\left(-\frac{2\pi}{3}\right) = -2$

$\tan\left(-\frac{2\pi}{3}\right) = \sqrt{3}$ $\cot\left(-\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$

46a. $-\frac{\sqrt{2}}{2}$

46b. -2

46c. $-\sqrt{3}$

46d. 1

46e. 0

46f. $-\frac{\sqrt{2}}{2}$

47.
$\sin(\theta) = \frac{3\sqrt{13}}{13}$ $\csc(\theta) = \frac{\sqrt{13}}{3}$

$\cos(\theta) = \frac{2\sqrt{13}}{13}$ $\sec(\theta) = \frac{\sqrt{13}}{2}$

$\tan(\theta) = \frac{3}{2}$ $\cot(\theta) = \frac{2}{3}$

53.
$\sin(\theta) = -\frac{2\sqrt{10}}{7}$ $\csc(\theta) = -\frac{7\sqrt{10}}{20}$

$\cos(\theta) = -\frac{3}{7}$ $\sec(\theta) = -\frac{7}{3}$

$\tan(\theta) = \frac{2\sqrt{10}}{3}$ $\cot(\theta) = \frac{3\sqrt{10}}{20}$

54.
$\sin(\theta) = -\frac{15}{17}$ $\csc(\theta) = -\frac{17}{15}$

$\cos(\theta) = \frac{8}{17}$ $\sec(\theta) = \frac{17}{8}$

$\tan(\theta) = -\frac{15}{8}$ $\cot(\theta) = -\frac{8}{15}$

Pre-Calculus UNIT 6
Chapter 4.5-4.7
Learning Objectives

Section 4.5

1. I can sketch a graph of a sine or cosine function that has been stretched horizontally/vertically, translated horizontally/vertically, and/or reflected. I can also state the domain and range of the function.

   Please remember, Sine & Cosine graphs should have 5 key points labeled on a period
   Remember to write in factored form if needed!

   MODELS: ____

   Steps:

   RECALL:

   a. Sketch $y = -3\sin(2x - 2\pi)$

   b. Sketch $y = 2\sin\left(\frac{1}{2}x + \pi\right) - 2$

3. I can write the equation of the trig graph based on its graph, given a max and min, or given a set of data. I can express the equation as a sine function and a cosine function.

   a. Find an equation of a sine wave with a peak of 12 and a minimum of 6, starts its cycle at 3π and completes one full cycle every 4π units.

4. I can use sine and cosine functions to model real life data. I can use models to make predictions.

   a. The water level in a city water storage tank oscillates in a simple harmonic motion. The water level varies depending on the time of day and the corresponding demand of the people. The low point of the water in the tank, 22 feet, occurs at 8am and 8pm when demand is highest. The high points occur at 2am and 2pm with a water level of 58 feet. Create a sinusoidal function that models the data and use it to predict the water height at 4pm.

5. I can state the domain and range

Section 4.7

8. I can evaluate inverse trig functions from memory or by using my calculator. I understand the restrictions on each trig function. . .

NOTES: SIN COS

a. $\arctan\left(\frac{\sqrt{3}}{3}\right)$

b. $\arcsin\left(-\frac{1}{2}\right)$

c. $\arcsin(2)$

d. $\arctan(-1)$

9. I can use properties of inverse trig functions to evaluate expressions.

a. sin(arcsin 1)

b. cos(arcos .3)

c. $\arctan(\tan \pi)$

10. I can find the exact value or an algebraic expression for a trig expression by using the "triangle technique."

a. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$

b. $\sin\left(\arctan\left(\frac{5}{6}\right)\right)$

c. $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$

Practice Problems

1. Graph the following trigonometric functions. (no calc)

a. $y = 3\cos\left(2x - \frac{\pi}{4}\right) - 1$ Factored form: ______________________

a = ________ b = __________

c = ________ d = __________

Amp.________ Per________

P.S.________ V.S.________

DOMAIN:

RANGE:

b. $y = -3\text{Sin}[\frac{\pi}{2}(x+2)] + 10$

a = ________ b = __________

c = ________ d = __________

Amp.________ Per________

P.S.________ V.S.________

DOMAIN

RANGE

3. A ferris wheel has a diameter of 30 meters. The center of the wheel is 18 feet off the ground. It makes two revolutions every MINUTE (60 seconds).

A. Sketch the trig graph of one complete cycle, assuming the rider gets on at the lowest point.

B. Find the cosine equation of the graph.

C. What is the height of the rider 52 seconds after he gets on the ride?

***D. At what times is the rider 20 meters above the graund?

4. One of the largest ferris wheels ever built is in the British Airways London Eye which was completed in 2000. The diameter is 135 m and passengers get on at the bottom 4 m above the ground. The wheel rotates once every three minutes.

