Lesson #2 Arithmetic Series NOTES & PRACTICE

Key

Warm Up/Review – DO NOW!

1. $\frac{3!8!}{9!} = \frac{3 \cdot 2}{9} = \frac{6}{9} = \frac{2}{3}$

2. Write a formula to find the nth term of the sequence -7, -2, 3, 8 . . .

Formula: $d = 5$, $a_1 = -7$

$a_n = a_1 + d(n-1)$

$a_n = -7 + 5(n-1)$

$a_n = 5n + \frac{-12}{2}$

3. Write the formula for the nth term of the arithmetic sequence with a third term of 21 and a 21st term of - 57. Leave in fractional form.

(3, 21) (21, -57)

Arithmetic → discrete linear. d = m.

① Find m → $d = \frac{-57-21}{21-3} = -\frac{78}{18} = -\frac{39}{9} = \frac{-13}{3} = d$

② pick one "point." Sub in to find a_1.

$a_n = a_1 + d(n-1)$

$21 = a_1 + \frac{-13}{3}(3-1)$

$21 = a_1 + \frac{-26}{3}$

$\frac{63}{3} + \frac{26}{3} = a_1$

$\frac{89}{3} = a_1$

4. Calculate the sum $\sum_{i=3}^{6} -2i + 5$

3 → -6 + 5 = -1

4 → -8 + 5 = -3

5 = -5

6 = -7

sum = -16

5. What is the common ratio for the nth term of a geometric sequence given below?

4, -8, 16, -32?

$\frac{-8}{4}$ $r = -2$

② write rule

$a_n = \frac{89}{3} + \frac{-13}{3}(n-1)$

Example 2: Find the indicated partial sum for each arithmetic series.

RULE: $S_n = \frac{n(a_1 + a_n)}{2}$

a) $a_1 = 5, a_{100} = 1200, S_{100} =$ 60,250

$S_{100} = \frac{100(5+1200)}{2}$

$S_{100} = 60,250$

b) $a_1 = 12, a_{20} = -83, S_{20} =$ -710

c) $5 + 10 + 15 + 20 + \ldots S_{15} =$ 600

d) $97 + 91 + 85 + 79 + \ldots S_{35} =$ -175

We know: $a_1 = 5$

$d = 5$

We need to find: $a_{15} = 5 + 5(15-1)$

$a_{15} = 5 + 70$

Summation: $a_{15} = 75$

$S_{15} = \frac{15(5+75)}{2} = 600$

e) $8 + 17 + 26 + \ldots + 71 =$ 316

f) $(-24) + (-21) + (-18) + \ldots + 300 =$ 15,042

We know: $a_1 = 8$

$d = 9$

We need to find n $a_n = 71$

Sequence Rule: $a_n = a_1 + d(n-1)$

$71 = 8 + 9(n-1)$

N = _____

$71 = 8 + 9n - 9$

Summation Rule: $71 = 9n - 1$

$72 = 9n$

$n = 8$

Solution:

$S_8 = \frac{8(8+71)}{2}$

$S_8 = 316$

Example 4: Write each arithmetic series in expanded form, and then find its sum.

a) $\sum_{n=1}^{4}(2n+1) =$ 24

$n=1 \rightarrow 3$
$n=2 \rightarrow 5$
$\rightarrow 7$
$\rightarrow 9$

$3+5+7+9$

b) $\sum_{k=1}^{5}(-3k) =$ -45

$-3 + -6 + -9 + -12 + -15$

c) $\sum_{k=3}^{7}(k+4) =$ 45

$K=3 \rightarrow 7$
$K=4 \rightarrow 8$
$5 \quad 9$
$6 \quad 10$
$7 \quad 11$

$7+8+9+10+11$

d) $\sum_{j=2}^{6}(j+7) =$ 55

9 10 11 12 13

Now, use the formula for an Arithmetic Series to find the following.

Recall, the number of terms = upper limit - lower limit + 1.

e) $\sum_{k=7}^{30} -2k+3 =$ -816

lower limit output: $-2(7)+3 = -11 = a_1$

upper limit output: $-2(30)+3 = -57 = a_{24}$

terms $30-7+1 = 24 = n$

Formula: $S_{24} = \frac{24(-11+-57)}{2}$

Solution: $= \frac{24(-68)}{2} = 12(-34) = -408$

f) $= \sum_{n=6}^{85}\left(\frac{1}{2}n-1\right)$ 1700

$a_1 = 2$
$a_{80} = 83/2$
#terms $= n = 80$

$S_{80} = \frac{80\left(\frac{4}{2}+\frac{83}{2}\right)}{2}$

$= 40\left(\frac{87}{2}\right)$

$= 20(87)$

$= 1740$

-816

Example 5: Find the indicated term for each arithmetic sequence

a) $a_1 = -4,\ S_{15} = 885,\ a_{15}$ 122

$n = 15$

Since we know S, we must use the Series Formula!

In order to find a_{15}:

$S_n = \frac{n(a_1+a_n)}{2}$

$885 = \frac{15(-4+a_{15})}{2}$

$885 = \frac{-60+15a_{15}}{2}$

$1770 = -60 + 15a_{15}$

$1830 = 15a_{15}$ $a_{15} = 122$

b) $a_1 = -12,\ S_{60} = -7800,\ a_{60}$

$a_{60} = -248$