4.1 Notes Day 4 Angular and Linear Velocity

Spinning Activity:

The angle that all members of the whip (radius) spun through was the same.

Therefore, angular velocity does not depend on the radius.

Angular Velocity = $1 \text{ rev} = 2\pi \text{ rads}$

Measured in Radians per unit of time.

Ex. The carousel at Disney makes 10 revolutions per minute. What is the angular speed of the carousel in radians per second?

$$\frac{10 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rads}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} = \frac{20\pi}{60} \frac{\text{rads}}{\text{sec}}$$

START = $= \frac{\pi}{3}$ rads/sec END

or 1.047 rads/sec.

The speed at which all members of the whip (radius) traveled while spinning depended on their location → distance from center.

The person on the outside of the whip spun faster than

The person on the origin.

Therefore, linear velocity depends on the length of the radius.

Linear Velocity =

Measured in unit of length per unit of time.

Examples: Ft/sec mph (miles per hour) meters per second.

Definition of the Day: 1 RADIAN = LENGTH OF RADIUS

Ex. You are on a horse 10 feet from the center. How fast are you traveling in mph?

$$\frac{1.047 \text{ rad}}{1 \text{ sec}} \cdot \frac{10 \text{ ft}}{1 \text{ rad}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

GOAL ✓ $\boxed{\frac{\text{miles}}{\text{hr}}}$

$$\cdot \frac{1 \text{ mile}}{5280 \text{ ft}} = \underline{7.14} \text{ mph}$$

WHEELS **1 REVOLUTION =** ____________________

1. Calculate the angular velocity in radians per minute of a ferris wheel with a diameter of 180 feet that rotates once every 25 seconds. .

goal

$$\frac{1 \text{ rev}}{25 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 15.08 \frac{\text{rad}}{\text{min}}$$

2. If you sat on the rim of the ferris wheel described above, what would your linear velocity be in feet per minutes? Miles per hour?

goal

$$\frac{15.08 \cancel{\text{rad}}}{1 \text{ min}} \cdot \frac{90 \text{ ft}}{1 \cancel{\text{rad}}} = 1357.2 \boxed{\frac{\text{ft}}{\text{min}}}$$

$$\frac{1357.2 \cancel{\text{ft}}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \cdot \frac{1 \text{ mile}}{5280 \cancel{\text{ft}}} = 15.42 \frac{\text{miles}}{\text{hr}}$$

STRATEGY:

(1) write ratio

(2) write what you want to get.

(3) Time 1st perhaps?

(4) 1 radius = 1 radian.

CONVERSIONS:

1 revolution = 1 rotation = 2pi radians = 360 degrees

1 minute = 60 seconds

1 mile = 5280 feet

1 radian = 1 radius

1 yard = 3 feet

1 foots = 12 inches

Name: Key Date: ___________

1. The second hand on a clock is 6 inches long. How far does the tip of the hand move in:

$2\pi \text{ rad} = 1\text{rev}$

a. 15 seconds?

$$\frac{15\text{sec}}{1}\cdot\frac{1\text{rev}}{60\text{sec}}\cdot\frac{2\pi\text{rad}}{1\text{rev}}\cdot\frac{6\text{inch}}{1\text{rad}} = 3\pi\text{radians} \approx 9.42\text{inches}$$

b. 2 minutes 15 seconds?

$$\frac{135\text{sec}}{1}\cdot\frac{1\text{rev}}{60\text{sec}}\cdot\frac{2\pi\text{rad}}{1\text{rev}}\cdot\frac{6\text{inch}}{1\text{rad}} = \frac{1620\pi}{60}\text{inch} = 27\pi\text{inch} \approx 84.82\text{inches}$$

2. Find the central angle measure (in radians) of an arc of length 5 cm on a circle with a radius of 3 cm.

$S = \theta r$ $\qquad \frac{5}{3}$ radians

$5 = \theta 3$

3. A merry-go-round makes 8 revolutions per minute

a. What is the exact angular speed of the merry-go-round in radians per minute?

