Precalculus Name Key

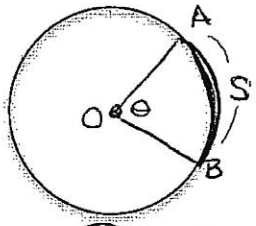
4.1 Notes: Radian and Degree Measure- *Day 3* *(Arc Length & DMS)*

Arc length: this is essentially a review from Geometry!

In Geometry, you learned that arc length was just a fraction of the circle's circumference. You found the length of an arc from the measure of the central angle that intercepts the arc in a circle with a specified radius. The angle measure you used was in degrees. Let's take that formula from Geometry and rearrange some factors to discover a formula for arc length using the angle measure in radians!

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{degree measure of arc}}{360^\circ}$$

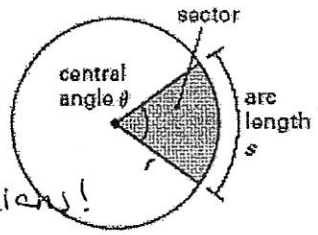
$$\frac{S}{2\pi r} = \frac{\theta}{360}$$



$m\overset{\frown}{AB} = \theta$

ARC LENGTH

The arc length s with radius r and central angle θ (measured in radians) is as follows.



2π rad $= 360^\circ$

$$\frac{S}{2\pi r} = \frac{\theta}{2\pi}$$

$$S = \frac{\theta}{2\pi} \cdot 2\pi r$$

Formula:

$$S = r\theta$$

* θ must be in radians!

Examples:

1. If r = 7 in. and the central angle of a circle is 215°, then find the length of the arc that subtends the circle.

$\theta = 215^\circ$

$\theta = 215^\circ \cdot \frac{\pi}{180^\circ}$ → $\theta = \frac{43\pi}{36}$

$S = r\theta$

$S = 7\left(\frac{43\pi}{36}\right)$

$S = \frac{301\pi}{36} \approx 26.247$ in

2. Suppose the central angle $\theta = \frac{5\pi}{6}$ and the radius of the circle is 5 ft. Find the length of the arc that meets this angle.

$S = r\theta$

$S = 5\left(\frac{5\pi}{6}\right)$

$S = \frac{25\pi}{6}$

$S = \frac{25\pi}{6} \approx 13.090$ ft

3. Find the arc length with given radius and angle: r = 10 m. $\theta = \frac{2\pi}{7}$

$S = r\theta$

$S = 10 \cdot \frac{2\pi}{7}$

$S = \frac{20\pi}{7} \approx 8.976$ m

4. Find the arc length with given radius and angle: $r = 5$ cm. $\theta = 75^\circ$

$\theta = 75^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{12}$

$S = r\theta$

$S = 5\left(\frac{5\pi}{12}\right) = \frac{25\pi}{12} \approx 6.545$ cm

5. Find the radius of the circle with given arc length and angle: $s = 3.5$ in. $\theta = 60°$

$\theta = 60° \cdot \frac{\pi}{180°} = \frac{\pi}{3}$ $\quad S = r\theta$

$3.5 = r \cdot \frac{\pi}{3}$

$\frac{3.5}{(\pi/3)} = r$

$r \approx 3.342$ inches

6. Find the central angle with given radius and arc length: $r = 4$ ft. $s = 6.25$ ft.

$S = r\theta$

$6.25 = 4\theta$

$6.25/4 = \theta$

$\theta = 1.5625$ radians

7. A pizza is cut into six pieces. Each piece represents an equivalent sector of the pizza. Find the measure of the angle formed by four pieces of pizza. Give the answer in both degrees and radians.

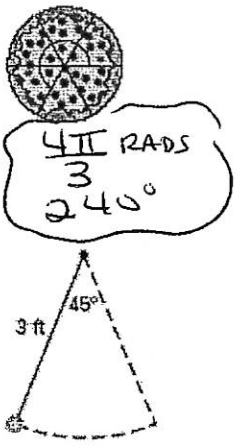
$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$ radians (each piece)

$\theta = \frac{360}{6} = 60°$

4 pieces

$4(\pi/3) = \frac{4\pi}{3}$ RADS

$4(60°) = 240°$



8. The pendulum of a grand father clock is 3 feet long and swings back and forth creating a 45° angle. Find the length of the arc created by the pendulum after one swing.

