

2016

TYPE 1 : FINANCIAL

Ex 1: Compound Interest $A = P \left(1 + \frac{r}{n} \right)^{nt}$

A = _____ P = _____ r = _____

n = _____ t = _____

- a) A credit union pays 6% interest on its savings account, compounded monthly. If you deposit \$1000, how long must you leave the money in the account before the balance reaches \$1500?

$$1500 = 1000 \left(1 + \frac{.06}{12} \right)^{12t}$$

$$1.5 = \left(1 + \frac{.06}{12} \right)^{12t}$$

$$\frac{\ln 1.5}{12 \ln 1.005} = \frac{12t \ln (1.005)}{\ln 1.005}$$

$$t = 6.775 \text{ yrs}$$

Ex. 2 The formula $A = Pe^{rt}$ is used when an amount of money P is invested at a rate of r and compounded continuously for t years.

A = _____ P = _____ r = _____ t = _____

- a) Suppose you invest \$500 in an account at 6% interest compounded continuously. How long will it take the account to double in value?

$$A_0 = 500$$

$$r = .06$$

$$A = 1000 \text{ (doubled)}$$

$$1000 = 500 e^{.06t}$$

$$2 = e^{.06t}$$

$$\ln 2 = .06t$$

$$\ln 2 \div .06 = t$$

$$t \approx 11.6 \text{ yrs}$$

2. Name: _____ More Exponential Growth and Decay

- 1) Low interest rates, easy credit, and strong demand from new immigrants have driven up the average sales price (the price increases exponentially) of new one-family houses in the United States. In 1995, the average sales price was \$158,700 and by 2000 it had increased to \$207,200. When will the average sales price of a new one-family house reach \$300,000?

$$207200 = 158700 e^{5k}$$

$$k \approx .0533$$

$$300000 = 158700 e^{.0533t}$$

$$t \approx 11.95 \text{ yrs}$$

$$+ \frac{1995}{2007}$$

- 2) A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once a population drops below 100, the situation is irreversible and the species will become extinct. At this rate, when will this happen (will the population reach 100)?

$$1000 = 1400 e^{5k}$$

$$\ln\left(\frac{5}{7}\right) = 5k$$

$$\ln\left(\frac{5}{7}\right) \div 5 = k$$

$$k \approx -.0673$$

$$100 = 1400 e^{-.0673t}$$

$$t \approx 39.21 \text{ yrs}$$

- 3) In ceramics class, you have decided to make a model of an ellipse for your mother on Mother's Day. It is removed from a heating oven with an initial temperature of 450° F and left sitting in a room that has a temperature of 70°. After 5 minutes, it has cooled to 300°.

Newton's Law of Cooling can be used to model the temperature as a function of time. Big I's Law of Cooling is $T = C + (T_0 - C)e^{kt}$, in which T_0 is the initial temperature, C is the constant temperature of the surrounding environment, t is the time in minutes, and T is the final temperature.

- a) What is the temperature of the model after 20 minutes?

- b) When will the temperature of the model be 80°?

① find k

$$300 = 70 + (450 - 70)e^{5k}$$

$$230 = 380e^{5k}$$

$$k \approx -.1004$$

$$T = 121.22^\circ \text{ F}$$

$$80 = 70 + 380e^{-.1004t}$$

$$10 = 380e^{-.1004t}$$

$$\frac{1}{38} = e^{-.1004t}$$

$$\ln\left(\frac{1}{38}\right) = -.1004t$$

$$t \approx 36.23 \text{ min}$$

TYPE 4: Newton's Law of Cooling is used to the temperature of an object as it cools or decreases exponentially over time. The formula is

$$T = C + (T_0 - C)e^{kt}$$

T = temp of heated object at time t

C = constant temp of surrounding medium

T_0 = initial temp of heated object t = time

k = negative constant associated with cooling

always negative

5. A cake removed from an oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

1) Find a model for the temperature of the cake, T, after t minutes.

Sub in what you know, solve for k:

T =

C =

T_0 =

t =

$$T_0 = 210$$

$$C = 70$$

$$t = 30$$

$$T = 140$$

$$140 = 70 + (210 - 70)e^{30k}$$

$$70 = 140e^{30k}$$

$$\frac{1}{2} = e^{30k}$$

$$\ln \frac{1}{2} = 30k$$

$$k = -.0231$$

1B) Create the exponential model for this problem:

Leave T and t as variables. $T = 70 + 140e^{-.0231t}$

2) What is the temperature of the cake after 40 minutes?

$$\text{sub in } t = 40$$

$$125.6^\circ$$

3) When will the temperature of the cake be 90°F?

$$90 = 70 + 140e^{-.0231t}$$

$$\frac{20}{140} = e^{-.0231t}$$

$$\ln \frac{1}{7} = -.0231t$$

$$\ln \left(\frac{1}{7} \right) \div -.0231 = t$$

~~30~~ minutes.
84

t = 84 mins.