$$\frac{8\text{rev}}{\text{min}}\cdot\frac{2\pi\text{rad}}{1\text{rev}} = 16\pi\text{ radians/min} \approx 50.27\text{rad/min}$$

b. How fast is a horse $\underbrace{\text{12 feet}}_{\text{1radian}}$ from the center traveling? (in mph)

$$\frac{16\pi\text{rad}}{\text{min}}\cdot\frac{12\text{ft}}{1\text{rad}}\cdot\frac{1\text{mi}}{5280\text{ft}}\cdot\frac{60\text{min}}{1\text{hour}} = 6.85\text{ mph}$$

c. How fast is a horse 4 feet from the center traveling? mph

$$\frac{16\pi\text{rad}}{\text{min}}\cdot\frac{4\text{ft}}{1\text{rad}}\cdot\frac{1\text{mi}}{5280\text{ft}}\cdot\frac{60\text{min}}{\text{hour}} = \frac{3840\pi}{5280} \approx 2.28\text{mph}$$

4. A riding lawn mower has wheels that are 15 inches in diameter, which are turning at 2.5 revolutions per second. $1\text{rad} = 7.5\text{inch}$

a. What is the angular speed of the wheel?

$$\frac{2.5\text{rev}}{\text{sec}}\cdot\frac{2\pi\text{rad}}{\text{rev}} = 5\pi\text{ rad/sec} \approx 15.71\text{ rad/sec}$$

b. How fast is the lawn mower traveling in miles per hour?

$$\frac{5\pi\text{rad}}{\text{sec}}\cdot\frac{60\text{sec}}{\text{min}}\cdot\frac{60\text{ min}}{\text{hour}}\cdot\frac{7.5\text{inch}}{1\text{rad}}\cdot\frac{1\text{ft}}{12\text{inch}}\cdot\frac{1\text{mi}}{5280\text{ft}} = \frac{135000\pi}{63360} = 6.69\text{mph}$$

5. A bicycle has wheels that are 26 inches in diameter. If the bike is traveling at 14 miles per hour, what is the angular speed of each wheel? $r = 13\text{inch}$

$$\frac{14\text{mi}}{\text{hour}} \rightarrow \frac{\text{rad}}{\text{sec}}$$

$$\frac{14\text{mi}}{\text{hour}}\cdot\frac{5280\text{ft}}{1\text{mi}}\cdot\frac{12\text{inch}}{1\text{ft}}\cdot\frac{1\text{rad}}{13\text{inch}}\cdot\frac{1\text{hour}}{3600\text{sec}} = 18.95\text{ rad/sec}$$

15. The minute hand of a clock moves from 12 to 2 o'clock, or $\frac{1}{6}$ of a complete revolution. (A) Through how many degrees does it move? (B) Through how many radians does it move?

$\frac{1}{6} \cdot 360 = 60^\circ \quad = \frac{\pi}{3}$

16. The minute hand of a clock is 6 inches long and moves from 12 to 4 o'clock. How far does the tip of the minute hand move? Express your answer in terms of π. [Hint: Find the arc length.]

$4 \cdot \frac{\pi}{6} = \frac{2\pi}{3}$

$\frac{1}{3}$ rev

$S = r\theta$

$S = 6 \cdot \frac{2\pi}{3} = 4\pi$ inches

17. Find the angular velocity in radians per second of the second hand of a clock. Express the answer in terms of π. Show work.

$\frac{1 \cancel{\text{rev}}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rads}}{1 \cancel{\text{rev}}} = \frac{\pi}{30} \frac{\text{rads}}{\text{sec}} \qquad \frac{\text{rads}}{\text{sec}}$ goal

18. Calculate the angular velocity in radians per minute of a Ferris wheel with a diameter of 208 feet that takes 25 seconds to rotate once. Express the answer in terms of π. Show work.

$r = 104$ ft

$\frac{1 \cancel{\text{rev}}}{25 \cancel{\text{sec}}} \cdot \frac{60 \cancel{\text{sec}}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}} \cdot \cancel{\frac{104 \text{ ft}}{1 \text{ rad}}} = \frac{120\pi}{25} \qquad \frac{\text{rad}}{\text{min}}$ goal

$= \frac{24\pi}{5}$ rad/min

19. If you sat on the rim of the Ferris wheel in #18, what would your linear velocity be, to the nearest foot per minute. Show work.