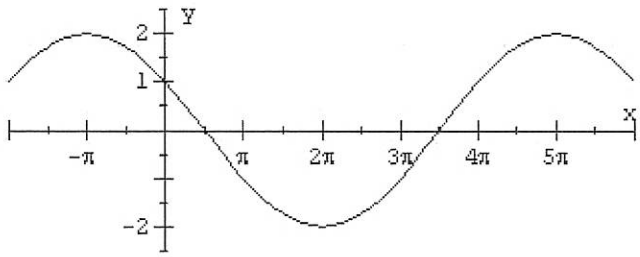
a) Draw a graph which represents the height of a passenger in metres as a function of time in minutes.

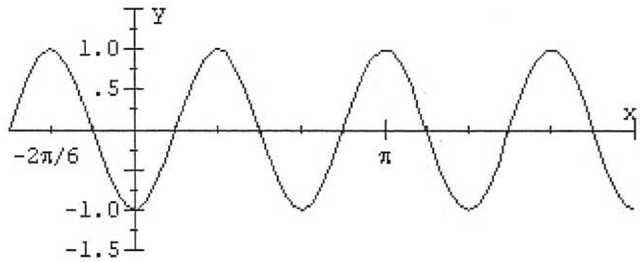
b) Determine the equation that expresses your height h as a function of elapsed time t

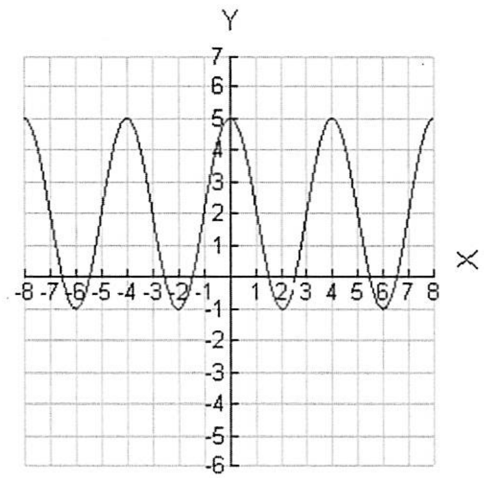
c) How high is a passenger 5 minutes after the wheel starts rotating?

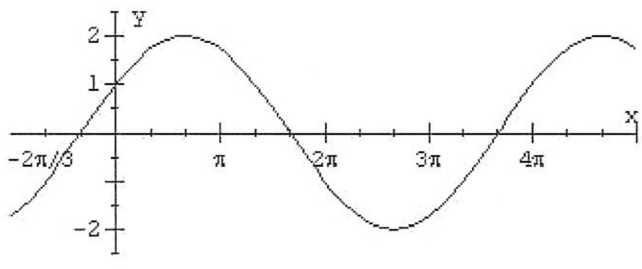
*d) How many seconds after the wheel starts rotating is a passenger 85 m above the ground for the first time. Answer to the nearest tenth.

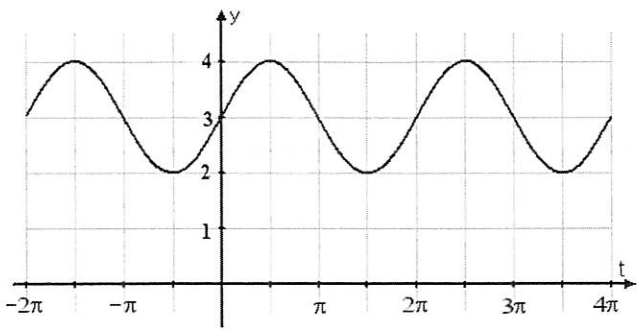
5. Write the equation of the graphs represented below. Where possible, write a sine and cosine function.











6. The temperature in an office is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function:

$$y = 19 + 6\sin\left(\frac{\pi}{12}(x - 11)\right)$$

where *y* is the temperature (ºC) and *x* is the time in hours past midnight.

a.) What is the temperature in the office at 9 A.M. when employees come to work?

b.) What are the maximum and minimum temperatures in the office?

C). By how much do the max and min temperatures vary?

7. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125 ° occurs 15 minutes after they start their examination. A minimum temperature of 99 ° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period and an equation of temperature as a function over time in minutes.

8. The height of the water in a bay varies sinusoidally over time. On a certain day off the coast of Maine, a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).

Practice Problems

Objectives: Evaluating trig function inverses, simplifying trig expressions, verifying, solving trig equations.

1. Find the exact value of each expression. Do not use a calculator. Remember the quadrant rules for inverses - COS Q1 and Q2, SIN Q1 and Q4.

a) $Tan^{-1}1$ ______________ b) $Cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ______________ c) $Sec^{-1}\sqrt{2}$ ______________

2. Find the exact value, if any, of each composite function.

a) $Sin^{-1}\left(\sin\frac{3\pi}{8}\right)$ ______________ b) $\tan\left(Sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ ______________ c) $\sin\left(Cot^{-1}\frac{3}{4}\right)$ ______________

3. **Simplify** each expression.