$\theta = 45 \cdot \frac{\pi}{180°} = \frac{\pi}{4}$

$S = r\theta$

$S = 3\left(\frac{\pi}{4}\right)$

$S = \frac{3\pi}{4}$ RADS

≈ 2.356 ft

Conversions with Degrees, Minutes & Seconds

$1° = 60'$ (minutes)
$1' = 60''$ (seconds)

Symbol for degree: °
Symbol for minute: '
Symbol for second: "

There is another way to state the size of an angle, one that subdivides a degree using a system different than the decimal number. The words minute and second used in this context have no immediate connection to how those words are usually used as amounts of time. 2nd Apps → Angle Menu to find seconds " → Alpha +

- *In a full circle there are 360 degrees.*
- *Each degree is split up into 60 parts, each part being 1/60 of a degree. These parts are called minutes.*
- *Each minute is split up into 60 parts, each part being 1/60 of a minute. These parts are called seconds.*

Convert from D°M'S" (Degrees/Minutes/Seconds) to decimal form:

9. 29°45'27"

$29 + \frac{45}{60} + \frac{27}{3600}$

$29.7575°$

10. −408°16'25"

$-\left(408 + \frac{16}{60} + \frac{25}{3600}\right)$

$-408.2736°$

Convert the angle measure to D°M'S" form.

11. 280.6°

$\cdot \frac{6}{1/60} = 36$ min

$280° 36' 0"$

12. −345.12°

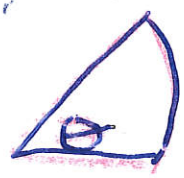
$\cdot 12(60) = 7.2$ min

$\cdot 2(60) = 12$ sec

$-345° 7' 12"$

4.1 Sector Area Name:_____ Key

Area of a Sector

	 The # of square units in one slice of pie.

Area of Sector

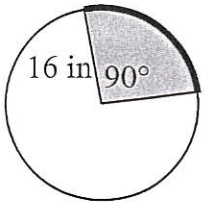
$A = \frac{\theta}{360°}\pi r^2$ where θ is in degrees

$A = \frac{1}{2}r^2\theta$ where θ is in radians

Area of Segment = Area of Sector – Area of Triangle

1. Calculate the sector area:

a. Using degrees



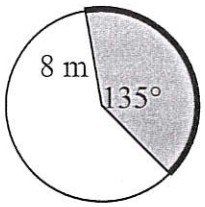
$A = \frac{90}{360}\pi(16)^2$

$A = \frac{1}{4}\pi(16)(16)$

$A = 64\pi \text{ in}^2$

$\approx$

b. Using radians



$135° \cdot \frac{\pi \text{ Rad}}{180°}$

$= \frac{135\pi}{180}$

$= \frac{3\pi}{4}$

$A = \frac{1}{2}(8)(8)\frac{3\pi}{4}$

$= (4)(2)(3\pi)$

$= 24\pi \text{ m}^2$

$\approx$

4.1 Sector Area Name:_____

Degrees

2. Find the measure of the central angle of a sector if its area is 5π and the radius is 6.

$A = \frac{\theta}{360} \cdot \pi r^2$

$5\pi = \frac{\theta}{360} \cdot 36\pi$

$5\pi = \frac{\theta\pi}{10}$

$50\pi = \theta\pi$

$50^\circ = \theta$

$50^\circ \cdot \frac{\pi}{180^\circ}\ \text{rad}$

$= \frac{5\pi}{18}\ \text{rad}$

3. The central angle of a sector is 72° and the sector has an area of 5π. Find the radius.

$A = \frac{72^\circ}{360^\circ} \cdot \pi r^2$

$5\pi = \frac{1}{5}\pi r^2$

$25\pi = \pi r^2$

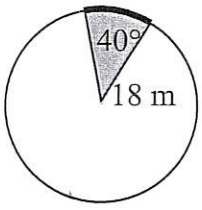
$25 = r^2$

$r = \pm\sqrt{25}$

$r = 5$

PRACTICE:

1. Find the area of the sector



$r = 18$

$\theta = 40^\circ$

$A = \frac{40}{360}\pi r^2$

$A = \frac{1}{9}\pi (18)^2$

$A = 324\pi/9$

$= 36\pi$

2. The area of a circle is 225π square inches. Find the area of the sector whose central angle is 45°.

$\pi r^2 = 225\pi$

$\theta = 45^\circ$

$A = \frac{45}{360} \cdot 225\pi$

$A = \frac{1}{8} \cdot 225\pi$

$\frac{225\pi}{8}$

3. A circle has a radius of 12. Find the area of the sector whose central angle is 120°.

OR $A = \frac{1}{3}\pi \cdot 144$

$r = 12$

$A_{sec} = 120^\circ$ or $\frac{2\pi}{3}$

$A = \frac{\frac{2\pi}{3}}{2} \cdot 144$

$= \pi/3 \cdot 144$

$= \frac{144\pi}{3}$

4. Find the radius of a circle which has a sector area of 9π whose central angle is 90°.

$A = 9\pi$

$\theta = 90$ or $\frac{\pi}{4}$

$A = 9\pi = \frac{1}{4}\pi r^2$

$36\pi = \pi r^2$

$36 = r^2$

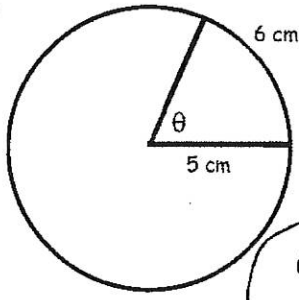
$r = 6$

Precalculus HW Name Key

4.1 Homework-Day 3 *(Arc Length & DMS)*

Find the angle in radians.

1.



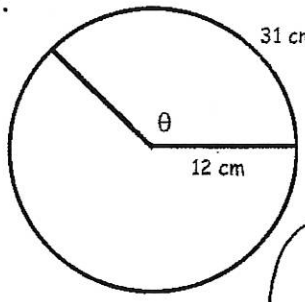
$S = r\theta$

$6 = 5\theta$

$\frac{6}{5} = \theta$

$\theta = \frac{6}{5}$ RADIANS or 1.2 RADS

2.



$S = r\theta$

$31 = 12\theta$

$\frac{31}{12} = \theta$

$\theta = \frac{31}{12}$ RADIANS OR 2.583 RADS

3. radius is 7 meters
arc length is 32 meters

$S = r\theta$

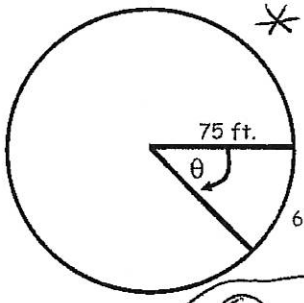
$32 = r\theta$

$32 = 7\theta$

$\frac{32}{7} = \theta$

$\theta = \frac{32}{7}$ RADIANS or 4.571 RADS

4.



* negative θ

$S = r\theta$

$60 = 75\theta$

$\frac{60}{75} = \theta$

$\theta = -\frac{4}{5}$ RADIANS OR −0.8 RADS

Find the length of the arc.

5. radius is 14 inches
central angle θ is 180°

$S = r\theta$

$180° \cdot \frac{\pi}{180°} = \pi$

$S = 14\pi$

$\theta = \pi$

$S = 14\pi \approx 43.982$ inches

6. radius is 12 centimeters
central angle θ is $\frac{3\pi}{4}$

$S = r\theta$

$S = \frac{12}{1}\left(\frac{3\pi}{4}\right)$

$S = 9\pi \approx 28.274$ cm

Find the radius.

7. arc length is 36 feet
central angle θ is $\frac{\pi}{2}$

$S = r\theta$

$36 = r\left(\frac{\pi}{2}\right)$

$\frac{2}{\pi} \cdot 36 = r$

$\frac{72}{\pi} = r$

$r = \frac{72}{\pi} \approx 22.918$ ft

8. arc length is 82 miles
central angle θ is 135°

$135° \cdot \frac{\pi}{180°}$

$\frac{135\pi}{180} = \frac{3\pi}{4}$

$S = r\theta$

$82 = r\left(\frac{3\pi}{4}\right)$

$\frac{4 \cdot 82}{3\pi} = r$

$\frac{328}{3\pi}$

$r = \frac{328}{3\pi} \approx 34.802$ mi

Use your graphing calculator to covert the angle measure to decimal degree form. Round your answer to three decimal places if necessary.

9. 64°45'	10. 85°18'30"	11. −125°36"
64.75°	85.308°	−(125° + 0' + 36") −125.01

Use your graphing calculator to covert the angle measure to *D°M'S''* form.

12. 280.6°	12. −345.12°	14. −0.355 ✱ RADIANS !
280° 36' 0"	−345° 7' 12"	$-.355\left(\frac{180^\circ}{\pi}\right) = (-20.340\,0173)$ Dms → −20° 20' 24.006"