$\frac{\frac{24\pi}{5} \cancel{\text{rad}}}{1 \text{ min}} \cdot \frac{104 \text{ ft}}{1 \cancel{\text{rad}}} \approx$ ~~[illegible]~~ feet per min

1568.28

20. A flywheel mounted on an engine crankshaft has a radius of 6 inches. If the engine is running at 2800 rpm, what is the linear velocity of a point on the outer edge of the flywheel in feet per second? Show work.

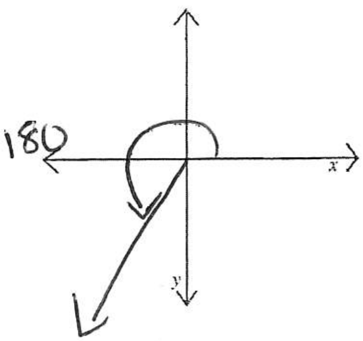
$\frac{2800 \cancel{\text{rev}}}{1 \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \cdot \frac{2\pi \cancel{\text{rad}}}{1 \cancel{\text{rev}}} \cdot \frac{6 \cancel{\text{in}}}{1 \cancel{\text{rad}}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in}}}$

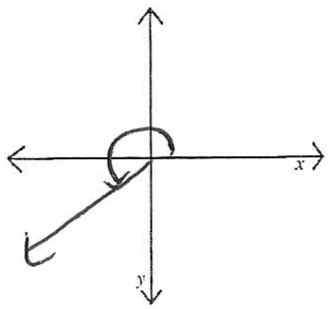
$= 146.6$ ft/sec

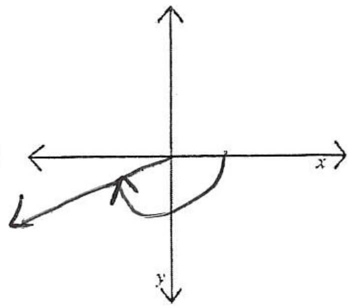
Unit Circle Day 3 WS   I am Reviewing & Labeling My Unit Circle, ____________________

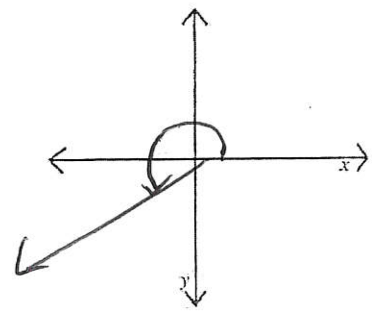
#1-4, Draw an angle with the given measure in ***standard position.***

1. 230°   2. $\frac{7\pi}{6}$ 3. $-\frac{11\pi}{12}$ 4. 4 radians









#5-9, Find a positive and negative ***coterminal angle*** for each given angle. Draw a picture if necessary.

± 360 or $\pm 2\pi$

5. 95°

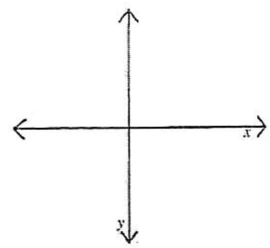
$95 + 360 = 455$

$95 - 360 = -265$

6. -210°

$-210 + 360 = 150^\circ$

$-210 - 360 = -570^\circ$



7. $\frac{5\pi}{3} \pm \frac{6\pi}{3}$

$\frac{11\pi}{3}, \frac{-\pi}{3}$

8. $-\frac{\pi}{4} \pm \frac{8\pi}{4}$

$\frac{7\pi}{4}, -\frac{9\pi}{4}$

9. $\frac{7\pi}{6} \pm \frac{12\pi}{6}$

$\frac{19\pi}{6}, -\frac{5\pi}{6}$

#10-13, Convert each degree measure into radians and each radian measure into degrees.

10. 225° $\cdot \frac{\pi}{180}$

$\frac{5\pi}{4}$

11. $\frac{7\cancel{\pi}}{6} \cdot \frac{180}{\cancel{\pi}}$

210°

12. 110° $\cdot \frac{\pi}{180}$

$\frac{11\pi}{18}$

13. $\frac{2\cancel{\pi}}{3} \cdot \frac{180}{\cancel{\pi}}$

120°

#14-18, Find the measure of the ***reference angle*** for each given angle, in degrees and radians.