a. $\dfrac{\sin x\cos x}{1-\cos^2 x}$

b. $(\sin x+\cos x)^2+(\sin x-\cos x)^2$

3 a.)______________ b.)______________

c. $\dfrac{\tan^2 x}{\sec x+1}+1$

d. $\sec x-\sin x\tan x$

3c.)______________ 3d.)______________

4. **Solve.** Give your answers in degrees $(0° \le \theta < 360°)$ and radians $(0 \le \theta < 2\pi)$ without using a calculator.

a. $\cot\theta = -\frac{\sqrt{3}}{3}$

b. $\sec\theta = \sqrt{2}$

4a.)____________________

4b.)____________________

5. Verify each identity.

a) $\tan\theta\cot\theta - \sin^2\theta = \cos^2\theta$

b) $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\csc\theta$

c) $\frac{1-\sin\theta}{\sec\theta} = \frac{\cos^3\theta}{1+\sin\theta}$

d) $\frac{\tan x - \sin x\cos x}{\sin^2 x} = \tan x$

e) $\frac{1}{1-\sin x}+\frac{1}{1+\sin x}=2\sec^2 x$

f) $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$

g) $\frac{1+\sin x}{\cos x}=\sec x+\tan x$

h). $\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$
DO NOT CROSS MULTIPLY

6. Solve each equation on the interval $0 \le \theta < 2\pi$.
Strategies: linear equation, plus or minus square root, "u", GCF, factoring, replacement, square both sides

a) $2\sin^2\theta-3\sin\theta+1=0$ ____________________

b.) $3\sin x-2=5\sin x-1$ ____________________

c.) $5\sin x = 3\sin x + \sqrt{3}$ ____________

d). tan3x = 1 ______________________________

e.) $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$ ____________________

f) $2\sin^2 x - 3\sin x + 1 = 0$

g.) $\tan x \sin^2 x = 3\tan x$

h.) $2\cos^2 x + 3\sin x = 0$

i). sinx - cosx = 1

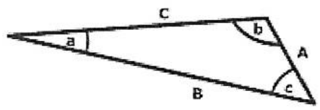
j.) $2\sec^2 x = 4$

UNIT 8

TOPIC 1: Law of Sines, Law of Cosines, Heron's Formula, Area formula, ambiguous case, angles of elevation and depression, applications of trig.

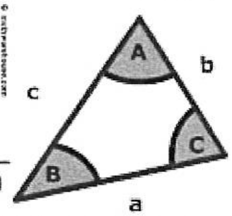
ASA SAA SSA – check ambiguous case! **SAS or SSS**

$$\frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$$



$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$$

Law of Cosines



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos(c)$$

Heron's Formula for area of a triangle given SSS

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

Area of a triangle given SAS

$$A_{\triangle} = \frac{1}{2}ab\sin C$$

Check for ambiguous case when you have SSA!
SSS – find angles in order, largest to smallest or smallest to largest.

1. Solve each triangle using the Law of Sines or the Law of Cosines. It may help to draw a picture. (Hint: Remember the ambiguous case!).

a. B = 10°, C = 20°, c = 33

A = ______________

b = ______________

a = ______________

b. B = 150°, a = 10, b = 3

A = ______________

C = ______________

c = ______________

c. $a = 2.5$, $b = 5.0$, $c = 4.5$

A = ______________

B = ______________

C = ______________

d. $B = 110°$, $a = 4$, $c = 4$

A = ______________

C = ______________

b = ______________

e. $B = 25°$, $a = 6.2$, $b = 4$

A = ______________

C = ______________

c = ______________

2. Find the area of each given triangle.

a. $B = 60°$, $a = 4$, $c = 8$ ________________

b. $a = 15$, $b = 8$, $c = 10$ ______________

3. You are skiing down a mountain with a vertical height of 1500 feet. The distance from the top of the mountain to the base is 3000 feet. What is the angle of elevation from the base to the top of the mountain?

4. A steel cable zip-line is being constructed for a competition on a reality television show. One end of the zip-line is attached to a platform on top of a 150-foot pole. The other end of the zip-line is attached to the top of a 5 foot stake. The angle of elevation of the platform is 23°.

a) How long is the zip-line?

b) How far is the stake from the pole?

5. To approximate the length of a marsh, a surveyor walks 425 meters from point A to point B. Then the surveyor turns 65° and walks 300 meters to point C. Approximate the length of AC of the marsh.

6. From a certain distance, the angle of elevation to the top of a building is 17°. At a point 50 meters closer to the building, the angle of elevation is 31°. Approximate the height of the building.

7.] Given triangle ABC with a = 10 inches, b = 8 inches, and c = 12 inches, find the measure of angle B.

10.] A large flagpole stands at the top of the Smythe Office Building. From the street, the angle of elevation to the top of the flagpole is 55°. The angle of elevation to the bottom of the flagpole is 50°. Find the height of the flagpole.

11. John walks 150 feet away from a fence post on his farm. He measures the angle of elevation from the ground to the top of the post to be 3°. How tall is the fence post?

12. Find the area of a regular pentagon inscribed in a circle with diameter 14 feet.

13. Find the area of a triangular garden with sides 10, 14 and 16.