14. $\theta = \frac{7\pi}{6}$

$\pi/6$

15. $\theta = -60°$

$\pi/3, 30^\circ$

16. $\theta = 230°$

150°

17. $\theta = \frac{2\pi}{3}$

$\frac{\pi}{3}$

18. $\theta = \frac{15\pi}{8}$

$\frac{\pi}{8}$

19. How many times with the length of a circle's radius fit around its circumference? ____________

≈ 6.28

20. An angle which measures 1 radian is equal to about __________ degrees.

1 radian $< 90^\circ$

1 rad. $\frac{180^\circ}{\pi} \approx 57.3^\circ$

Unit Circle WS Day 2 Degrees v. Radians Conversions Initials: ____________

#1-8, Convert each degree measure into radians and each radian measure into degrees.

1. $45°$

2. $\frac{\pi}{6}$

3. $330°$

4. $\frac{3\pi}{4}$

5. $\frac{2\pi}{3}$

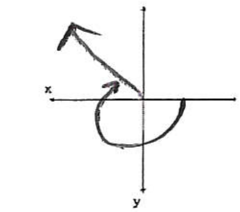
6. $270°$

7. $\frac{10\pi}{3}$

8. $225°$

#9-14, Draw each angle in *standard position*.

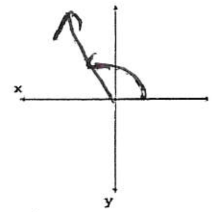
9. $-200°$



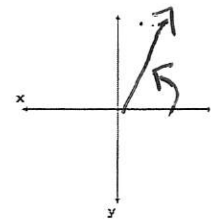
ref ∠ $= 20°$

10. $\frac{7\pi}{12}$

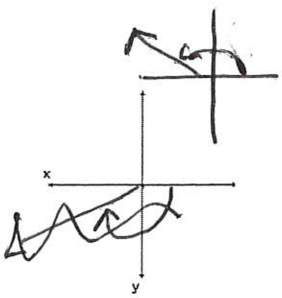
$6\pi/12$



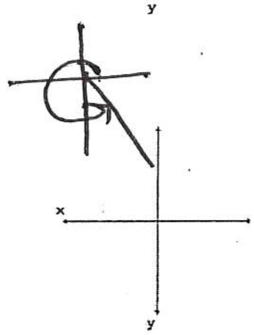
11. 1 rad.



12. 3 rad.



13. 5 rad.



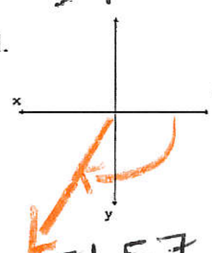
14. -2 rad.

-4.71

-6.28

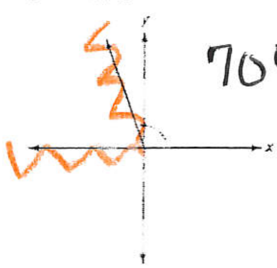
-3.14

-1.57



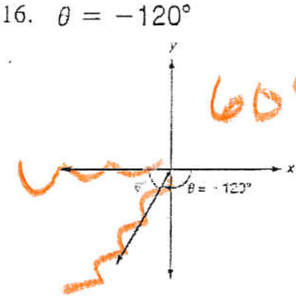
#15-22, Find the *reference angle*. Use the appropriate unit

15. $\theta = 110°$



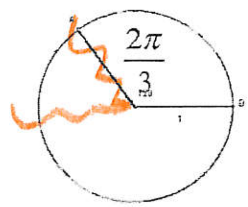
$70°$

16. $\theta = -120°$



$60°$

17.



$\pi/3$

18. $\theta = \frac{7\pi}{6}$ $\pi/6$

19. $\theta = 400°$ $40°$

20. $\theta = 315°$ $45°$

21. $\theta = \frac{11\pi}{6}$ $\pi/6$

22. $\theta = \frac{5\pi}{4}$ $\pi/4$

#23-25, Find a *positive* and *negative* coterminal angle for each given angle.

23. $135°$ ± 360

$495°$ $-225°$

24. $\frac{-3\pi}{4}$ $\pm \frac{8\pi}{4}$

$\frac{5\pi}{4}$ $\frac{-11\pi}{4}$

25. $50°$

$410°$ $-310°$